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Design of low energy space missions using dynamical systems theory

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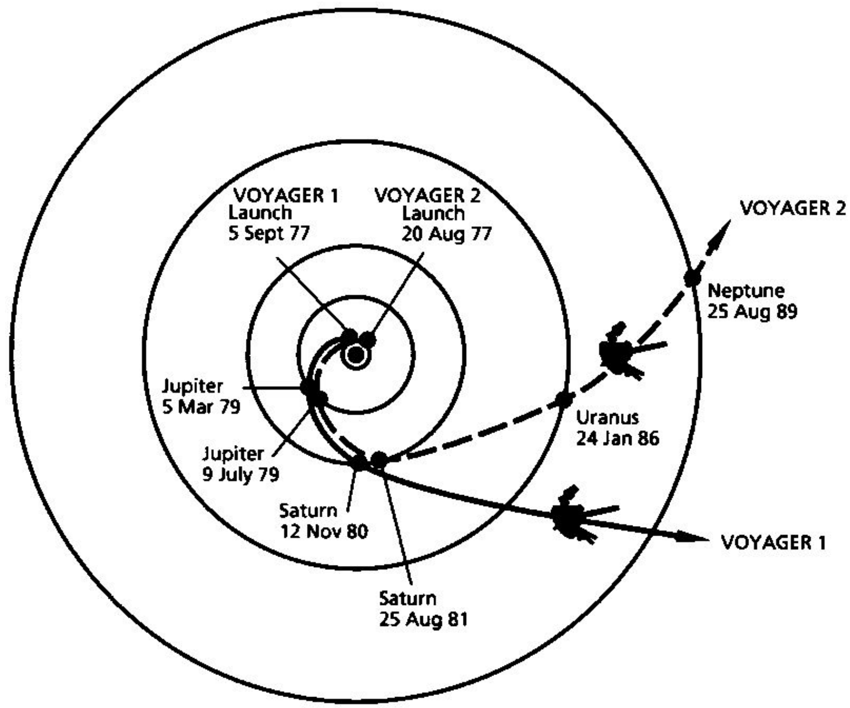
Low Energy Trajectory Design

- *Motivation: future missions*
- *What is the design problem?*
- *Solution space of 3-body problem*
- *Patching two 3-body trajectories: Mission to orbit multiple Jupiter moons*
- *Current and Ongoing Work*

Motivation: Future Missions

- Classical approaches to spacecraft trajectory design have been successful in the past: Hohmann transfers for *Apollo*, swingbys of planets for *Voyager*
- Costly in terms of fuel, e.g., large burns for orbit entry



Swingbys: Voyager Tour

Motivation: Future Missions

- **Low energy trajectories** $\longrightarrow$ large savings in fuel cost (as compared to classical approaches)
- Achieved using natural dynamics arising from the presence of a third body (or more)

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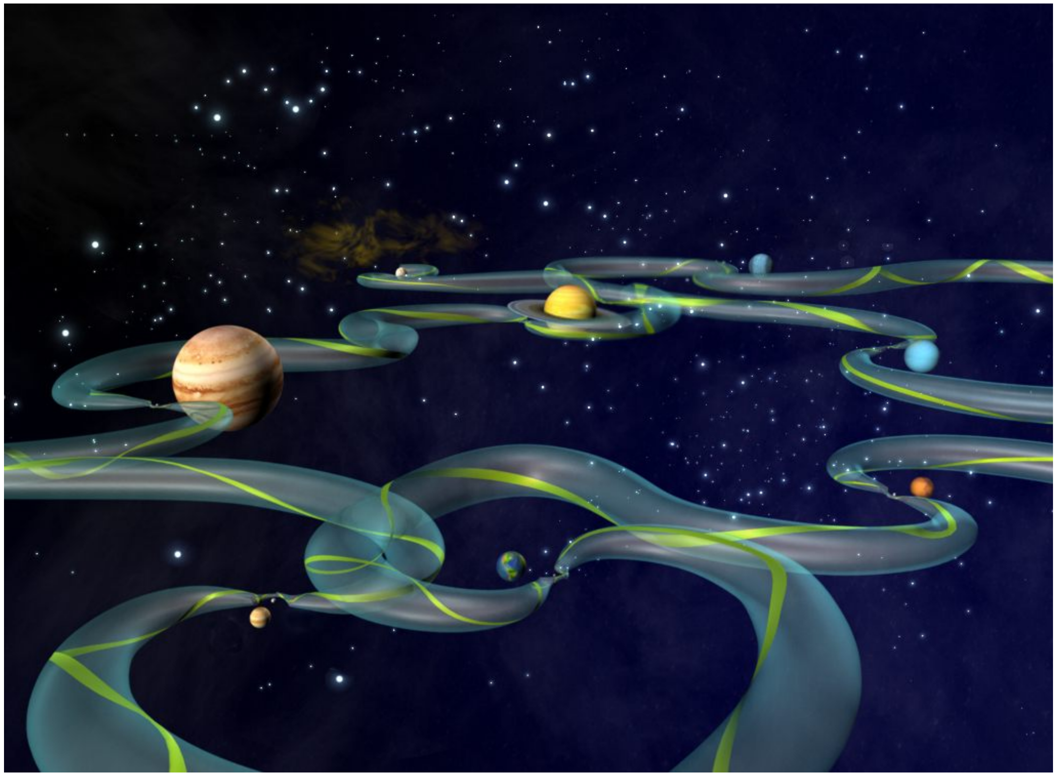
- **Low energy trajectories** $\longrightarrow$ large savings in fuel cost (as compared to classical approaches)
- Achieved using natural dynamics arising from the presence of a third body (or more)
- **New possibilities** $\longrightarrow$ long duration observations and/or constellations of spacecraft using little fuel

Motivation: Future Missions

- **Approach:** Apply **dynamical systems techniques** to space mission trajectory design
- Find **dynamical channels** in phase space



Dynamical channels exist throughout the Solar System

Motivation: Future Missions

Current research importance

- development of some NASA mission trajectories, such as lunar missions and **Jupiter Icy Moon Orbiter**
- Low thrust missions **must** consider multi-body effects

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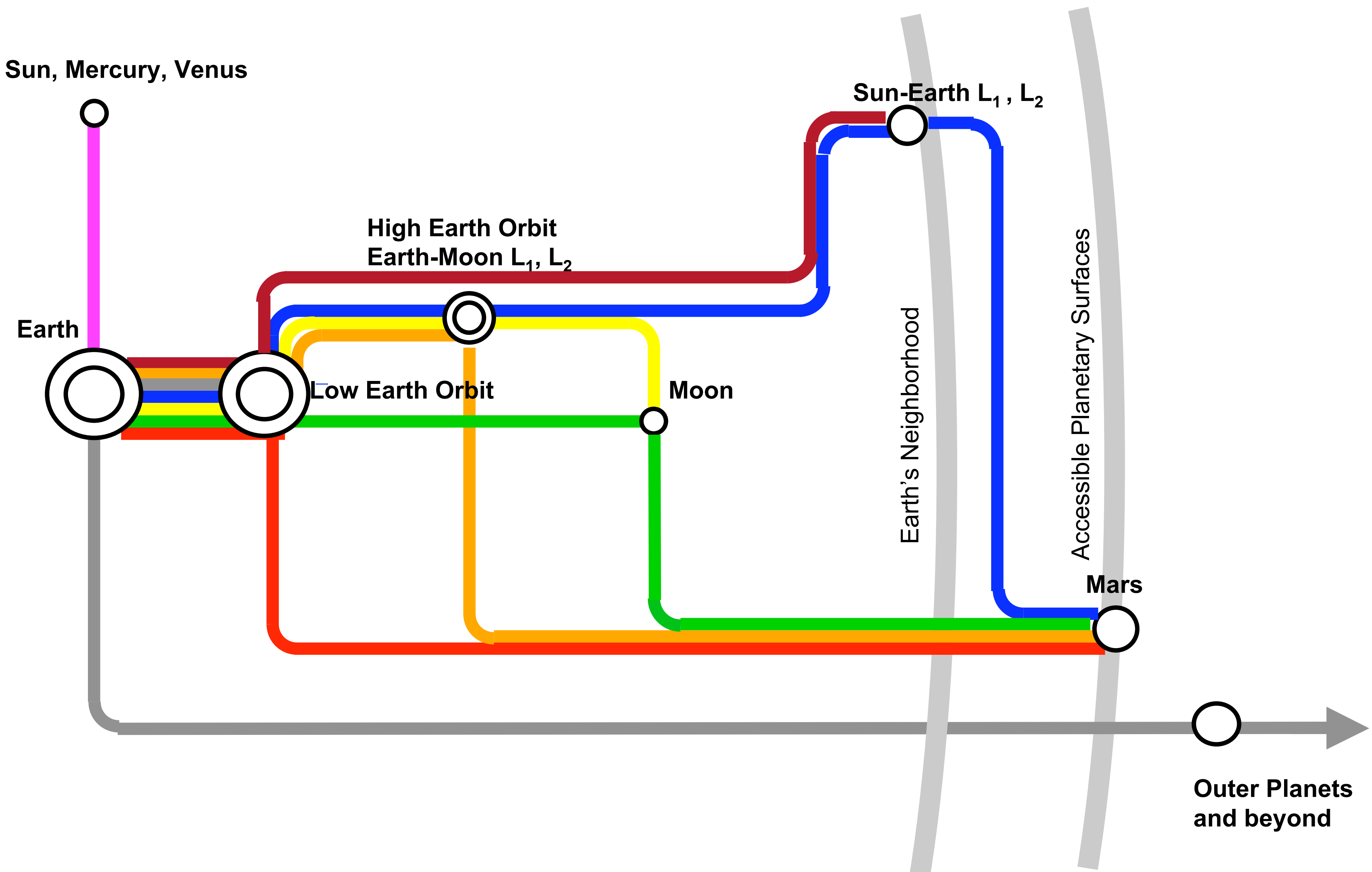
- development of some NASA mission trajectories, such as lunar missions and **Jupiter Icy Moon Orbiter**
- Low thrust missions **must** consider multi-body effects
- **Spin-off:** results also apply to mathematically similar problems in chemistry, astrophysics, and fluid dynamics.
- Let's consider some missions...

Solar System Metro Map

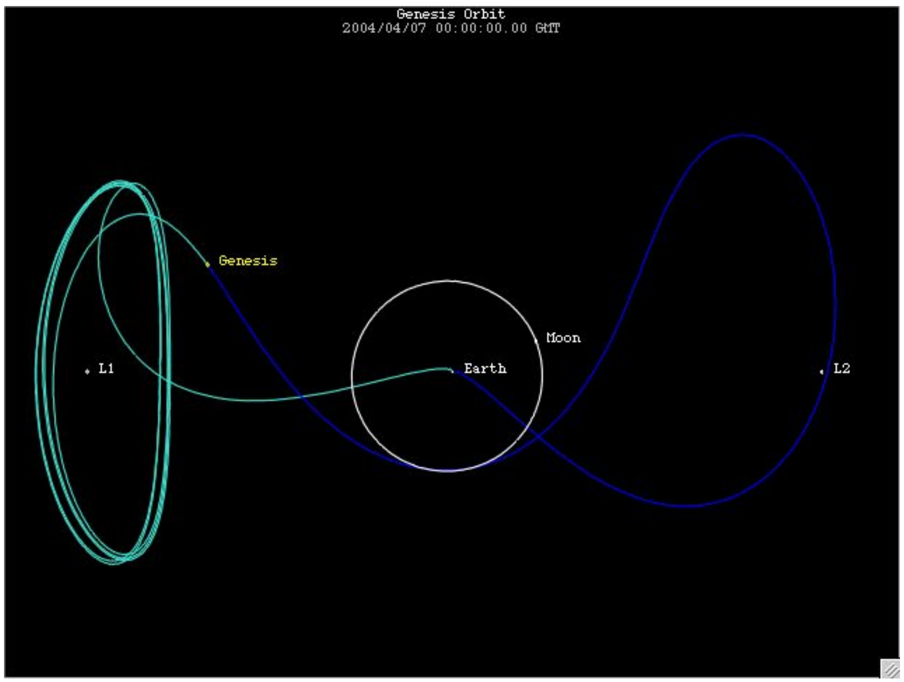


Source: Gary L. Martin, NASA Space Architect

Genesis Discovery Mission

- **Genesis** has collected solar wind samples at the Sun-Earth L1 and will return them to Earth this September.
- First mission designed using dynamical systems theory.

Genesis Spacecraft



Genesis Trajectory

New Mission Architectures

☐ Lunar L1 Gateway Station

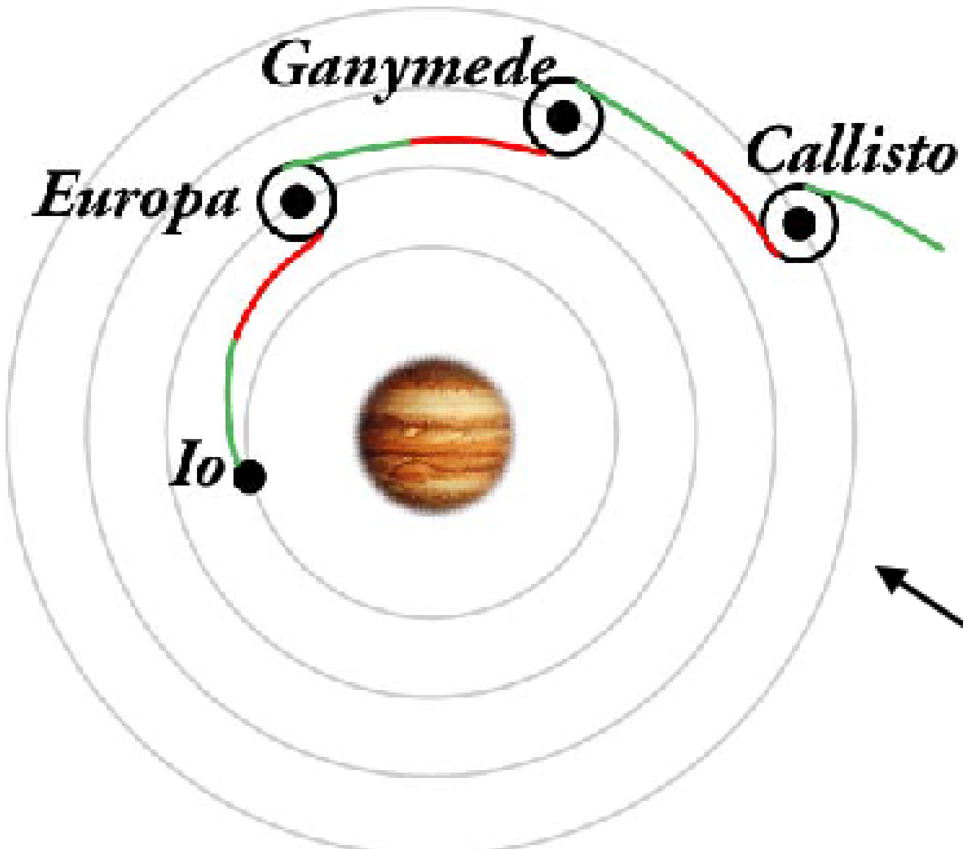
- transportation hub, servicing, commercial uses

Lunar L1 Gateway

Multi-Moon Orbiter

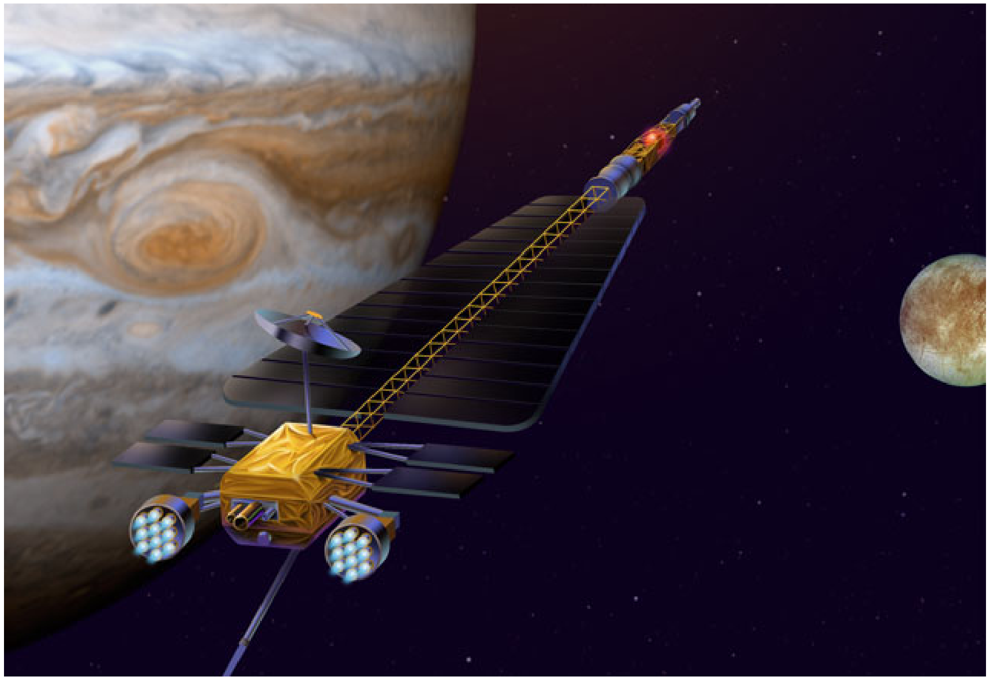
□ Multi-Moon Orbiter

- Jovian, Saturnian, Uranian systems by Ross et al. [1999-2003]
- e.g., orbit Europa, Ganymede, and Callisto in one mission



Jupiter Icy Moons Orbiter

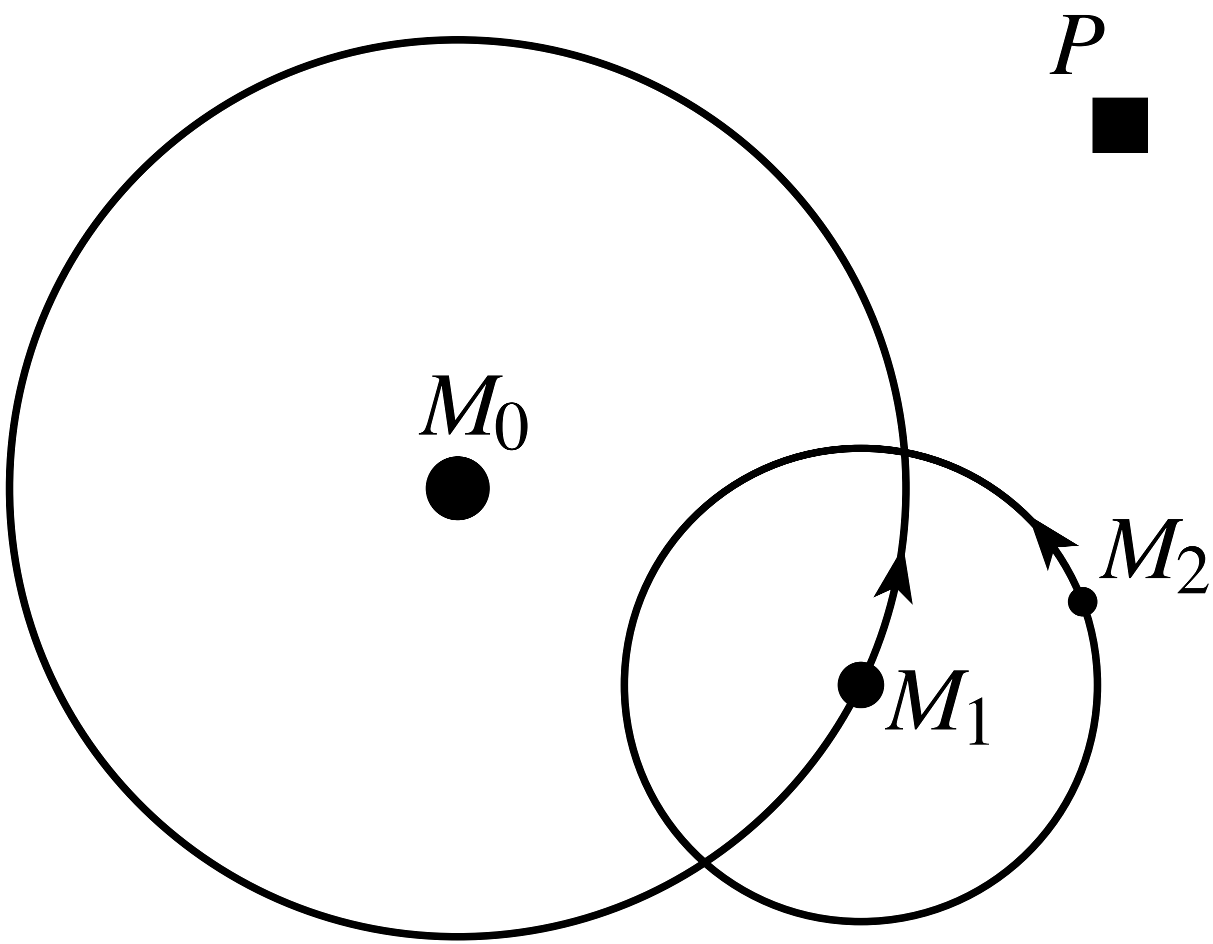
- NASA is considering a **Jupiter Icy Moons Orbiter**, inspired by this work on multi-moon orbiters
  - Earliest launch: 2011



Jupiter Icy Moons Orbiter

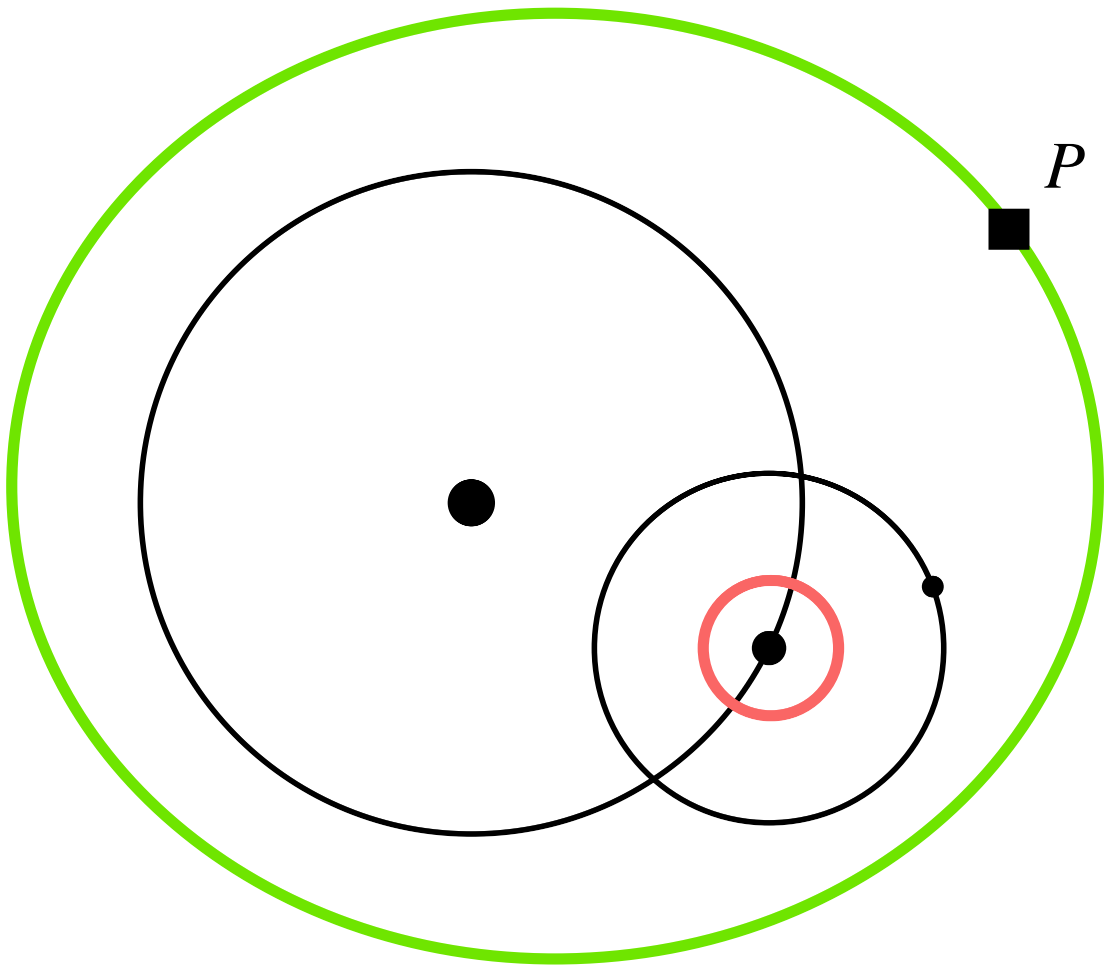
Design Problem Description

- Spacecraft P in gravity field of N massive bodies
- N massive bodies move in prescribed orbits



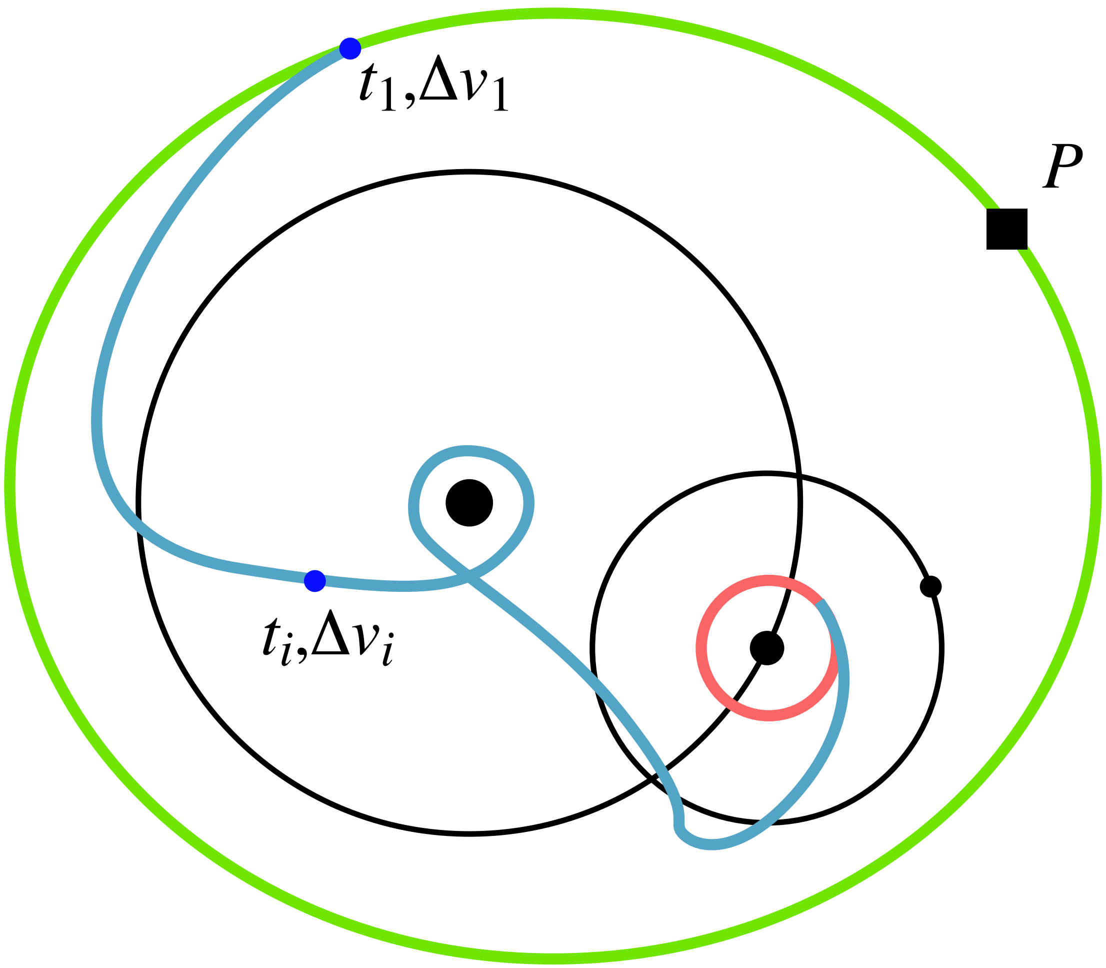
Design Problem Description

- **Goal:** initial orbit $\longrightarrow$ final orbit
- Controls: impulsive or low thrust



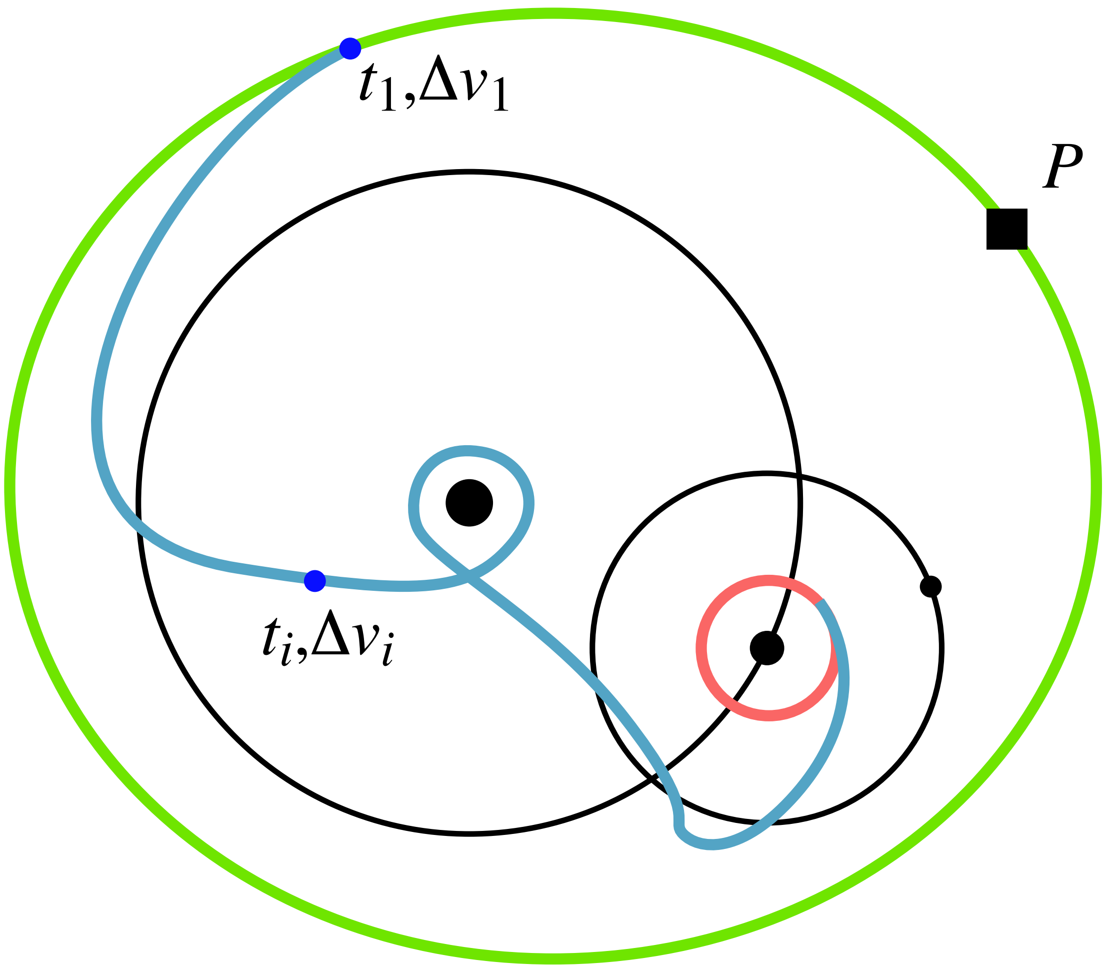
Design Problem Description

- **Impulsive controls:** instantaneous changes in spacecraft velocity, with norm Δv_i at time t_i



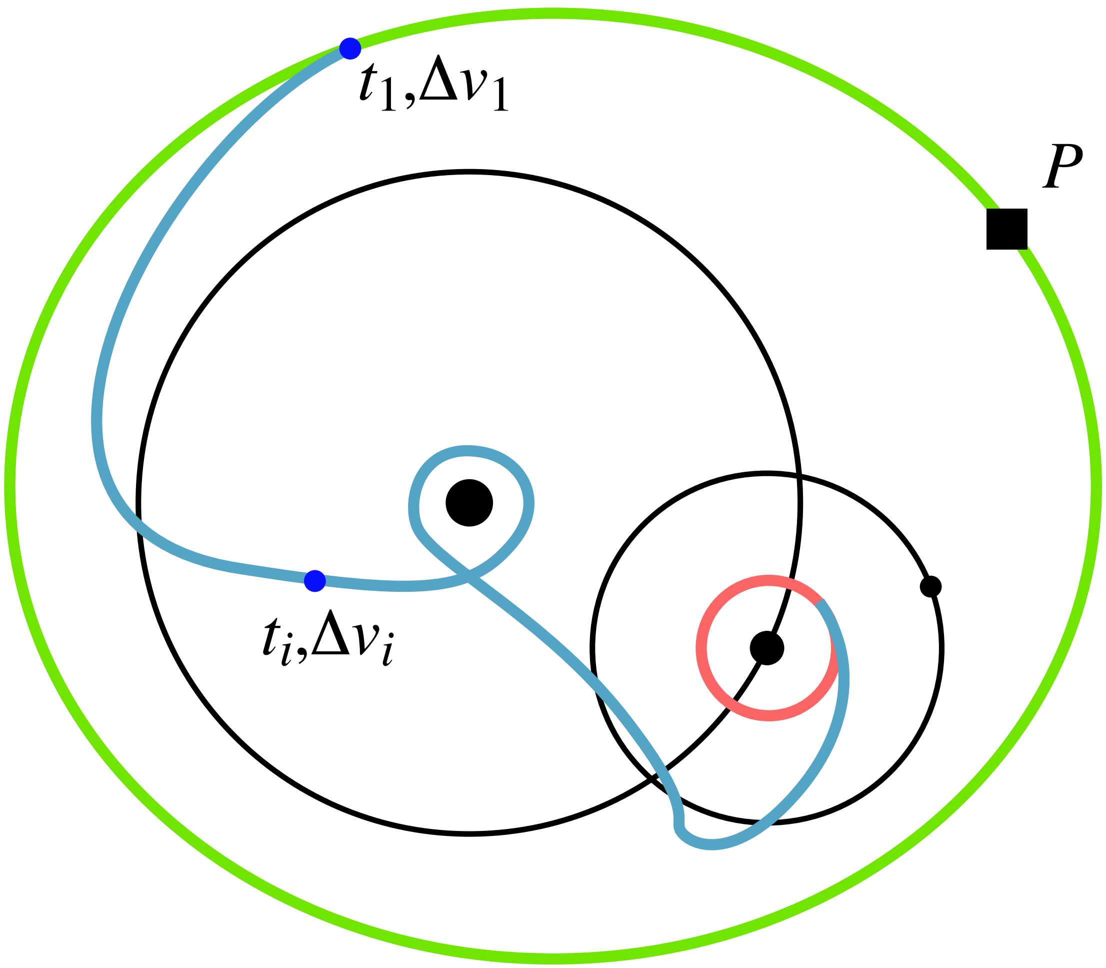
Design Problem Description

- corresponds to high-thrust engine burn maneuvers
- proportional to fuel consumption via rocket equation



Design Problem Description

- **Minimize Fuel/Energy:** find the maneuver times t_i and sizes Δv_i to minimize $\sum_i \Delta v_i$ = total ΔV



Tools Used in Solution

- **Hint:** Use natural dynamics as much as possible i.e., lanes of fast travel

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- **Hierarchy of models**
  – simple model $\longrightarrow$ initial guess for complex model

Tools Used in Solution

- **Patched 3-body approximation**

  $N+1$ body system decomposed into 3-body subsystems:

  spacecraft P + two massive bodies M_i & M_j

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- **3-body problem nonlinear dynamics**
  - phase space $\longrightarrow$ tubes, resonance structures, ballistic capture
  - patched solutions $\longrightarrow$ **first guess solution** in realistic model
  - Numerical continuation yields fast convergence to real sol'n

Tools Used in Solution

- **Patched 3-body approximation**

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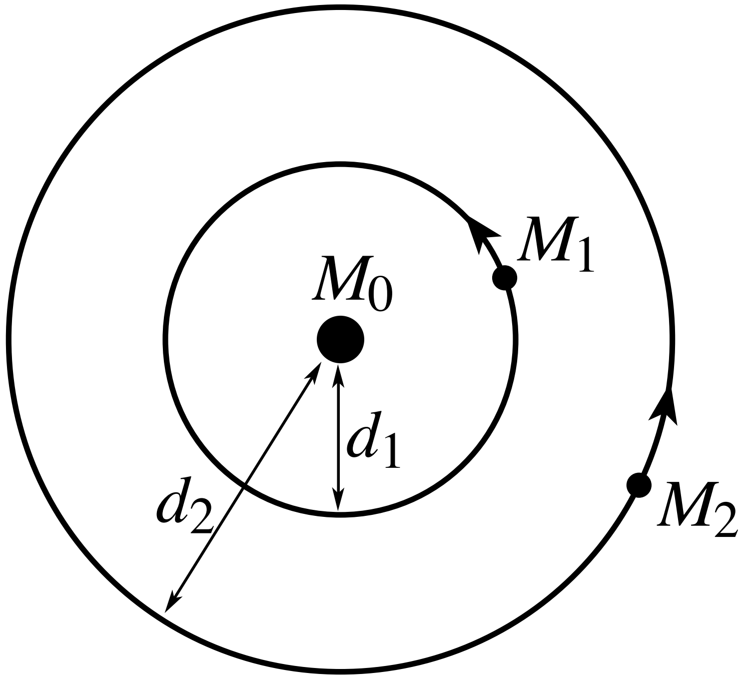
- Further refinements
  - – optimal control and parametric trade studies
  - – trajectory correction: work with natural dynamics
    - e.g., trajectory correction maneuvers for **Genesis** (Ross et al. [2002])

Patched 3-Body Approx.

- Consider spacecraft P in field of 3 massive bodies, M_0, M_1, M_2 e.g., Jupiter and two moons



Central mass M_0 and two massive orbiting bodies, M_1 and M_2

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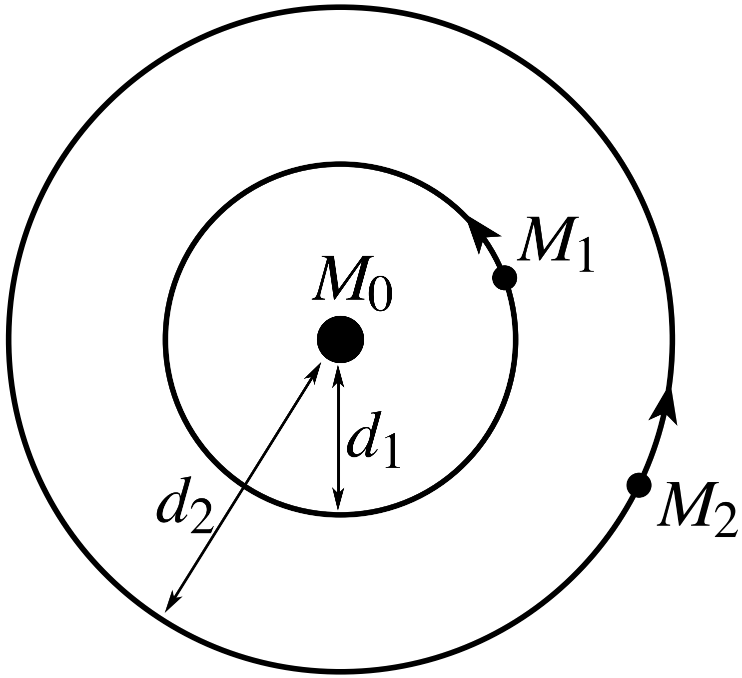

Central mass M_0 and two massive orbiting bodies, M_1 and M_2

- Assumption: Only one 3-body interaction dominates at a time (found to hold quite well)

Patched 3-Body Approx.

- **Initial approximation**

  4-body system approximated as two 3-body subsystems

- for $t < 0$, model as P-M_0-M_1

  for $t \geq 0$, model as P-M_0-M_2

  i.e., we "patch" two 3-body solutions

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- **Initial approximation**

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- for $t < 0$, model as P-M_0-M_1

  for $t \geq 0$, model as P-M_0-M_2

  i.e., we "patch" two 3-body solutions

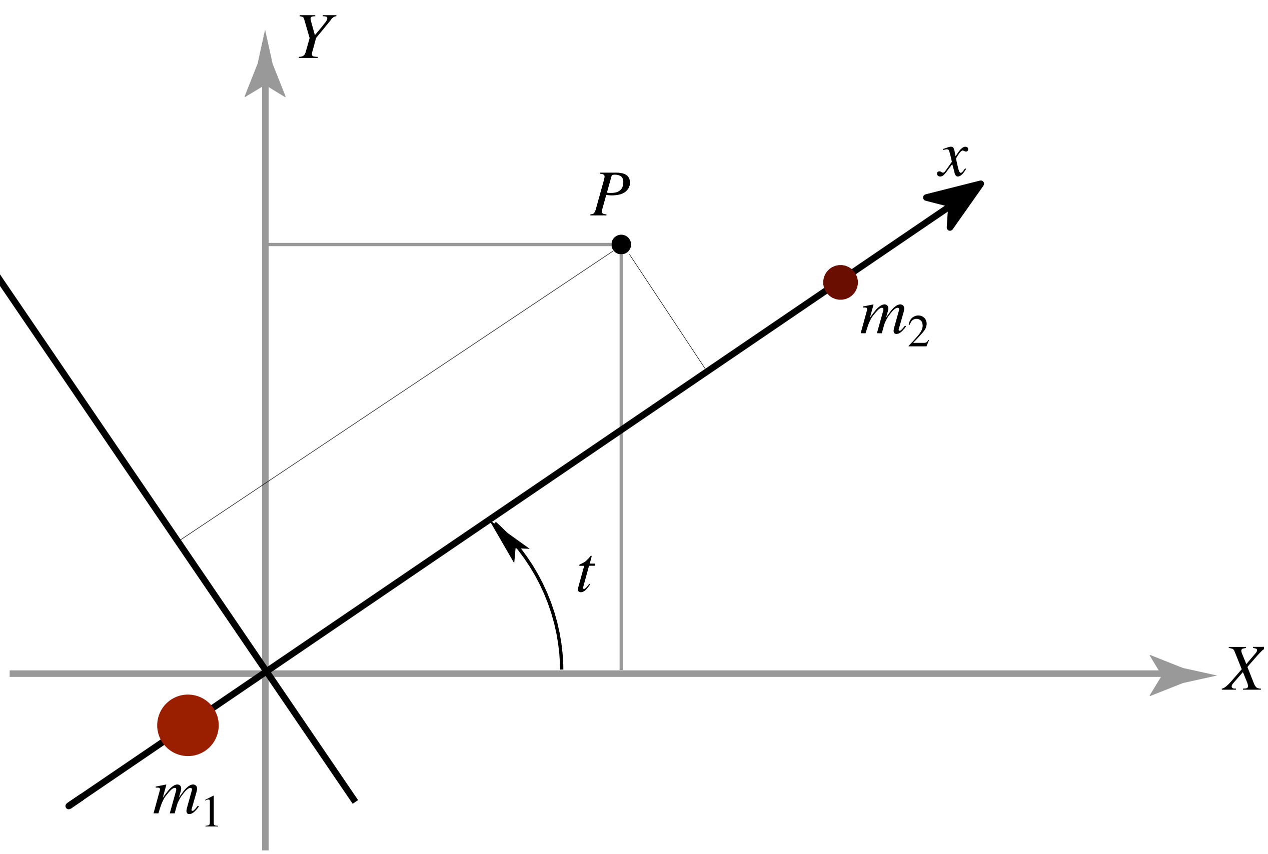
- 3-body solutions are now known quite well

  (Ross [2004]; Koon, Lo, Marsden, Ross [2004], ...)

  Consider the 3-body problem...

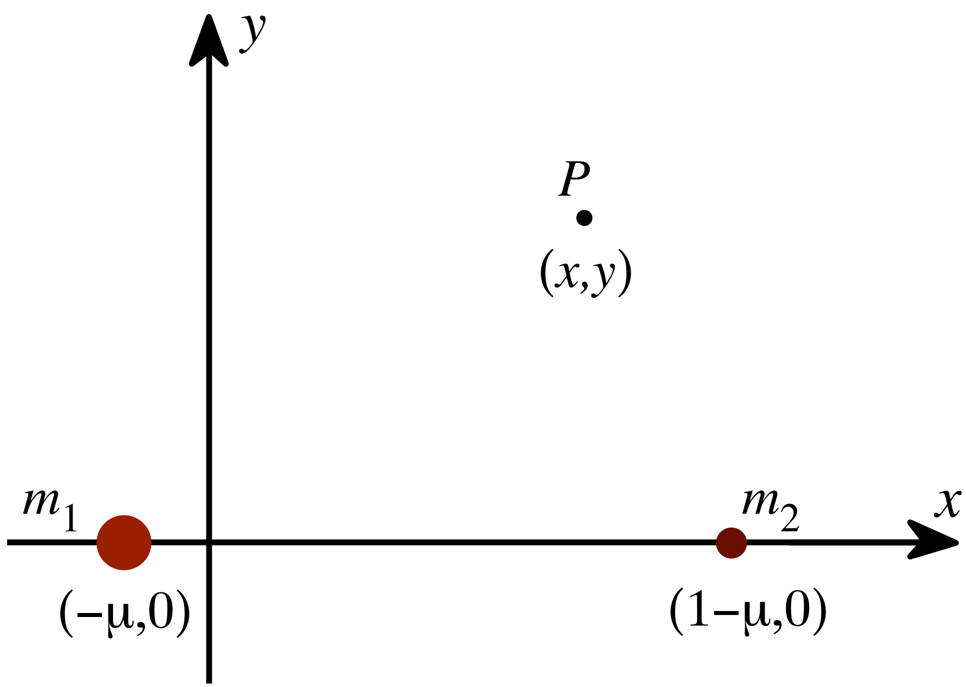
3-Body Problem

- **Planar, circular, restricted 3-body problem**
  - P in field of two bodies, m_1 and m_2
  - x-y frame rotates w.r.t. X-Y inertial frame

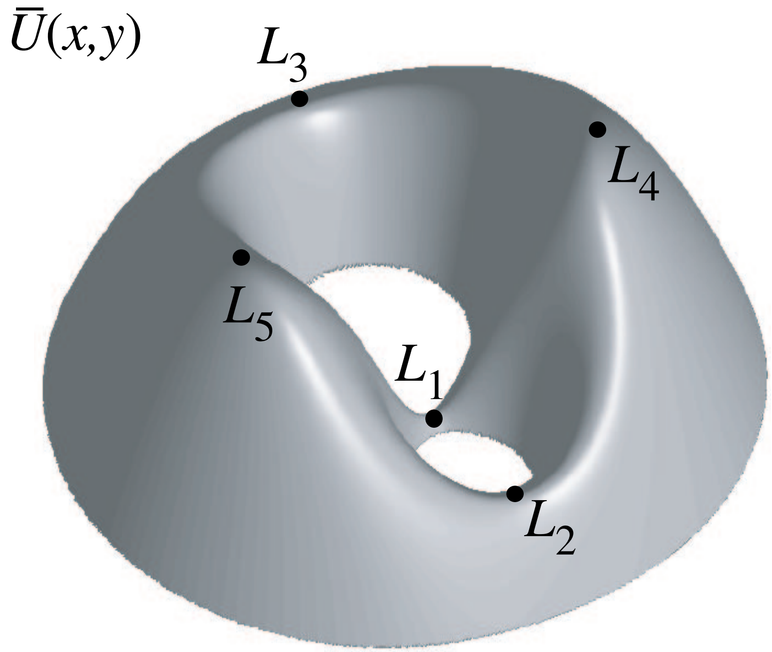


3-Body Problem

- Equations of motion describe P moving in an effective potential plus a coriolis force



Position Space



Effective Potential

Hamiltonian System

- Hamiltonian function

$$H(x, y, p_x, p_y) = \frac{1}{2}((p_x + y)^2 + (p_y - x)^2) + \bar{U}(x, y),$$

where p_x and p_y are the conjugate momenta, and

$$\bar{U}(x, y) = -\frac{1}{2}(x^2 + y^2) - \frac{1 - \mu}{r_1} - \frac{\mu}{r_2}$$

where r_1 and r_2 are the distances of P from m_1 and m_2 and

$$\mu = \frac{m_2}{m_1 + m_2} \in (0, 0.5].$$

- Eqs. of motion in 4D phase space.

Motion within Energy Surface

- For fixed μ, an energy surface of energy ε is

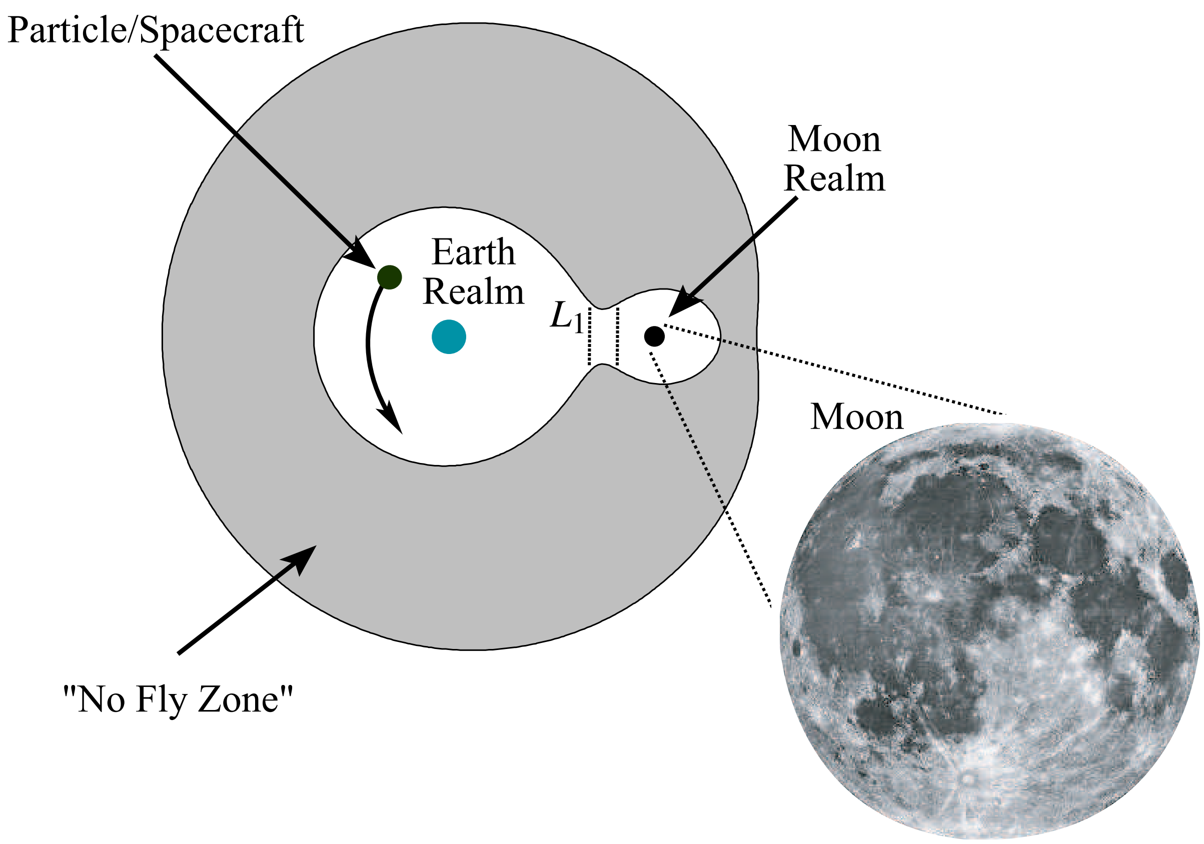
$$\mathcal{M}_\mu(\varepsilon) = \{(x, y, p_x, p_y) \mid H(x, y, p_x, p_y) = \varepsilon\}.$$

  In the 2 d.o.f. problem, these are 3D surfaces foliating the 4D phase space.

- In 3 d.o.f., 5D energy surfaces.

Realms of Possible Motion

- $\mathcal{M}_\mu(\varepsilon)$ partitioned into three **realms**
  e.g., Earth realm = phase space around Earth
- ε determines their connectivity



Multi-Scale Dynamics

- **n $\geq$ 2 d.o.f. Hamiltonian systems**
  - Phase space has structures mediating transport
  - Controls can take use of these for efficiency

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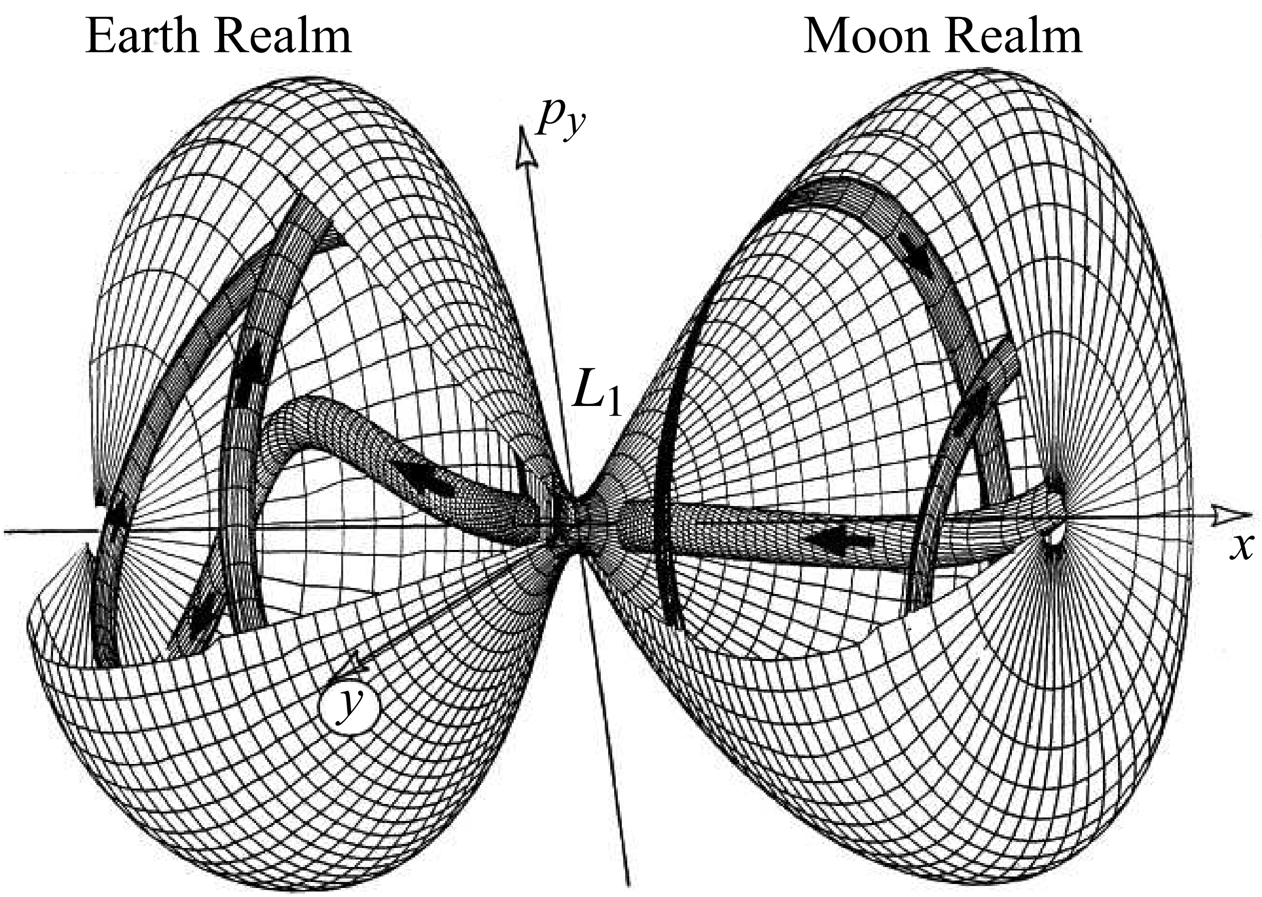
- **n $\geq$ 2 d.o.f. Hamiltonian systems**
  - Phase space has structures mediating transport
  - Controls can take use of these for efficiency
- Multi-scale approach
  - **Tube dynamics** : motion between **realms**
  - **Lobe dynamics** : motion between **regions** in a realm

Multi-Scale Dynamics

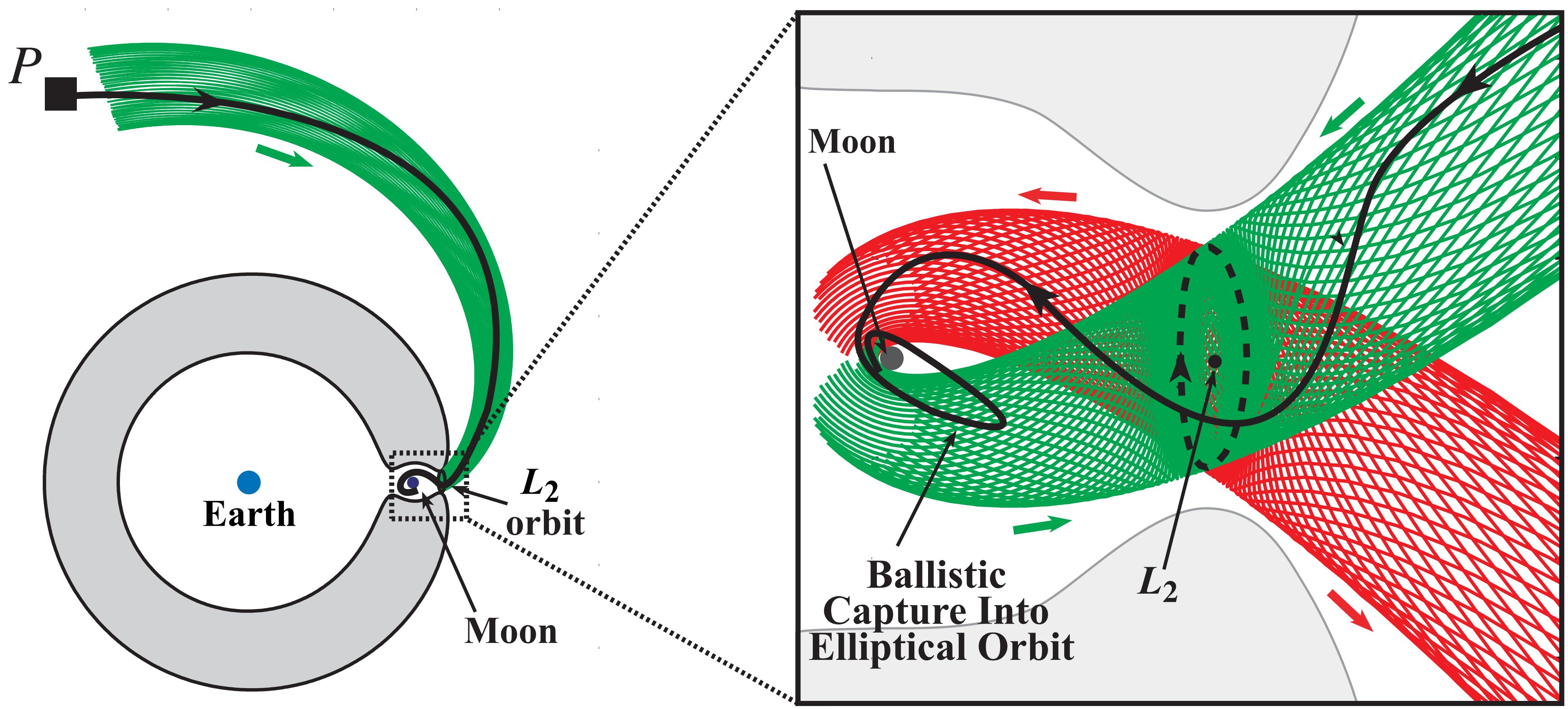
- Realms connected by **tubes** in the phase space



Phase Space (Position + Velocity)

Multi-Scale Dynamics

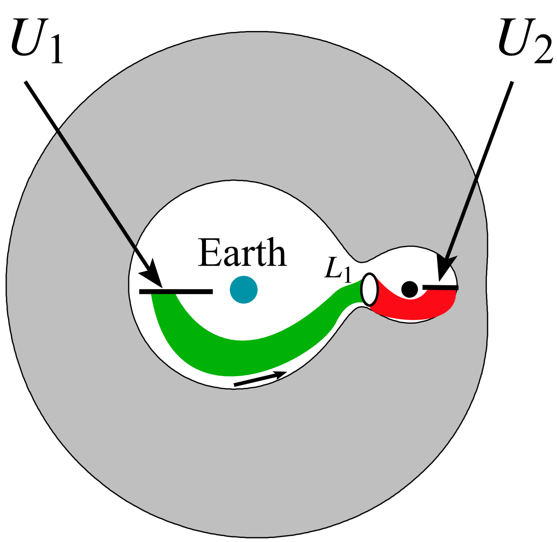
- Tubes associated with periodic orbits about L_1, L_2
  - Control ballistic capture and escape



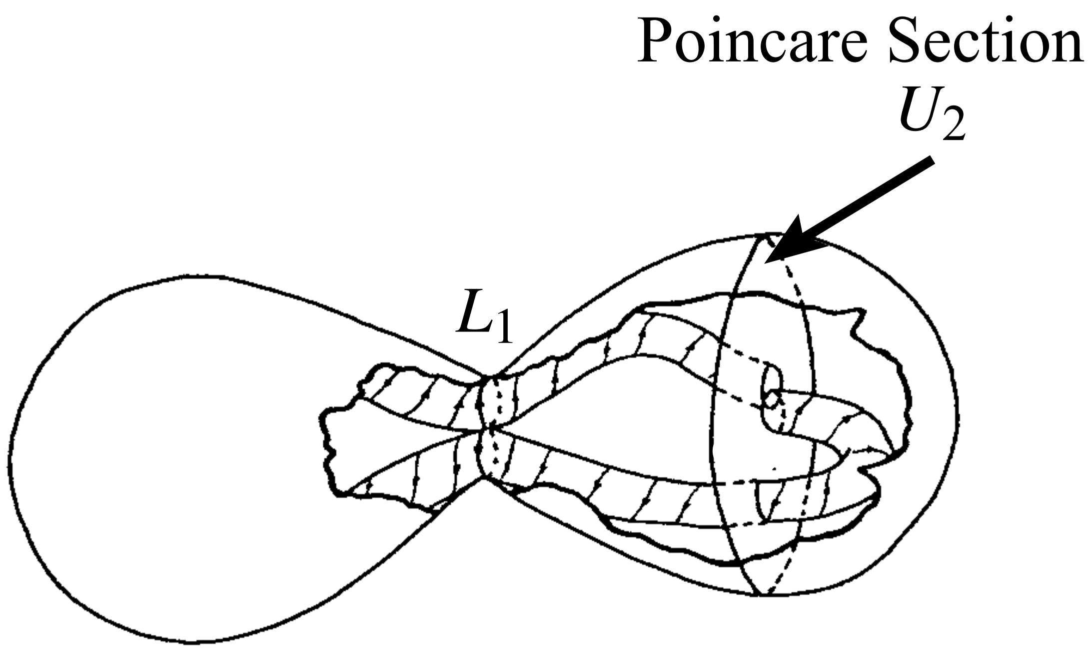
Tube leading to ballistic capture around the Moon (seen in rotating frame)

Multi-Scale Dynamics

- Poincaré section U_i in Realm i, $i = 1, \ldots, k$
- Lobe dynamics: evolution **on** U_i
- Tube dynamics: evolution **between** U_i



Position Space



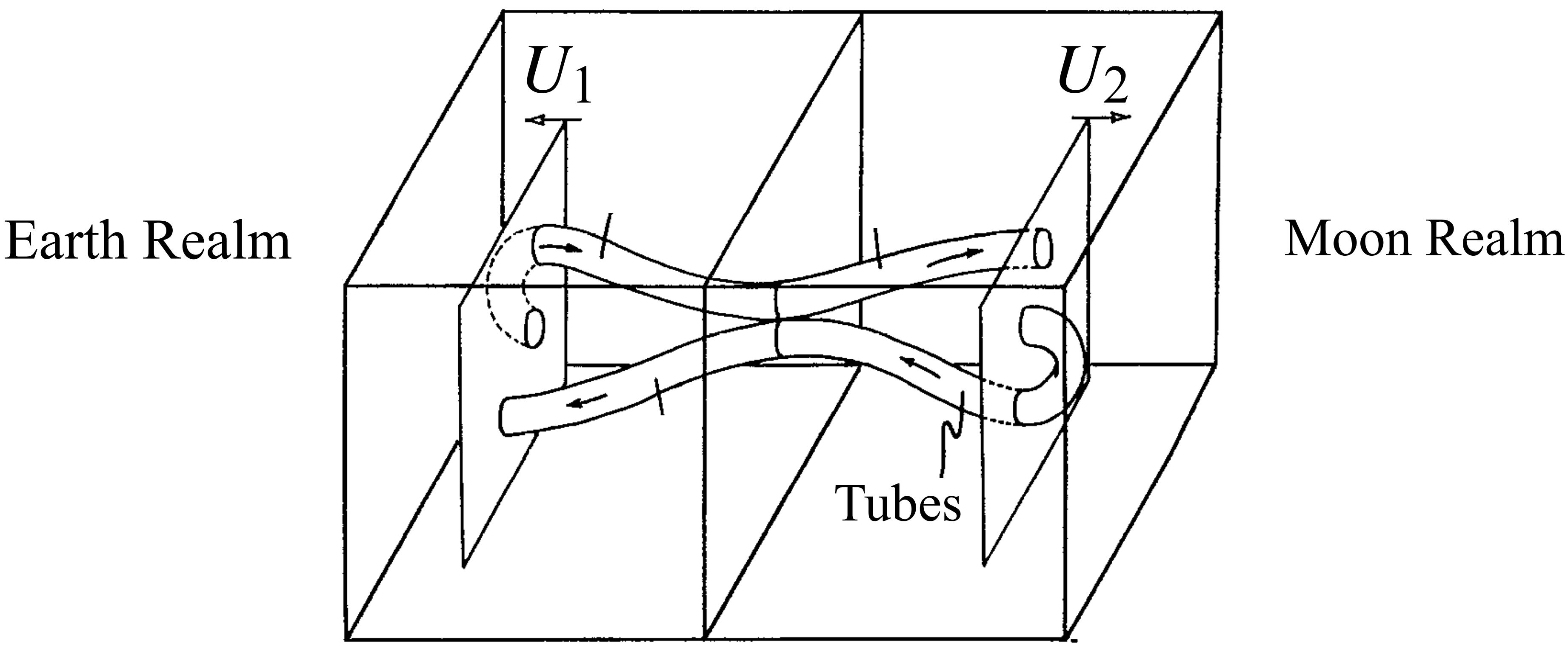
Phase Space

Tube Dynamics

- Motion between Poincaré section on $\mathcal{M}_\mu(\varepsilon)$:

$$U_i = \{(x, p_x) | y = \text{const} \in \text{Realm } i, p_y = g(x, p_x, y; \mu, \varepsilon) > 0\}.$$

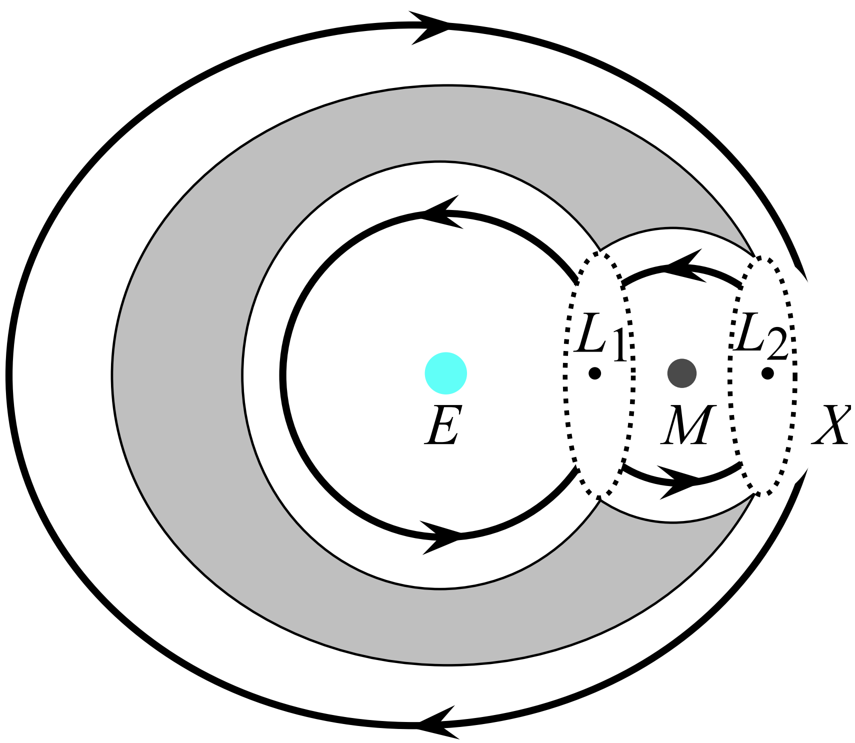
  System reduced to area-preserving k-map dynamics between the k U_i.



Poincaré surfaces-of-section U_1 & U_2 linked by tubes

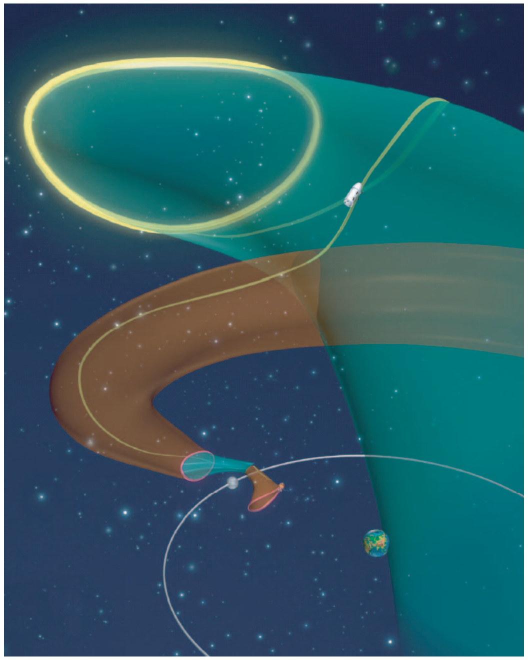
Tube Dynamics: Theorem

- *Theorem of global orbit structure*
  - says we can construct an orbit with any **itinerary**, eg $(\ldots, M, X, M, E, M, E, \ldots)$, where X, M and E denote the different realms (symbolic dynamics)
  - Main theorem of Ross et al. [2000]



Construction of Trajectories

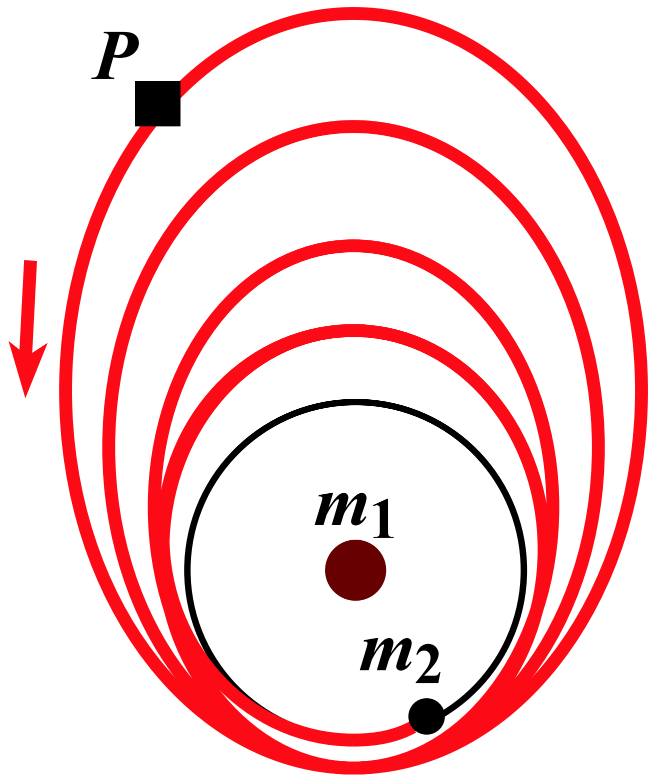
- Systematic construction of trajectories with desired itineraries – trajectories which use **little or no fuel.**
  - by linking tubes in the right order $\longrightarrow$ **tube hopping**
- Itineraries for multiple 3-body systems possible too.



Tube hopping

Resonant Flybys

- Tubes do not give the full picture...
- Considerable fuel savings can be achieved by using **resonant flybys**

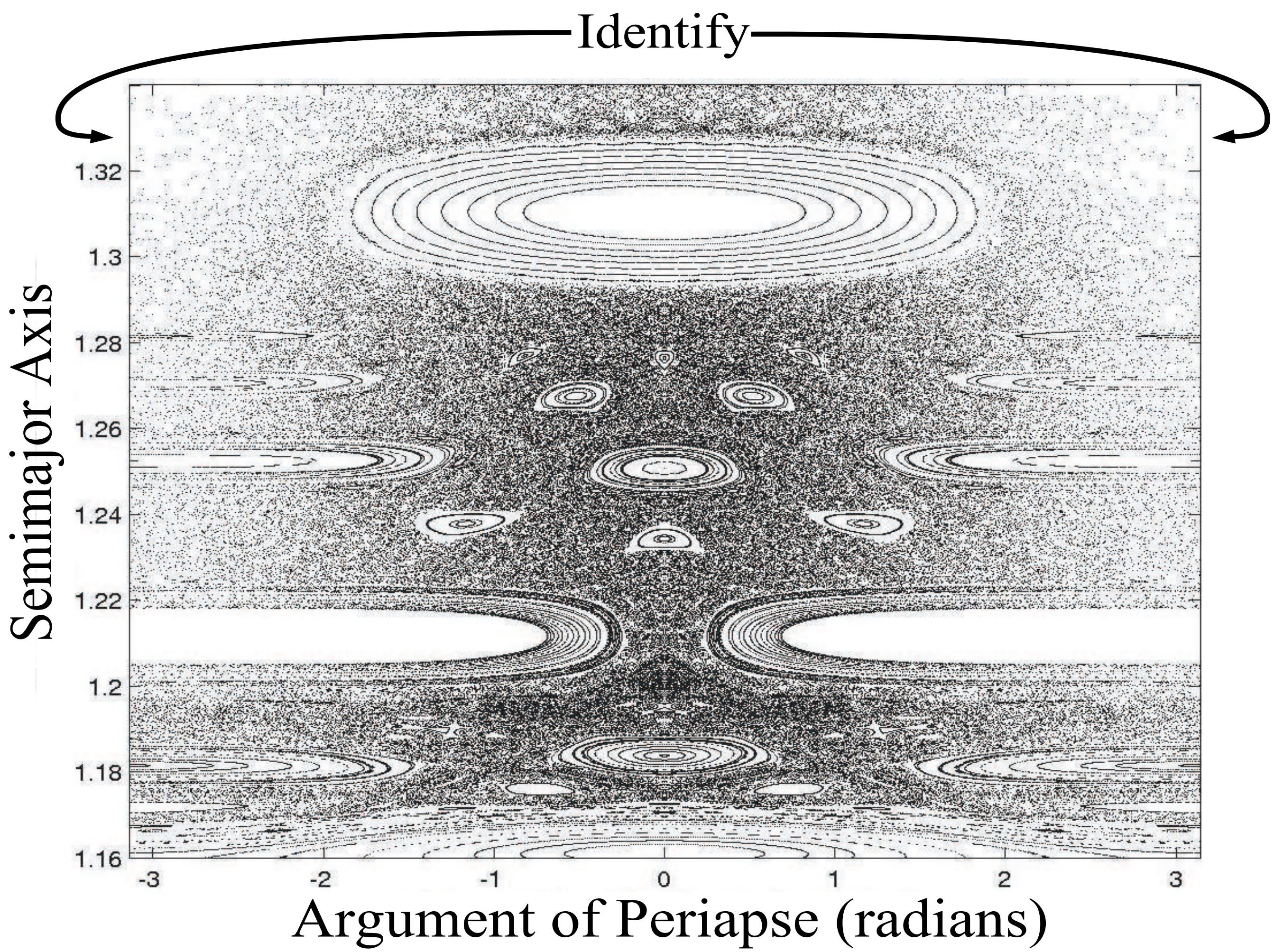


Underlying mechanism:
overlap of resonance regions, understood using lobe dynamics.

Goal: an optimal sequence of flybys.

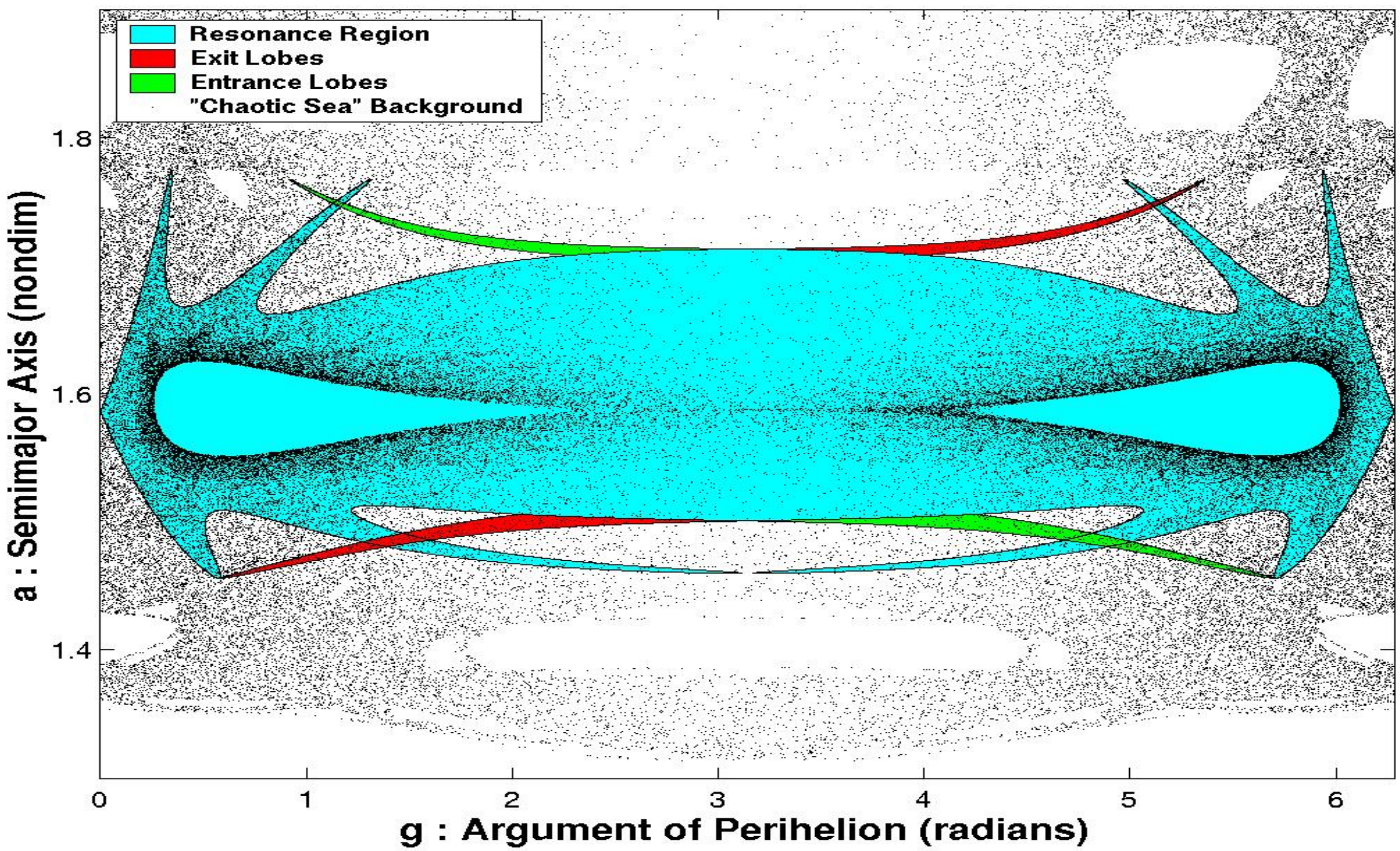
Resonance Structure

- Poincaré section reveals "chaotic zone"
  – unstable periodic points govern chaotic motion



Resonance Structure & Lobes

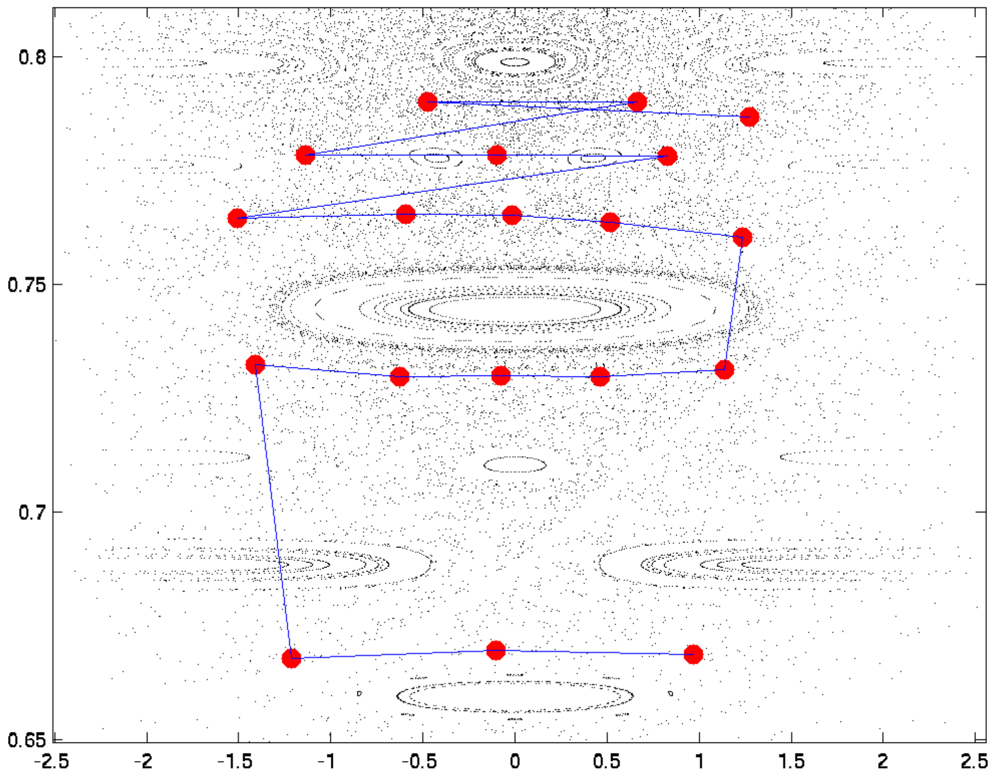
- Their stable & unstable manifolds bound **resonance regions**
  - Lobes associated with motion around it
  - Orbit changes for zero fuel cost



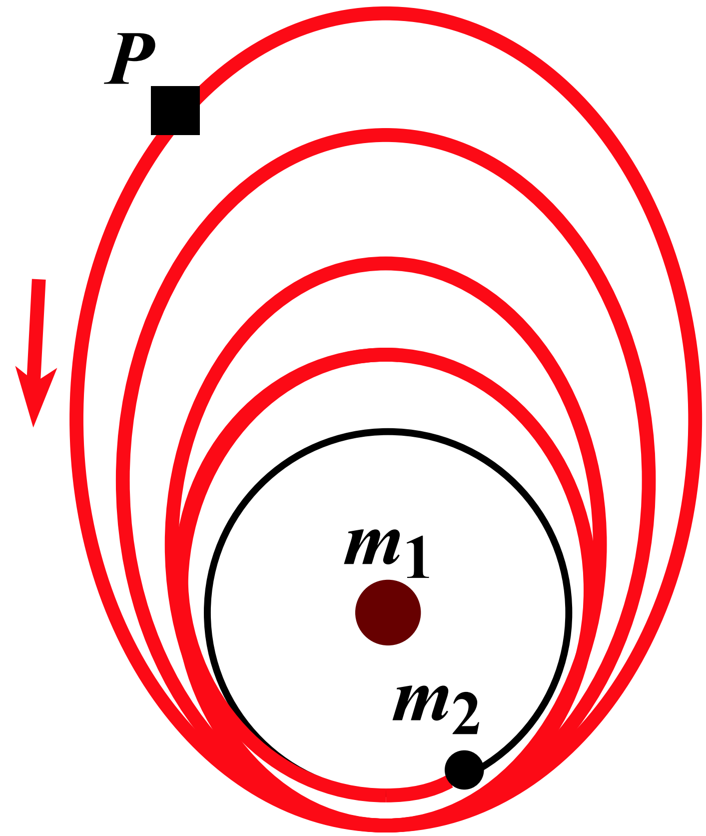
Resonance Structure & Lobes

- Trajectory construction:

  Large orbit changes with little or no fuel via **resonant flybys.**



Surface-of-section



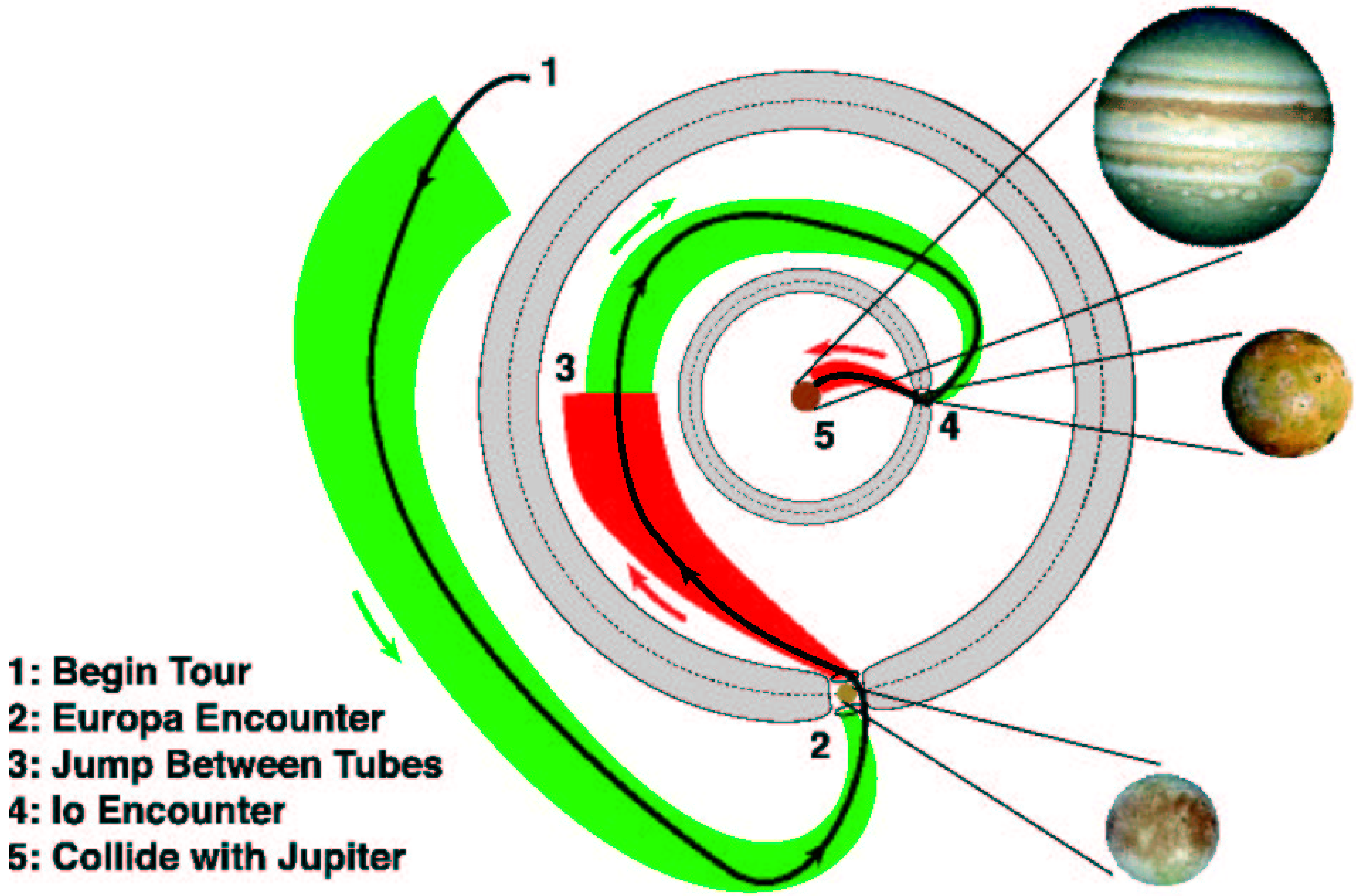
Large orbit changes

Patching Two 3-Body Sol'ns

- ***Multi-Moon Orbiter (e.g., JIMO)***
  - Orbit multiple moons with a single spacecraft
  - Advantage: Longer observations
  - Disadvantage: Standard "patched-conics" won't work – yields prohibitively high ΔV
  - **But:** Patched three-body approx. works – yields lower, technically feasible ΔV

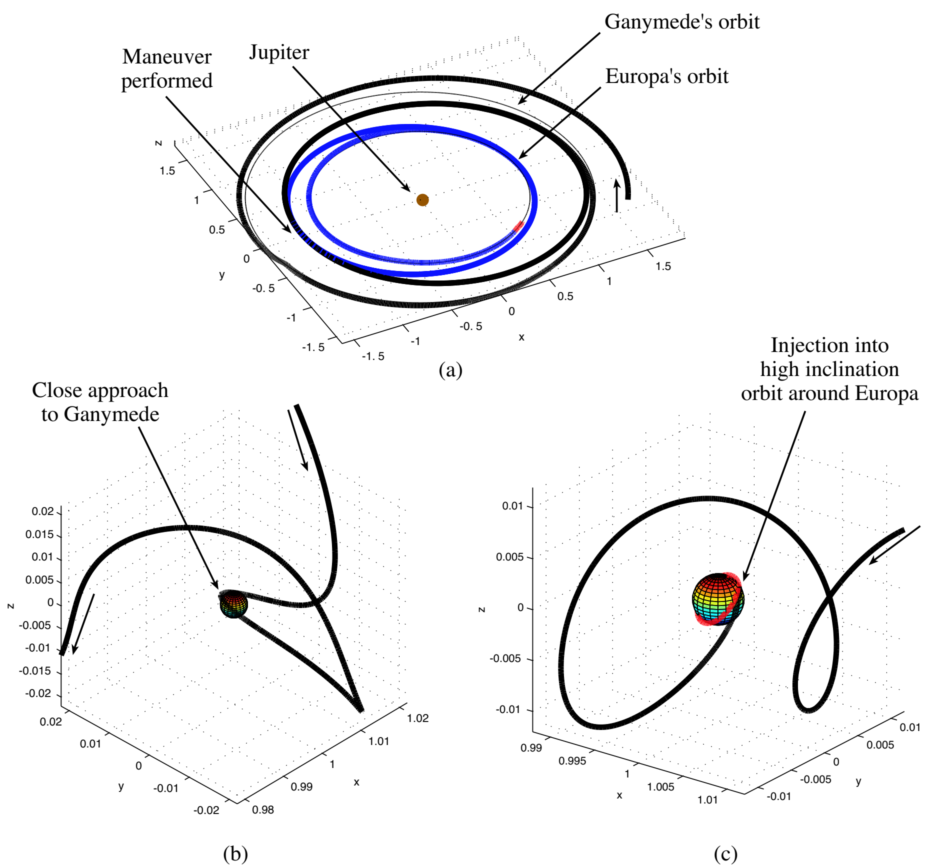
Multi-Moon Orbiters

- *Example 1:* Europa → Io → Jupiter



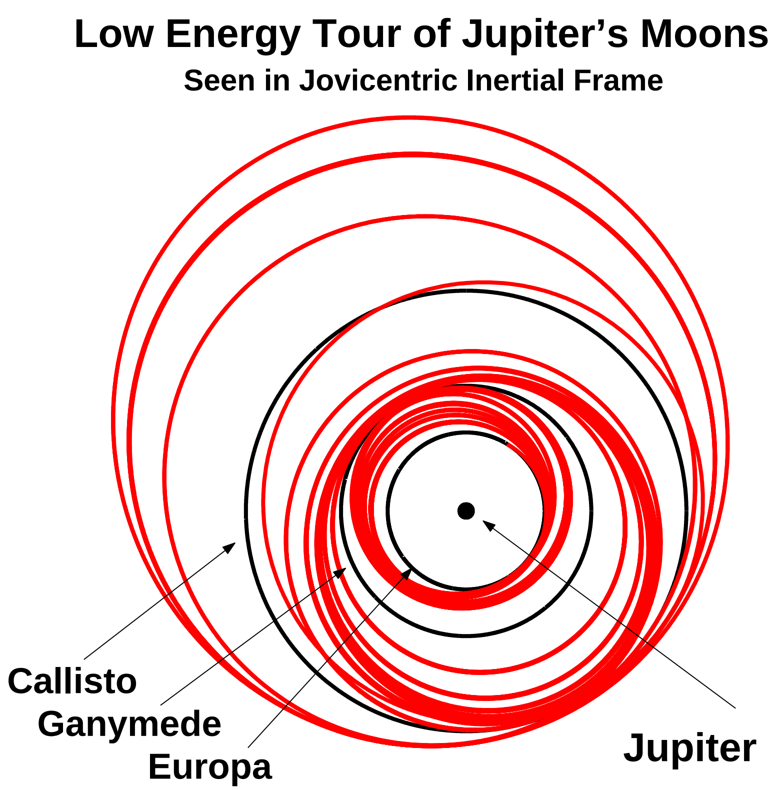
Multi-Moon Orbiters

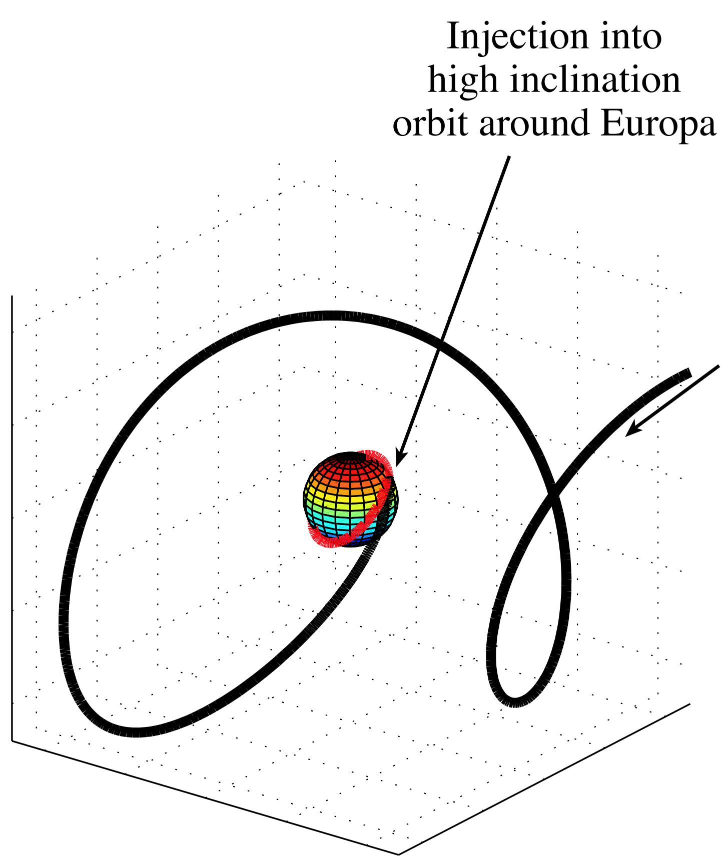
- *Example 2:* Ganymede-Europa Orbiter
  - ΔV of 1400 m/s was half the Hohmann transfer
  - Ross et al. [2001]



JIMO Prototype

- *Example 3:* Callisto-Ganymede-Europa Orbiter
  - Visit all icy moons: $\Delta V \sim 0$, flight time $\sim$ 30 months
  - Uses resonant flybys, tubes for capture/escape
  - Ross [2001], Ross et al. [2003]





Current and Ongoing Work

- Fully automated algorithm for trajectory generation
- Consider model uncertainty, unmodeled dynamics, noise
- Trajectory correction when errors occur
  – Re-targeting of original (nominal) trajectory vs. re-generation of nominal trajectory
  – Trajectory correction work for *Genesis* is a first step

Current and Ongoing Work

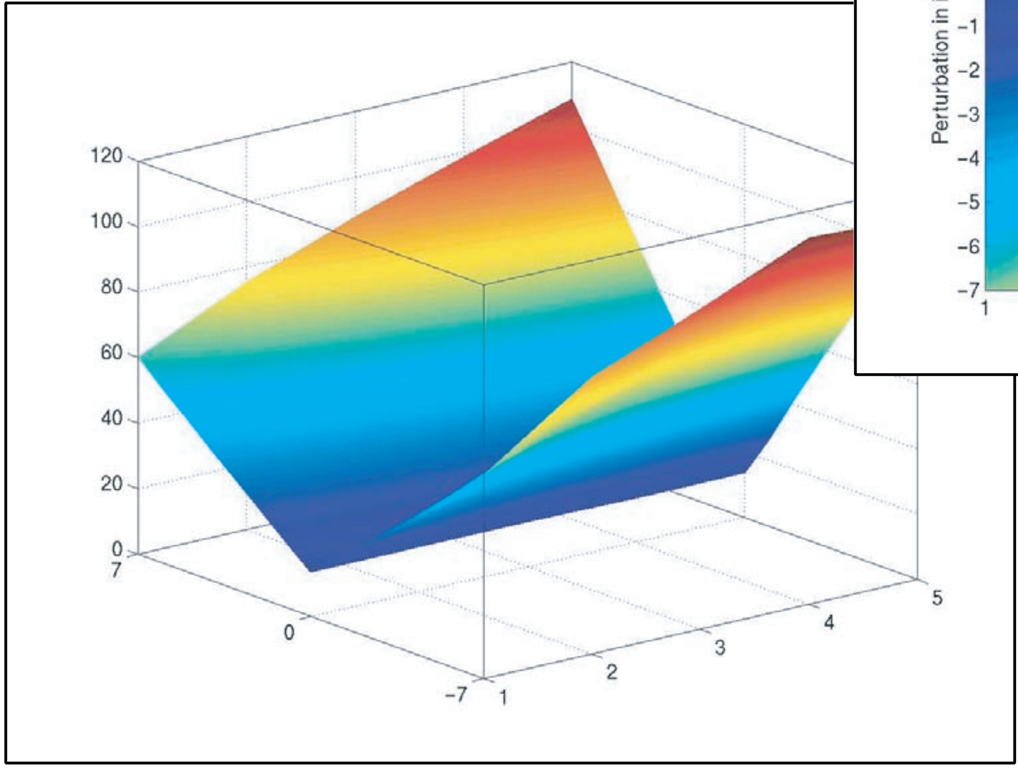
- Getting *Genesis* onto the destination orbit at the right time, while minimizing fuel consumption

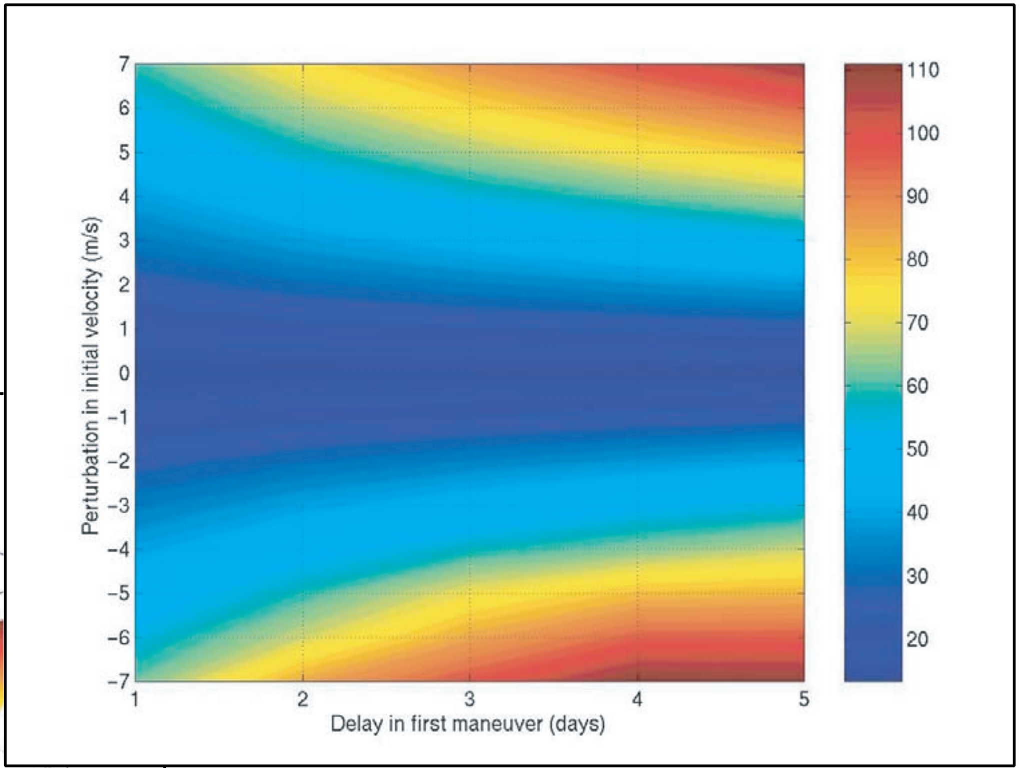
from Serban, Koon, Lo, Marsden, Petzold, Ross, and Wilson [2002]

Current and Ongoing Work

Parametric Studies of
Optimal Correction Solutions:

- A mixture of dynamical systems theory and optimal control





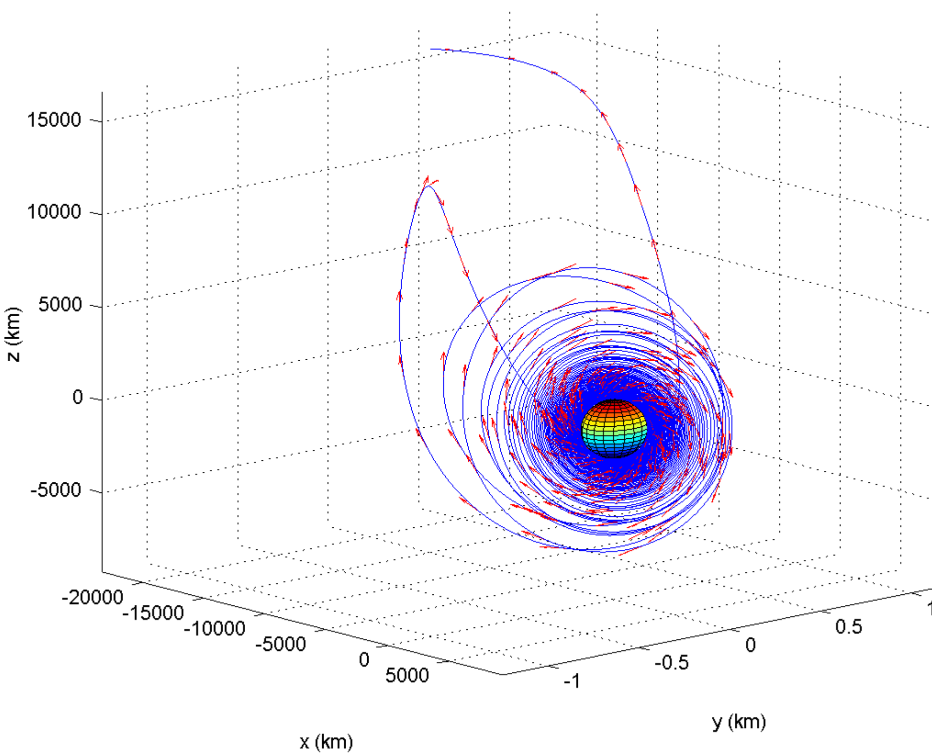
Influence of:

- **Delay in TCM1**
- **Perturbation in launching velocity**

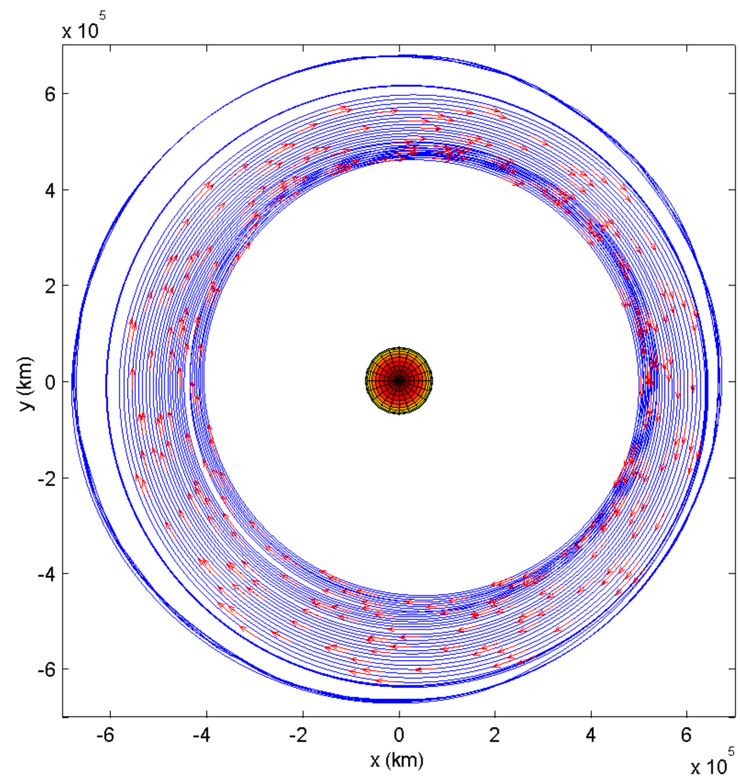
Optimal solutions found for all cases

Current and Ongoing Work

- Incorporation of low thrust
- Design to take best advantage of natural dynamics



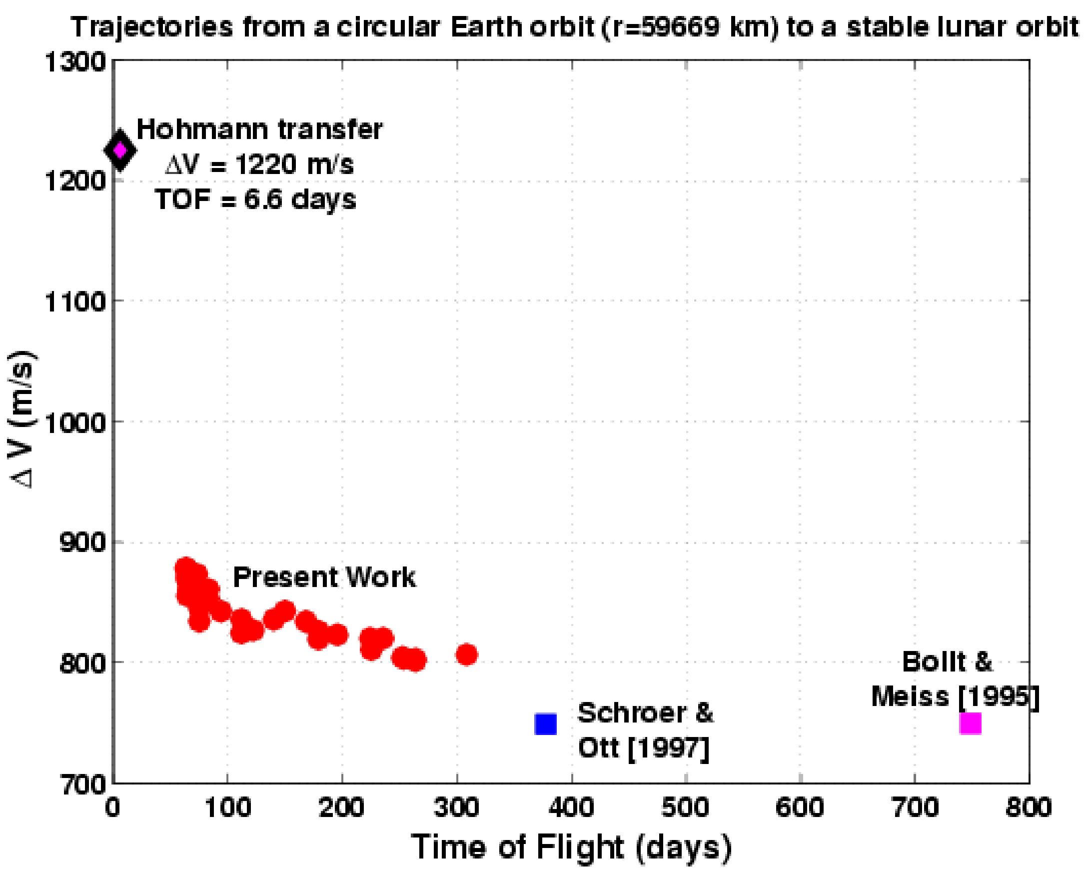
Spiral out from Europa



Europa to Io transfer

Current and Ongoing Work

- Meet goals/constraints of real missions
  e.g., desired orbit/duration at each moon, radiation dose
- Decrease flight time: evidence suggests large decrease in time for small increase in ΔV



Current and Ongoing Work

- Spin-off: Results also apply to mathematically similar problems in astrodynamics, chemistry, fluids, ...
  - phase space transport
  - networks of full body problems
- Applications
  - **asteroid collision prediction** (Ross [2003])
  - **underwater vehicle navigation** (Lekien, Ross [2003])
  - **atmospheric mixing** (Bhat, Fung, Ross [2003])
  - **biomolecular design** (Gabern, Marsden, Ross [2004])

The End

Some References

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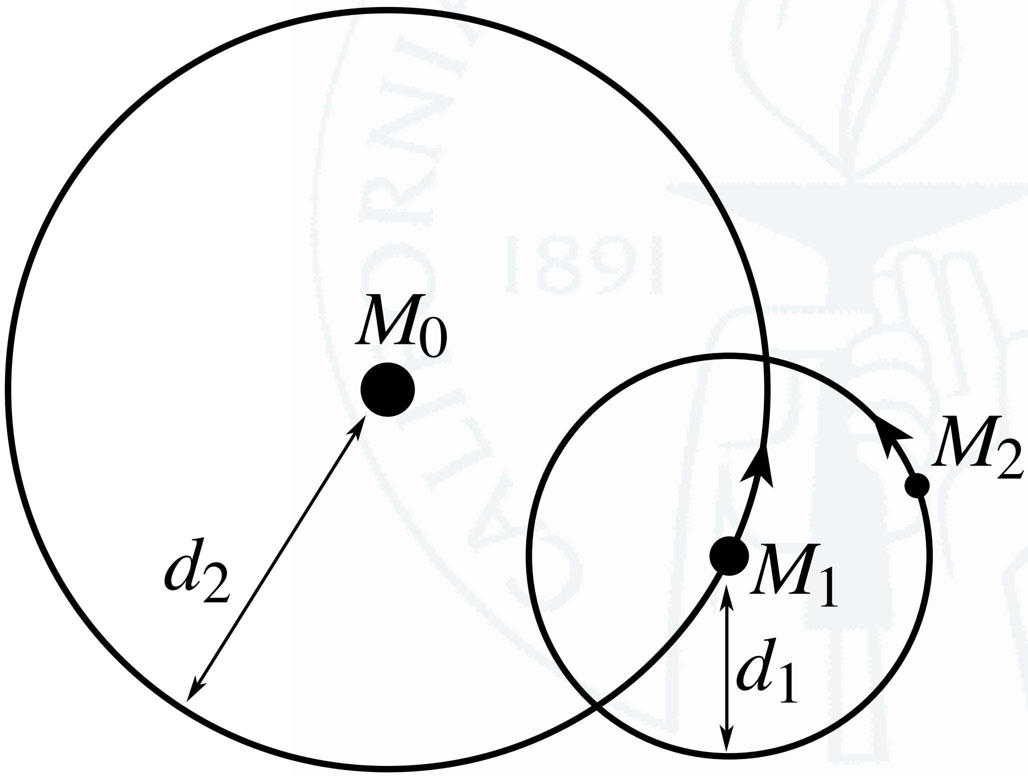
Extra Slides

Other Trajectory Studies

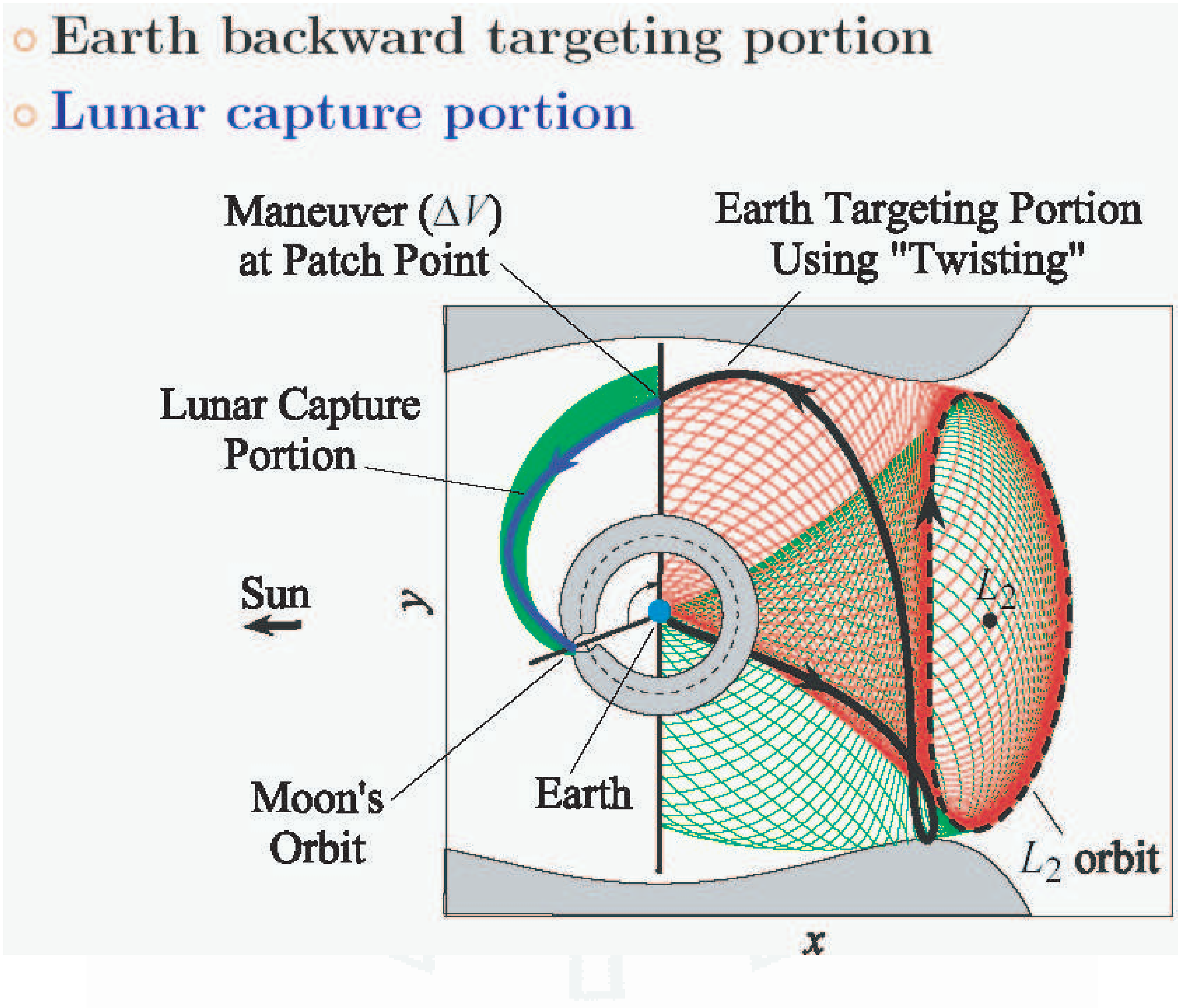
- *Many other trajectories can be designed using similar procedures*
  - One system of particular interest is the Earth-Moon vicinity, with the Sun's perturbation



M_2 in orbit around M_1; both in orbit about M_0

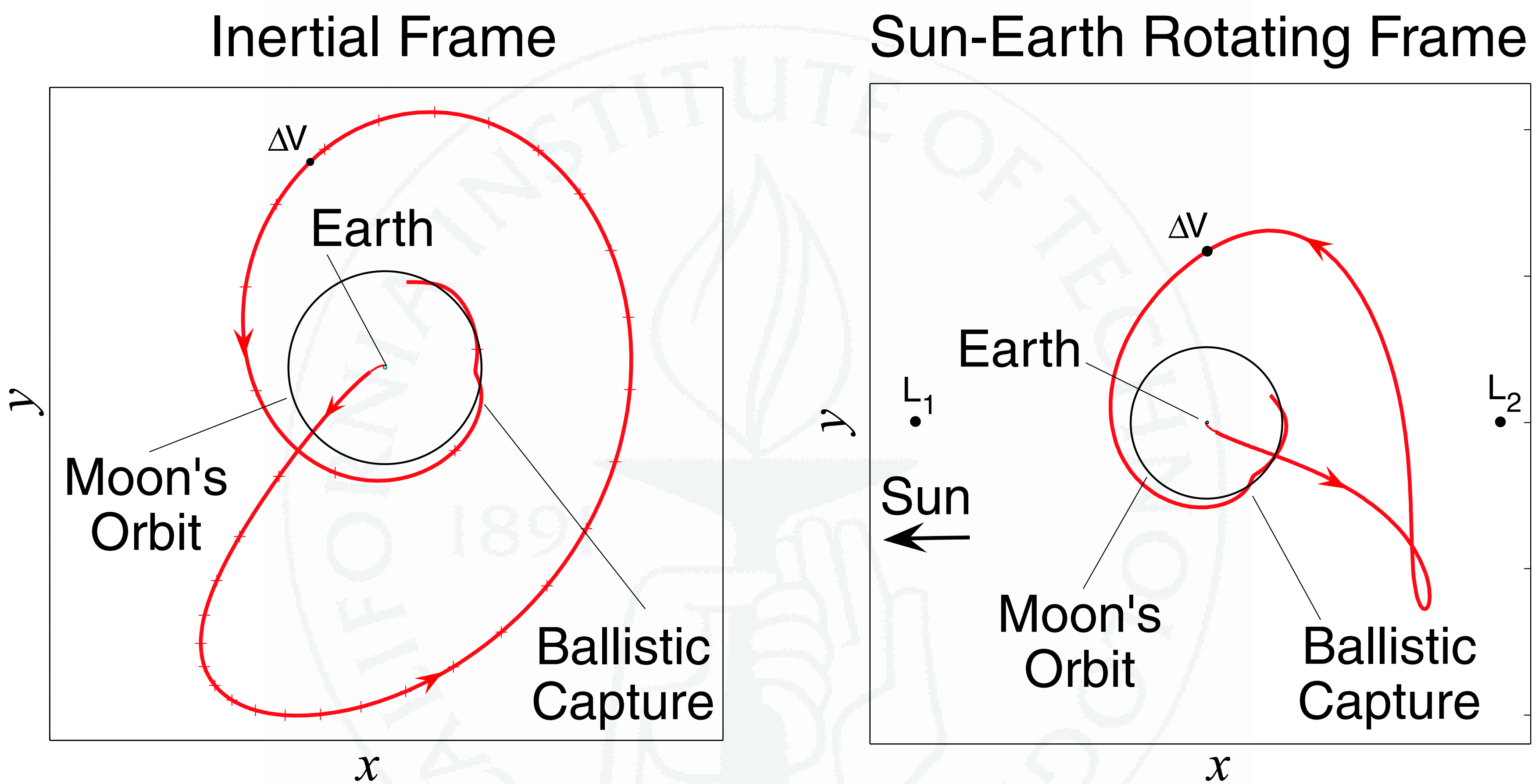
Sun-Earth-Moon Trajectories

- Fuel efficient paths to the Moon



Sun-Earth-Moon Trajectories

- 20% more fuel efficient than Apollo-like transfer

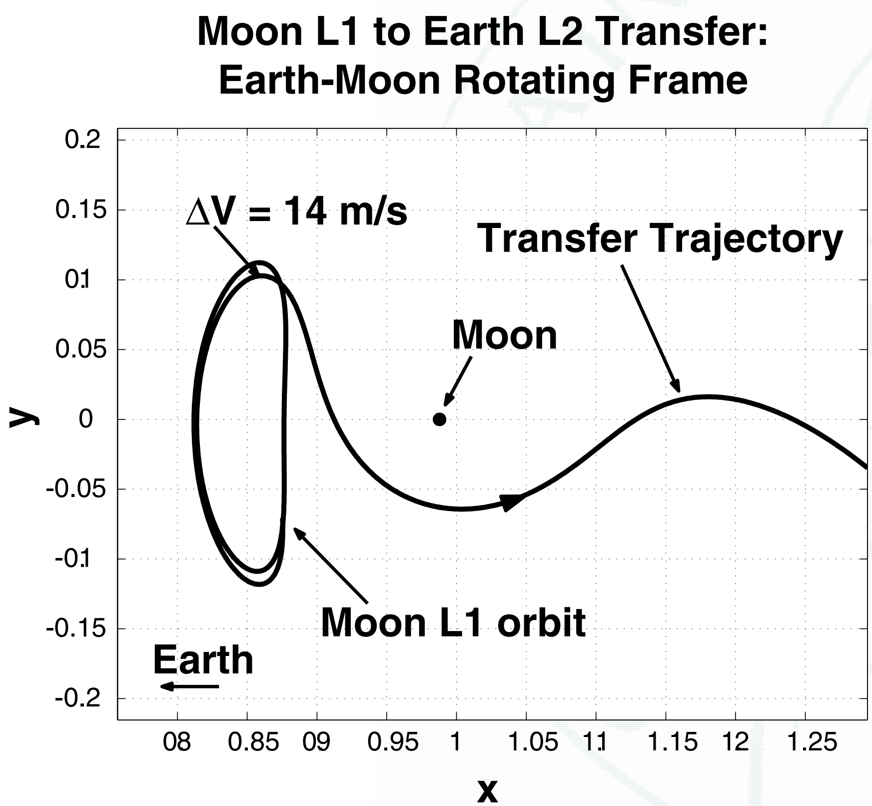


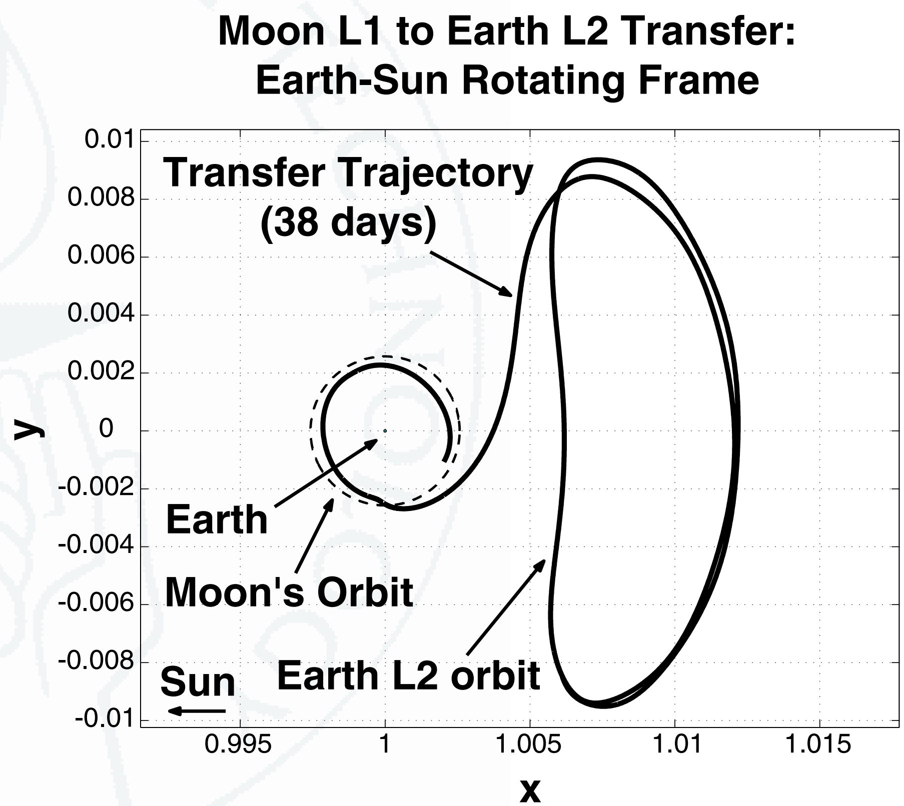
Sun-Earth-Moon Trajectories

shootthemoon-rotating.qt

Sun-Earth-Moon Trajectories

- Below is a fuel-optimal transfer between the Lunar L_1 Gateway station and a Sun-Earth L_2 orbit





Sun-Earth-Moon Trajectories

Sun-Earth frame movie

Inter-Moon Transfer

- The transfer between three-body systems occurs when energy surfaces intersect; can be seen on semimajor axis vs. eccentricity diagram (similar to Tisserand curves of Longuski et al.)

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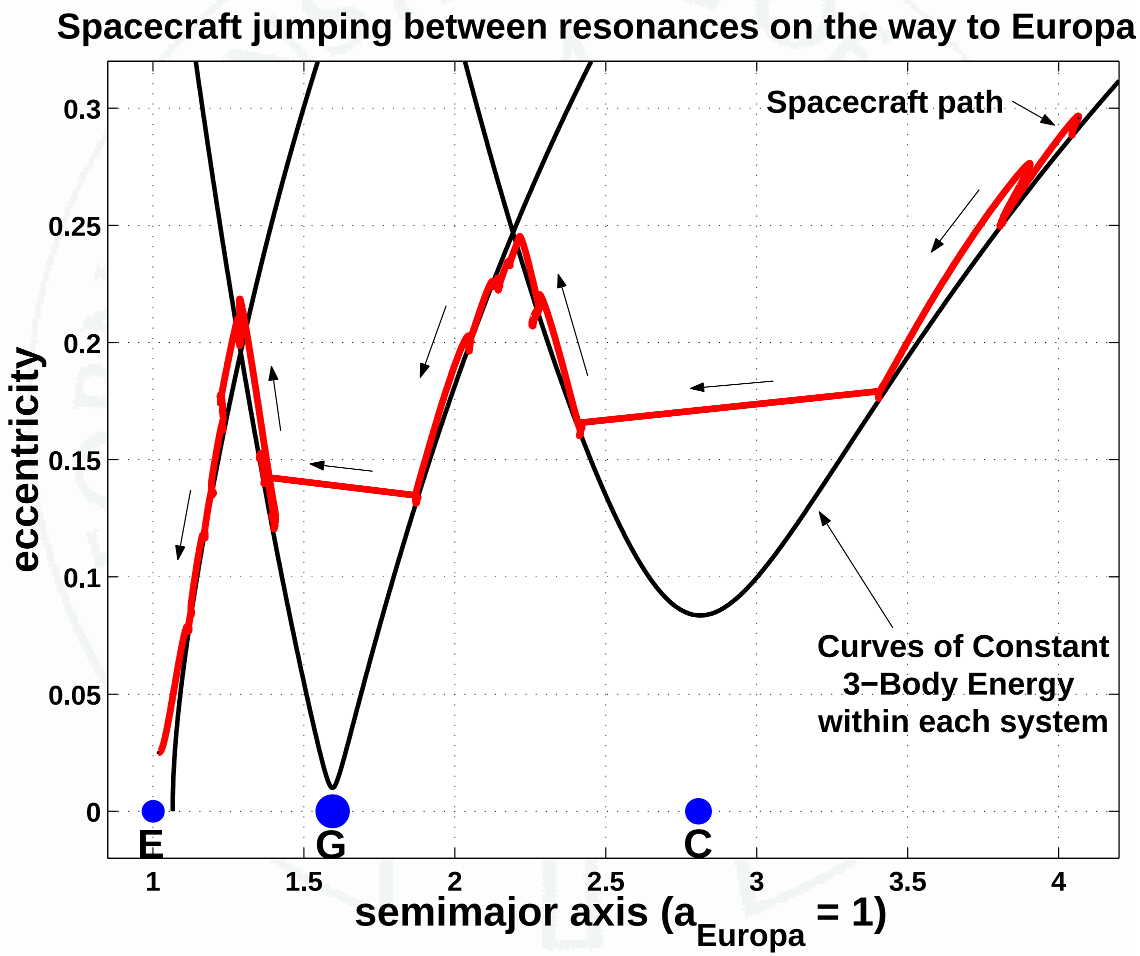


Lobe Dynamics: Partition Σ

- Let $\Sigma = U_i$, then our Poincaré map is a diffeomorphism

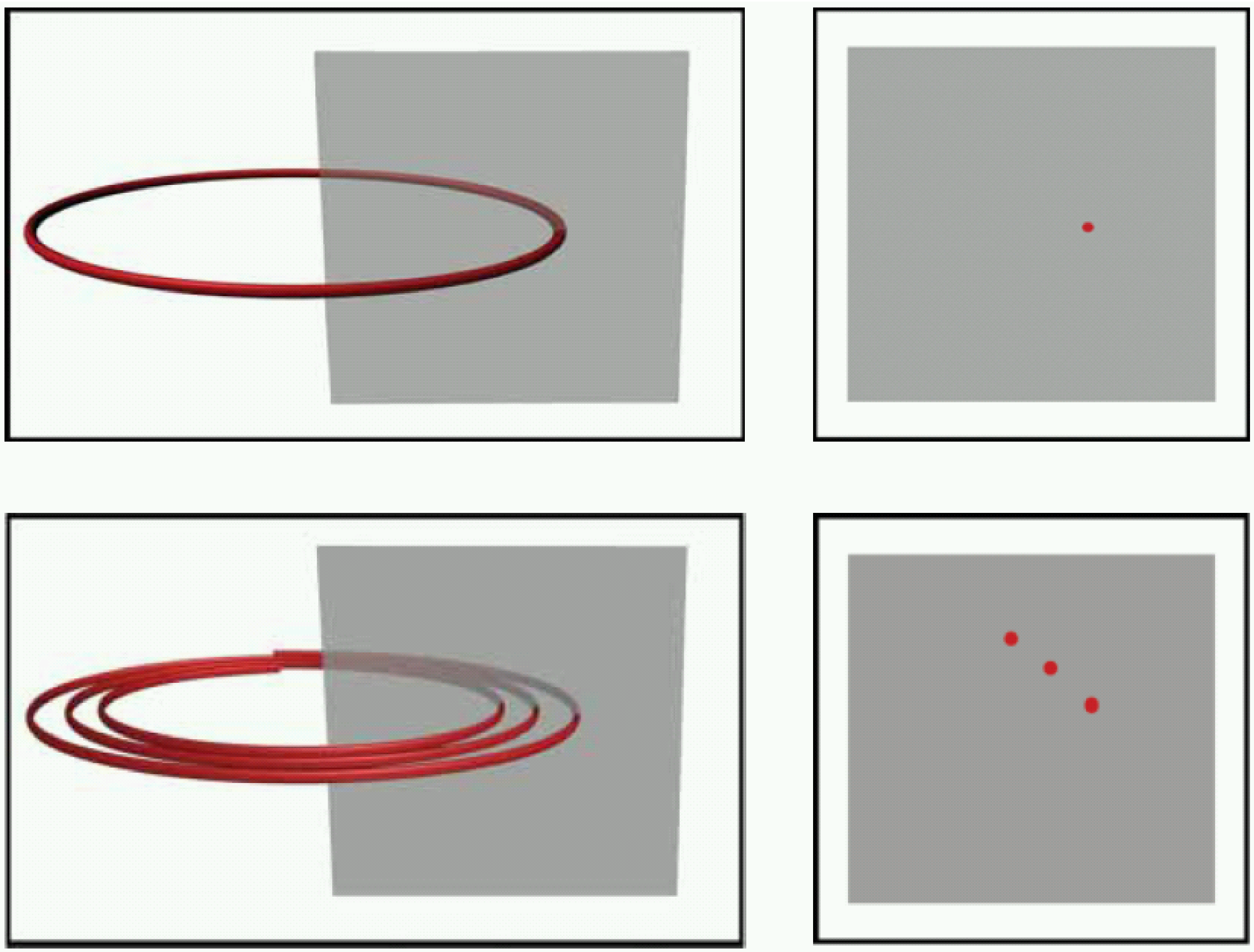
$$f : \Sigma \longrightarrow \Sigma,$$

- f is **orientation-preserving** and **area-preserving**

- Let $p_i, i = 1, \ldots, N_p$, denote a collection of saddle-type hyperbolic periodic points for f.

Lobe Dynamics: Partition Σ

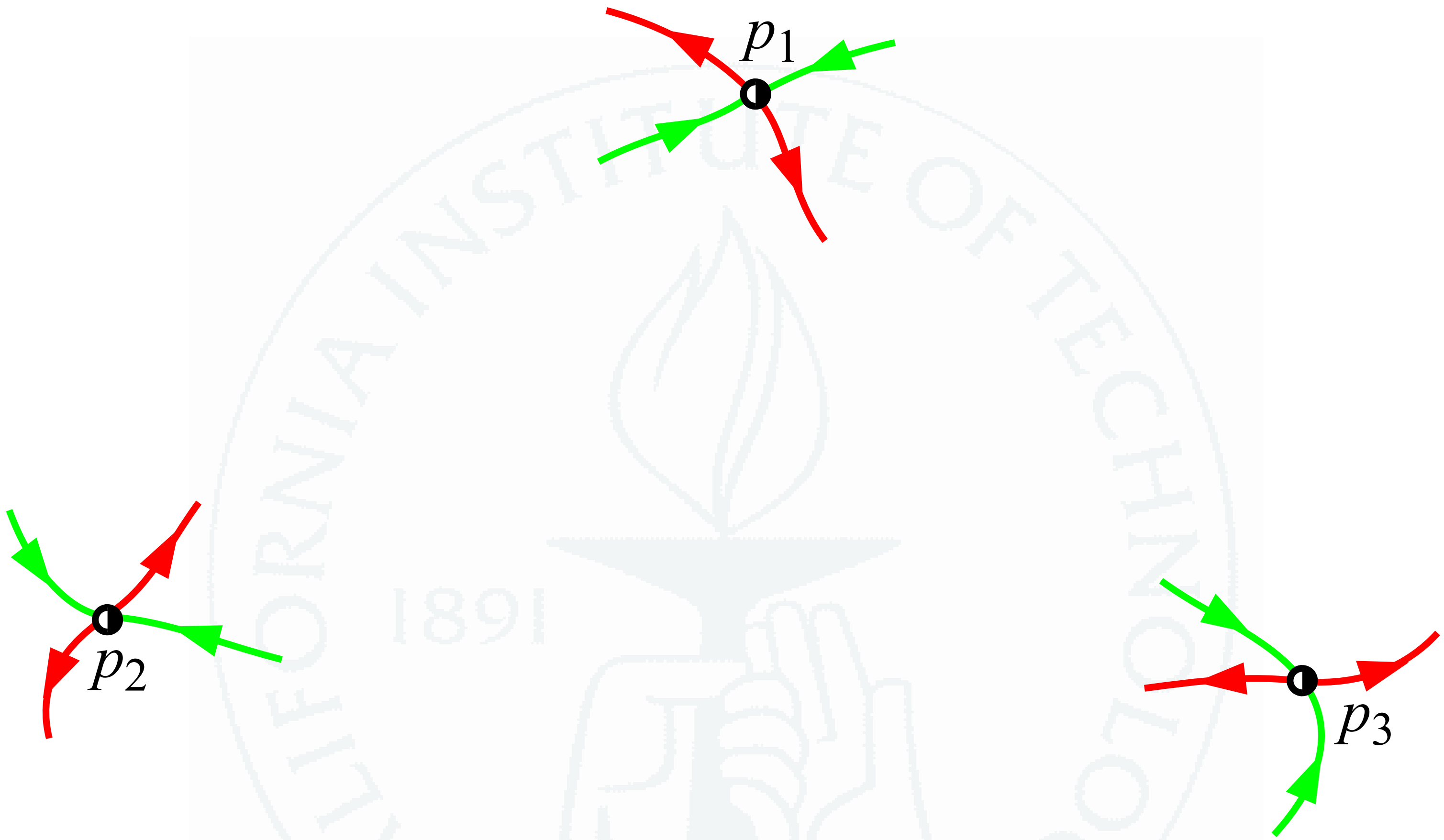
These are the unstable resonances reduced to Σ.



Poincaré surface of section

Lobe Dynamics: Partition Σ

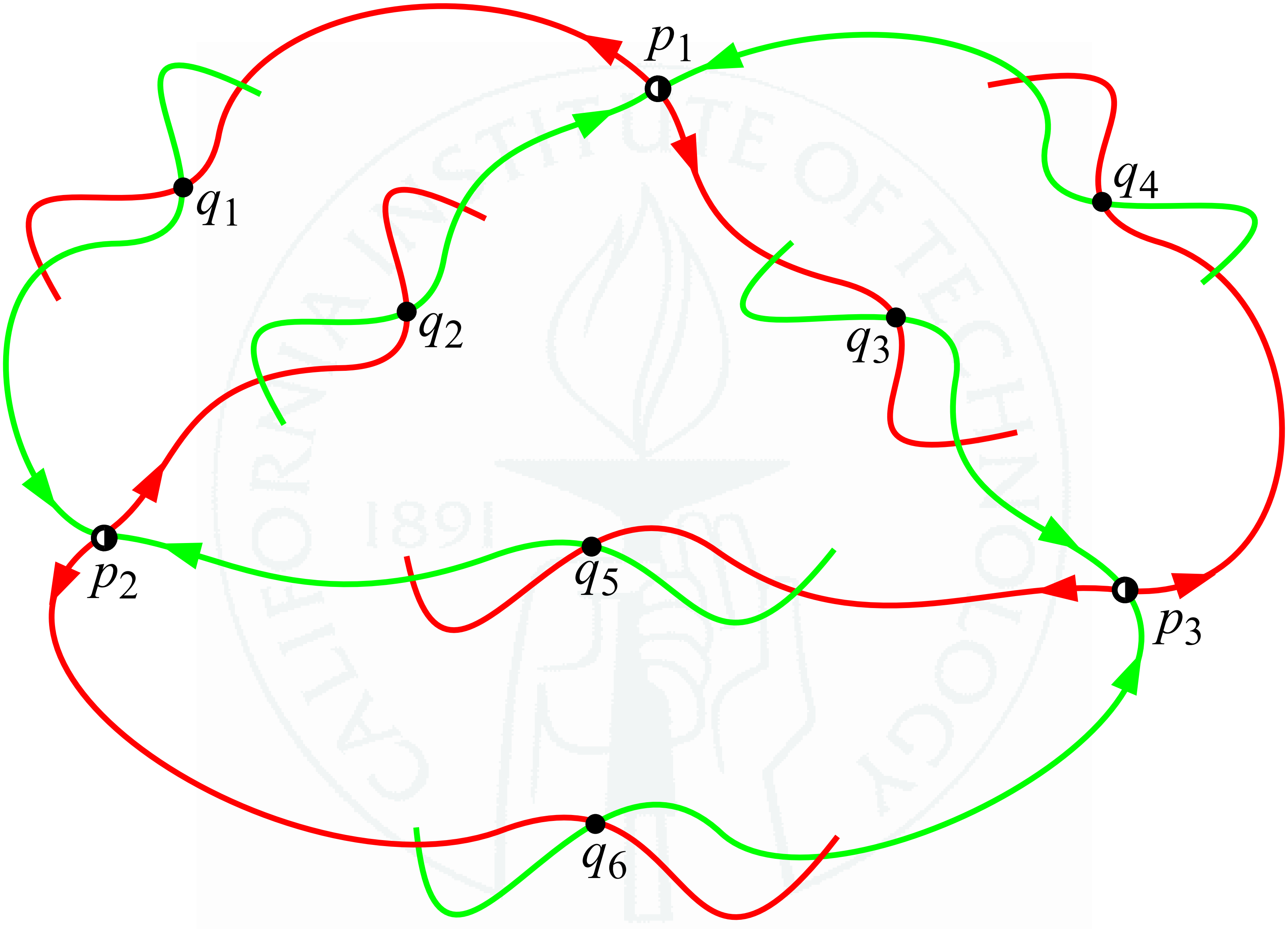
- Pieces of $W^u(p_i)$ and $W^s(p_i)$ partition Σ



Unstable and stable manifolds in **red** and **green**, resp.

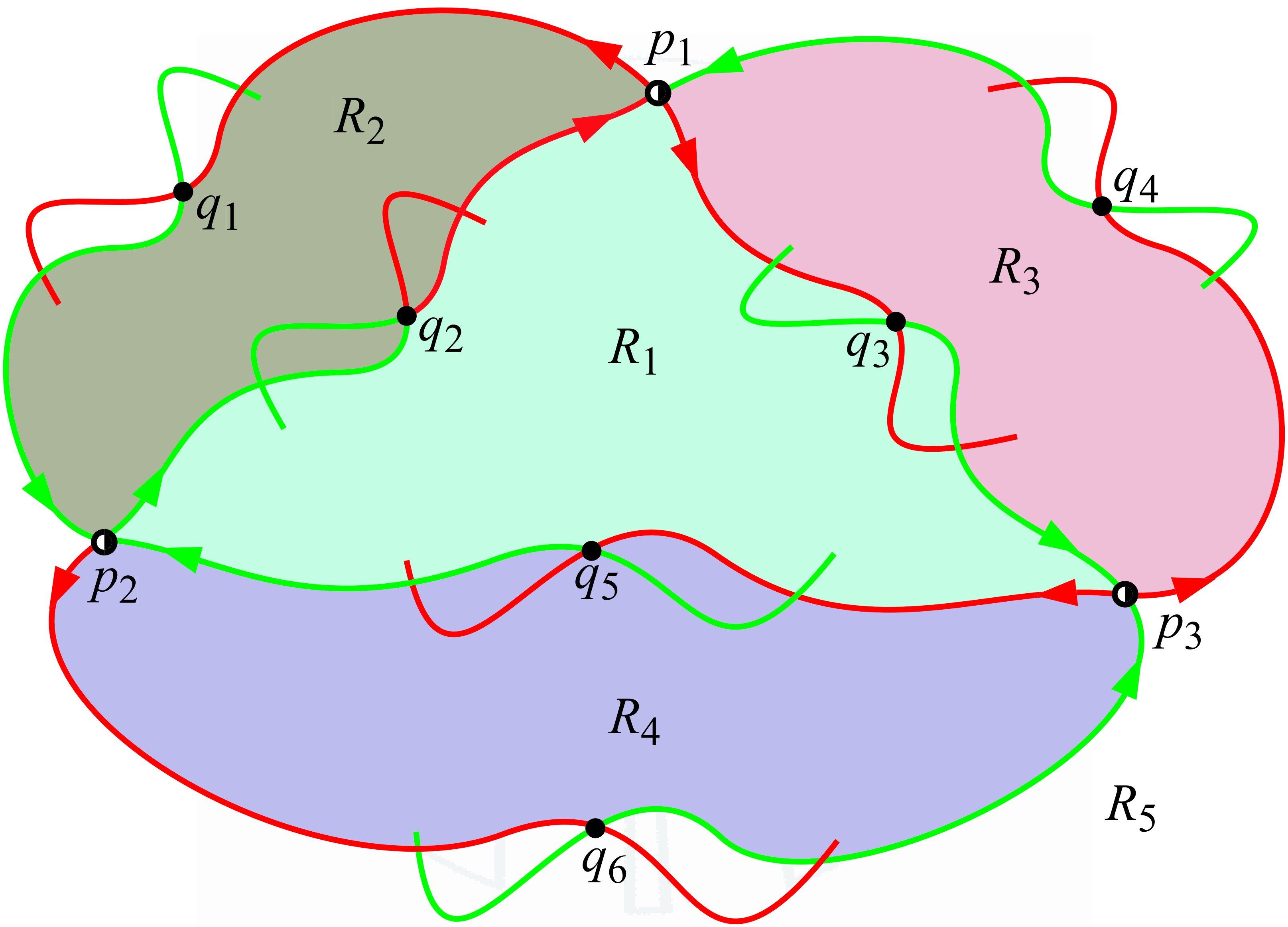
Lobe Dynamics: Partition Σ

- Intersection of unstable and stable manifolds define **boundaries**.



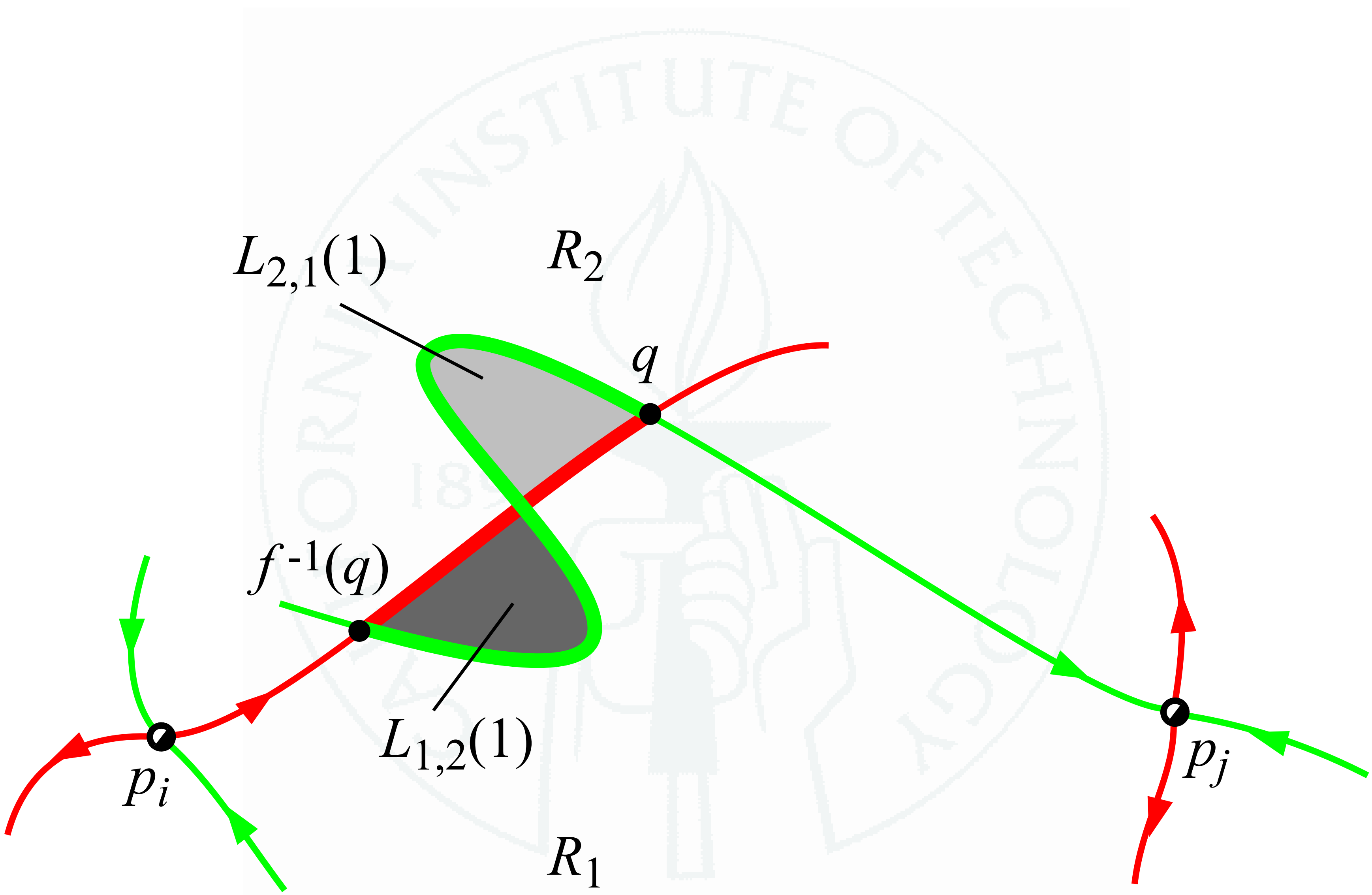
Lobe Dynamics: Partition Σ

- These boundaries divide phase space into **regions**, $R_i, i = 1, \ldots, N_R$



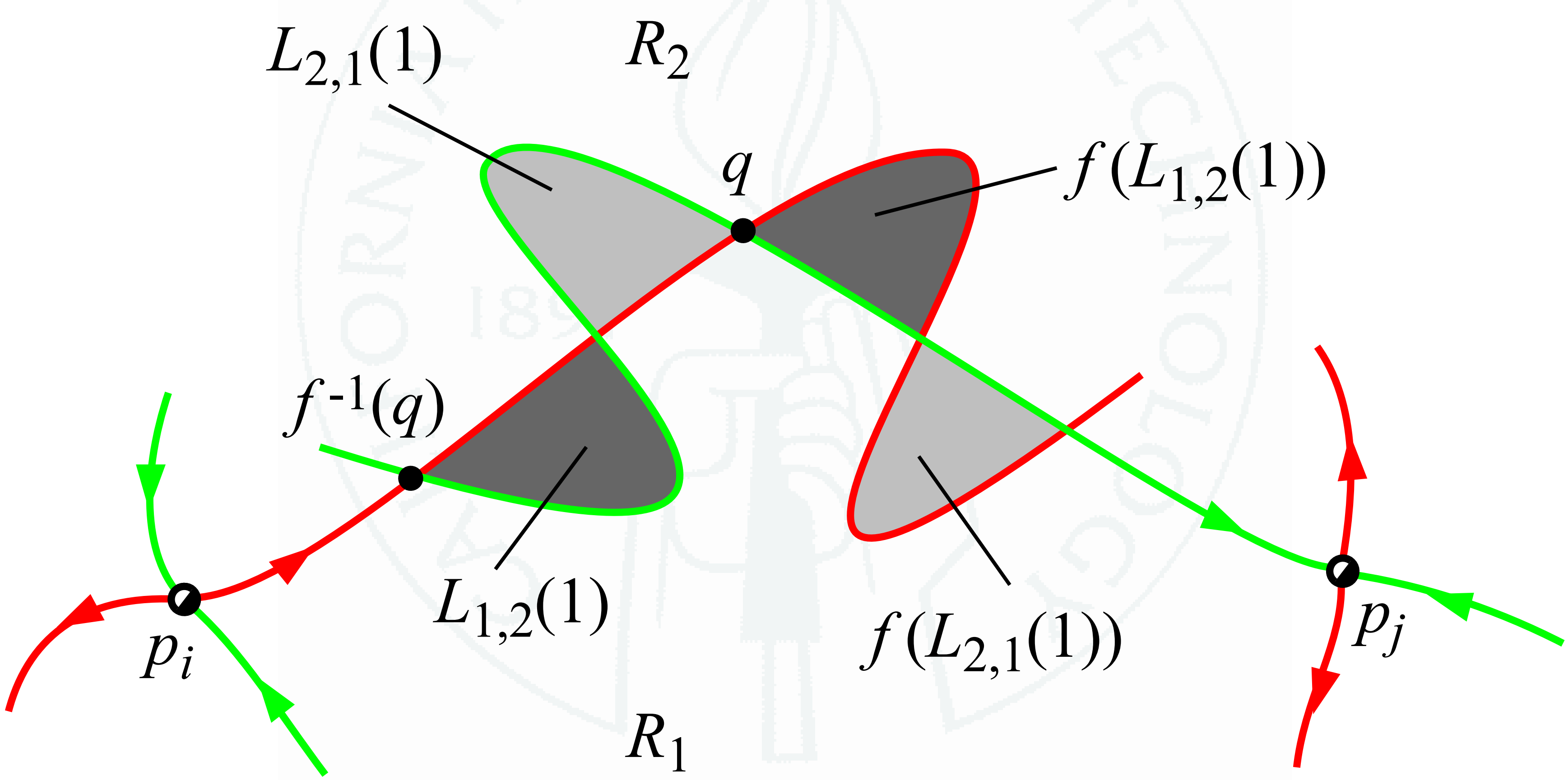
Lobe Dynamics: Turnstile

- $L_{1,2}(1)$ and $L_{2,1}(1)$ are called a **turnstile**



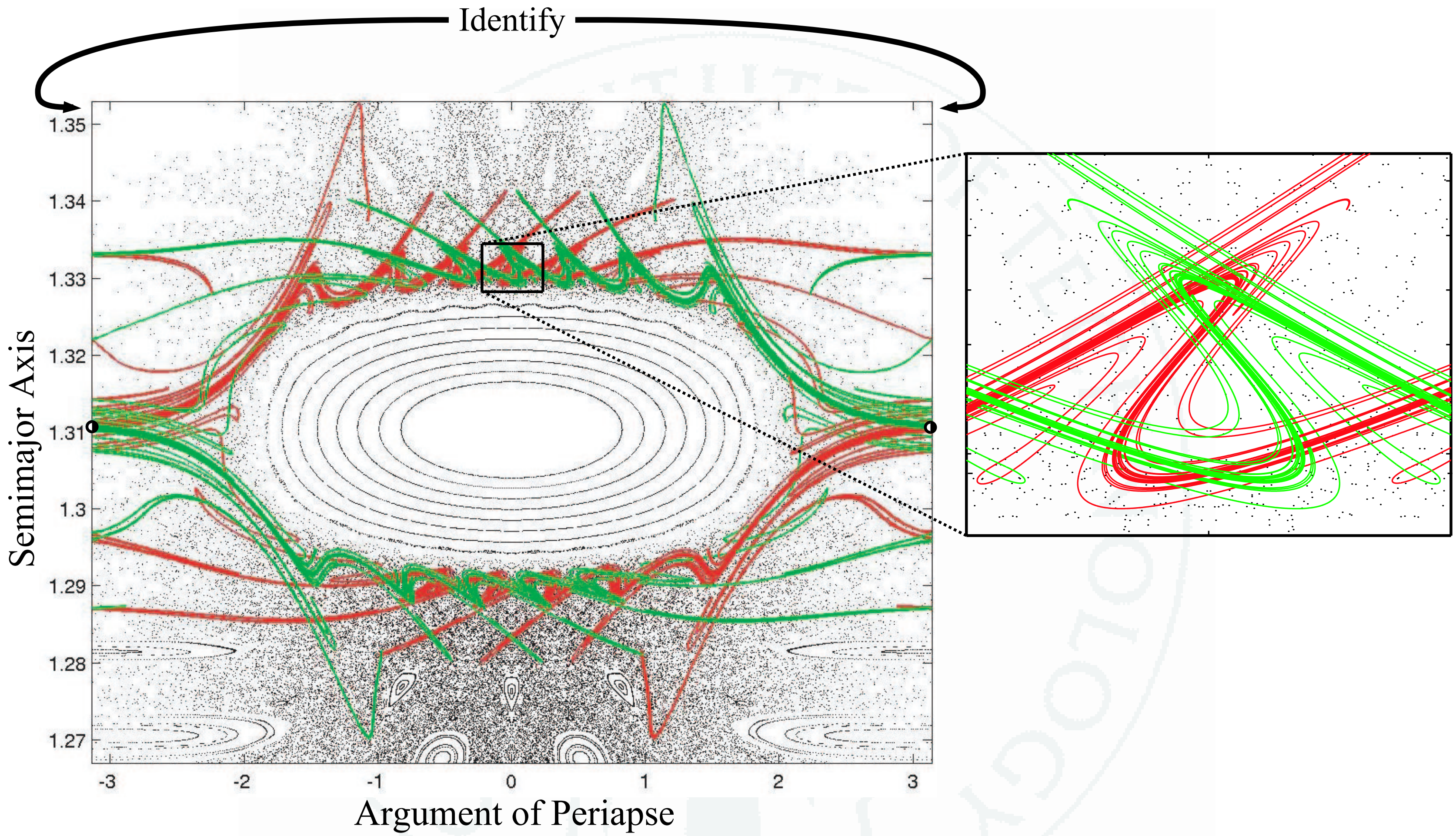
Lobe Dynamics: Turnstile

- They map from entirely in one region to another under one iteration of f



Move Amongst Resonances

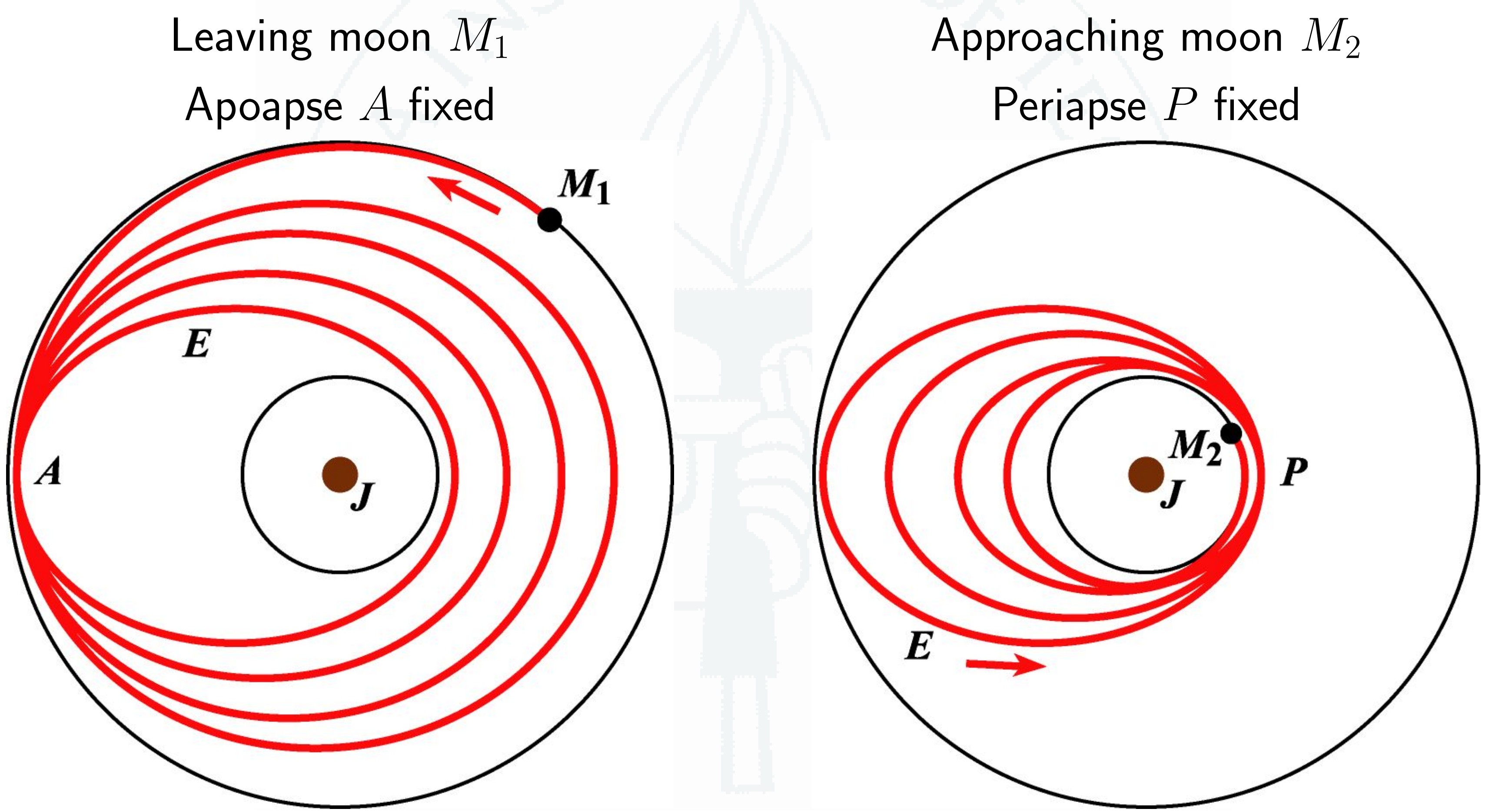
◦ Numerics: regions and lobes can be efficiently computed (MANGEN).



Unstable and stable manifolds in **red** and **green**, resp.

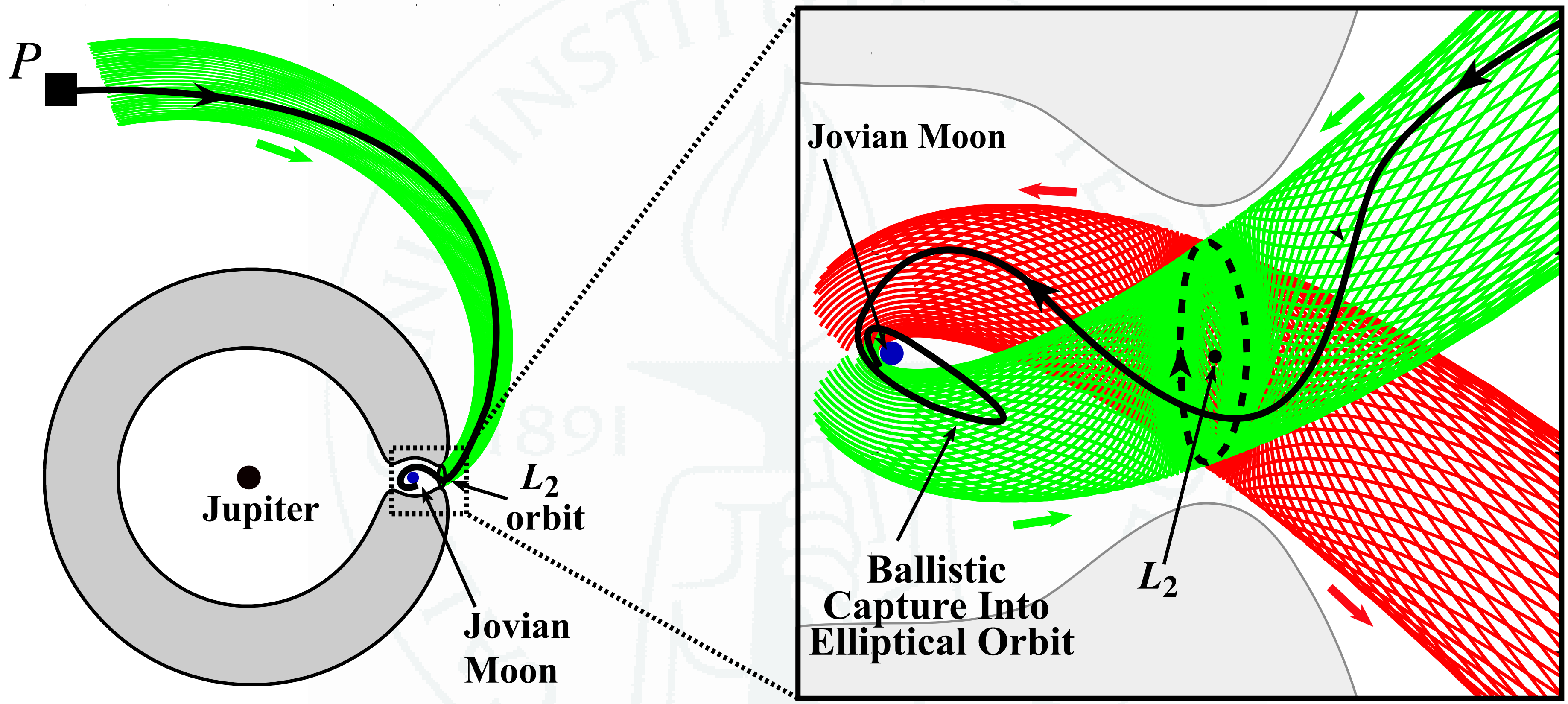
Inter-Moon Transfer

- Resonant gravity assists with outer moon M_1
- When periapse close to inner moon M_2's orbit is reached, J-M_2 system dynamics "take over"



Ballistic Capture

- Final phase of inter-moon transfer $\longrightarrow$ enter tube leading to ballistic capture



Tube leading to ballistic capture around a moon (seen in rotating frame)

Resulting Trajectory

- $\Sigma_i \Delta v_i = 22$ m/s (!!!), but flight time $\approx$ 3 years