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Control & Dynamical Systems

Trade-Off Between Fuel and Time Optimization

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Interplanetary Mission Design

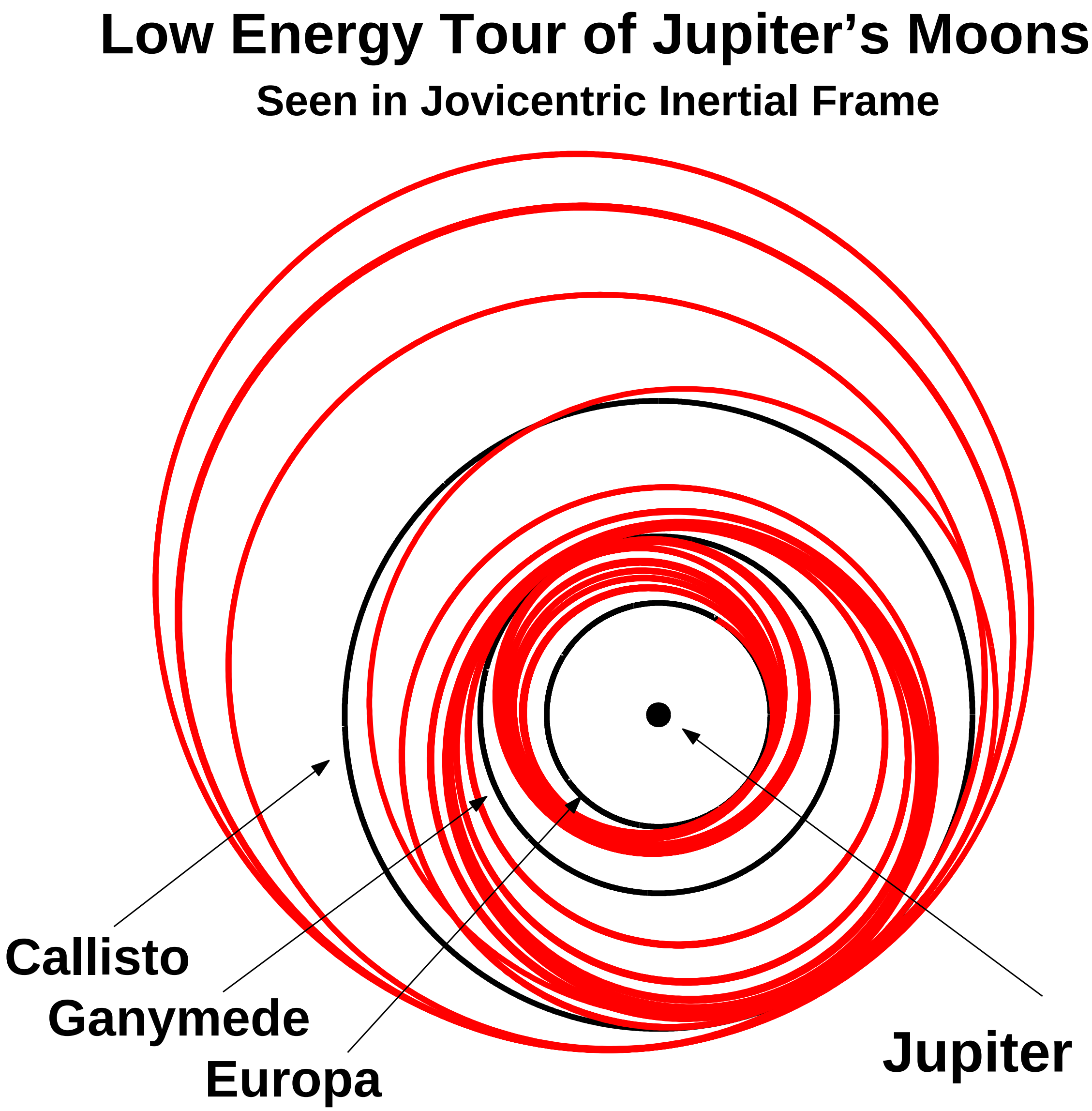
- ***Use natural dynamics for fuel efficiency***
  - **Dynamical channels** connecting planets and moons
  - Trajectory generation using invariant manifolds in the 3-body problem suggests new numerical algorithms for interplanetary missions
- ***How to balance fuel efficiency with reasonable flight times?***
  - Gravity assists, ballistic captures can take a long time
  - Short flight times important for challenging missions, e.g. *Multi-Moon Orbiter* to multiple Jovian moons

Multi-Moon Orbiter

- ***Orbit each moon in a single mission***
  - Other Jovian moons are also worthy of study
    - All may have oceans, evidence from *Galileo* suggests

Multi-Moon Orbiter

- ΔV is low (~ 20 m/s), but flight time $\sim$ 4 years



Fuel vs. Time Trade-Off

- ***Motivating Example***
- ***Earth to Moon Trajectories***
  - Consider a transfer from Earth orbit to lunar orbit
  - Previously addressed by Belbruno and Miller, where nonlinear n-body effects were used to lower ΔV at the expense of a longer time of flight compared to Hohmann
  - A good example problem for seeking efficient numerical algorithms to find fuel vs. time trade-off, applicable for many other situations

Fuel vs. Time Trade-Off

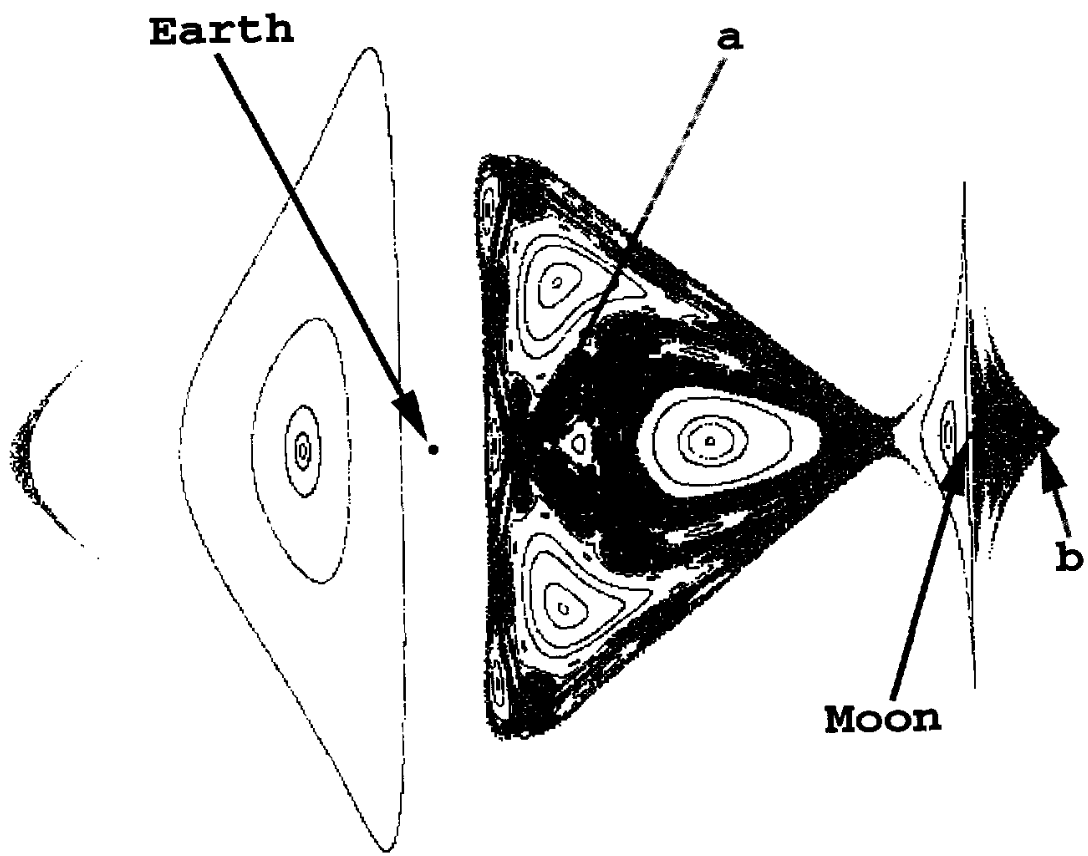
- Use planar circular restricted 3-body model
  - Consider the effect of only the Earth and Moon
- Compare with earlier methods; Hohmann, Bollt and Meiss, Schroer and Ott
  - Hohmann: simple and fast, but fuel-expensive
  - Remove recurrent loops in a chaotic trajectory
  - Target passes between mean motion resonances
- We will give some background on these methods

Some Background

- ***Starting/final orbital parameters***
  - Starting orbit: 59559 km circular Earth orbit
  - Final orbit: lunar orbit with perilune 13970 km
- ***Transfer trajectory***
  - Classical method: Hohmann transfer
    - Two maneuvers: ΔV_1 and ΔV_2, both large
  - Total $\Delta V = 1220$ m/s, TOF $= 6.6$ days
  - **First Goal:** use the same orbital parameters, but lower the total ΔV
  - **Second Goal:** keep time of flight reasonable

Some Background

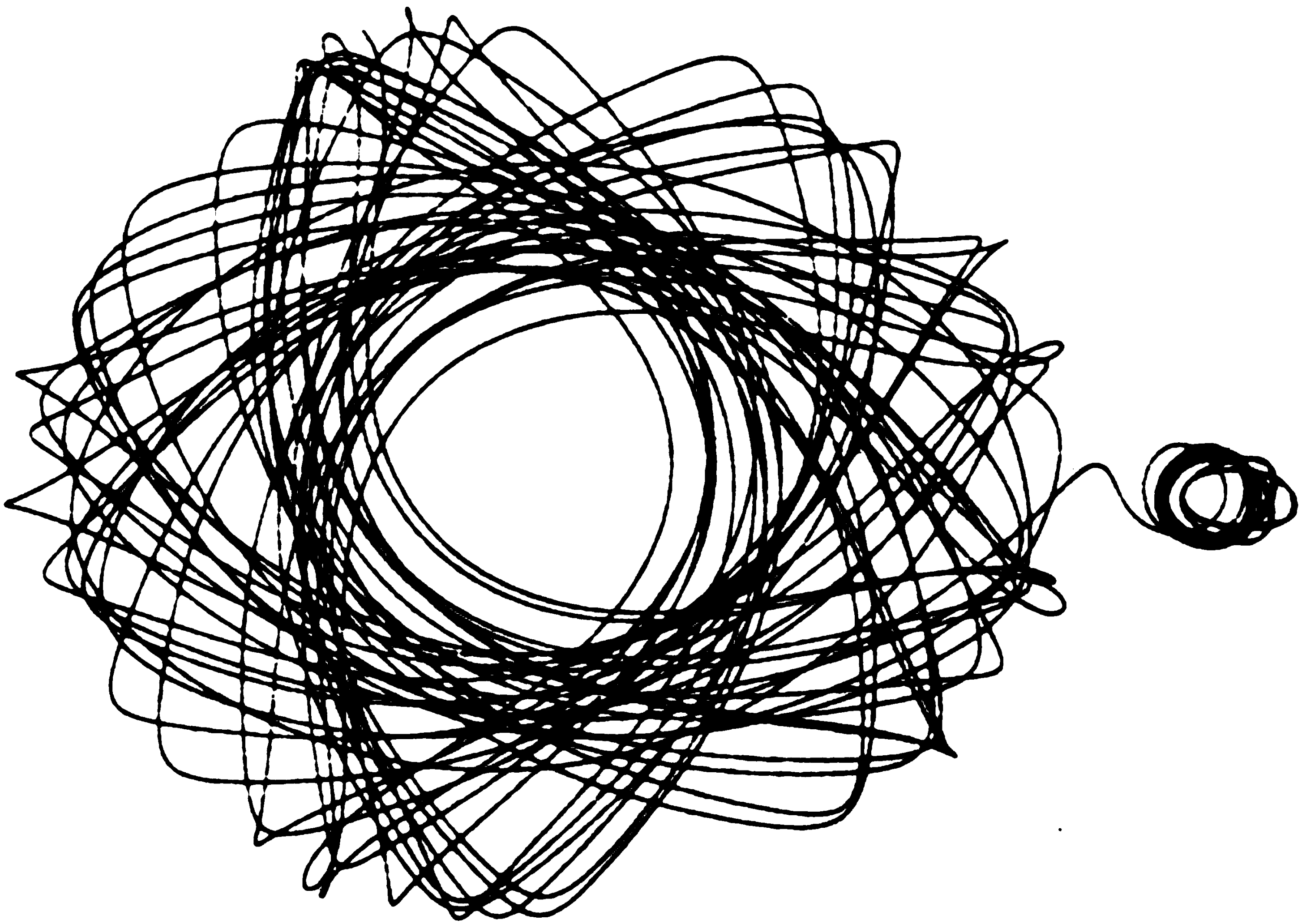
- Bollt & Meiss [1995] : target through recurrence
  - Find chaotic solution for fixed energy, remove recurrent loops with very small ΔV's



Poincare section: chaotic and regular motion intermixed.

Some Background

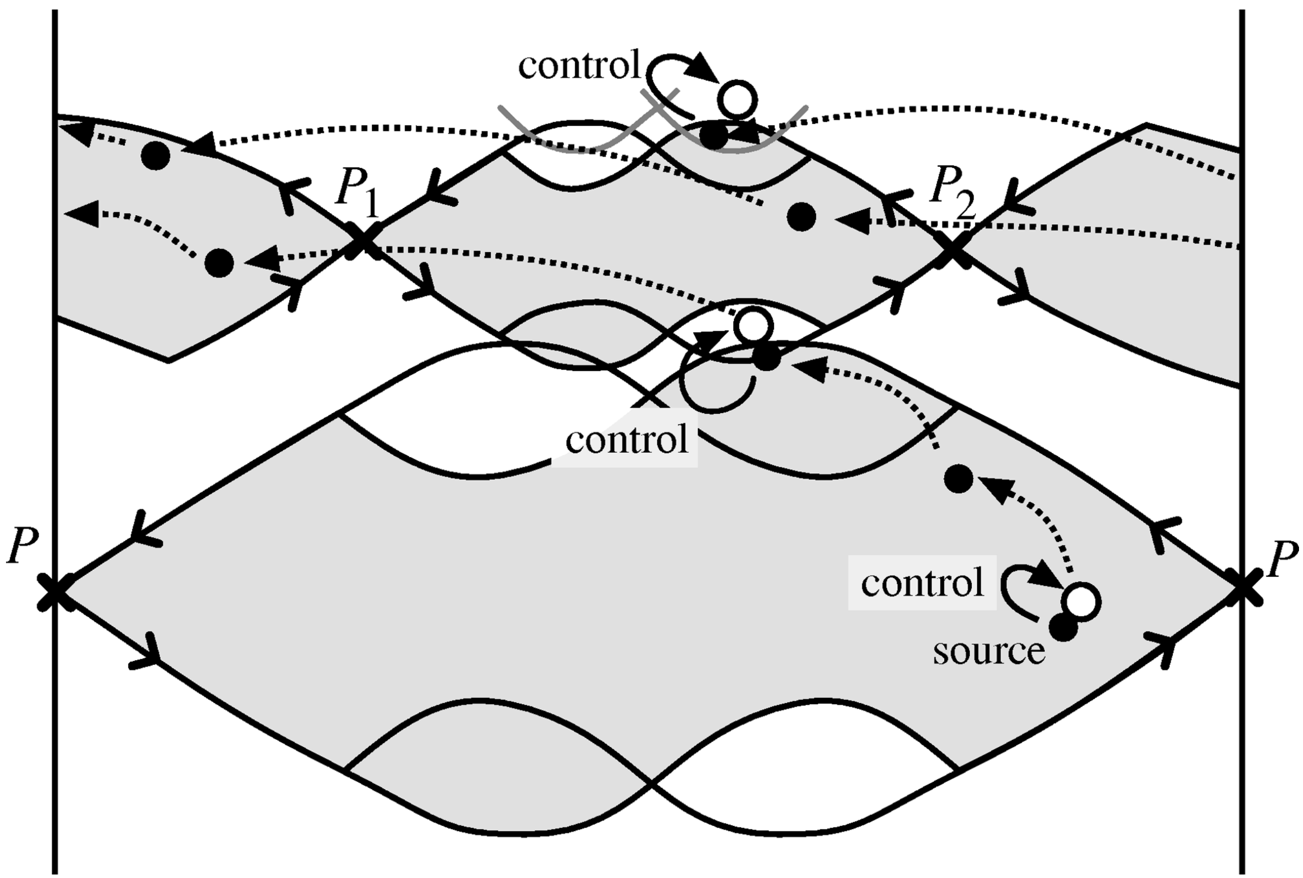
- Trajectory found : 750 m/s, 748 days



Spacecraft trajectory in the rotating frame

Some Background

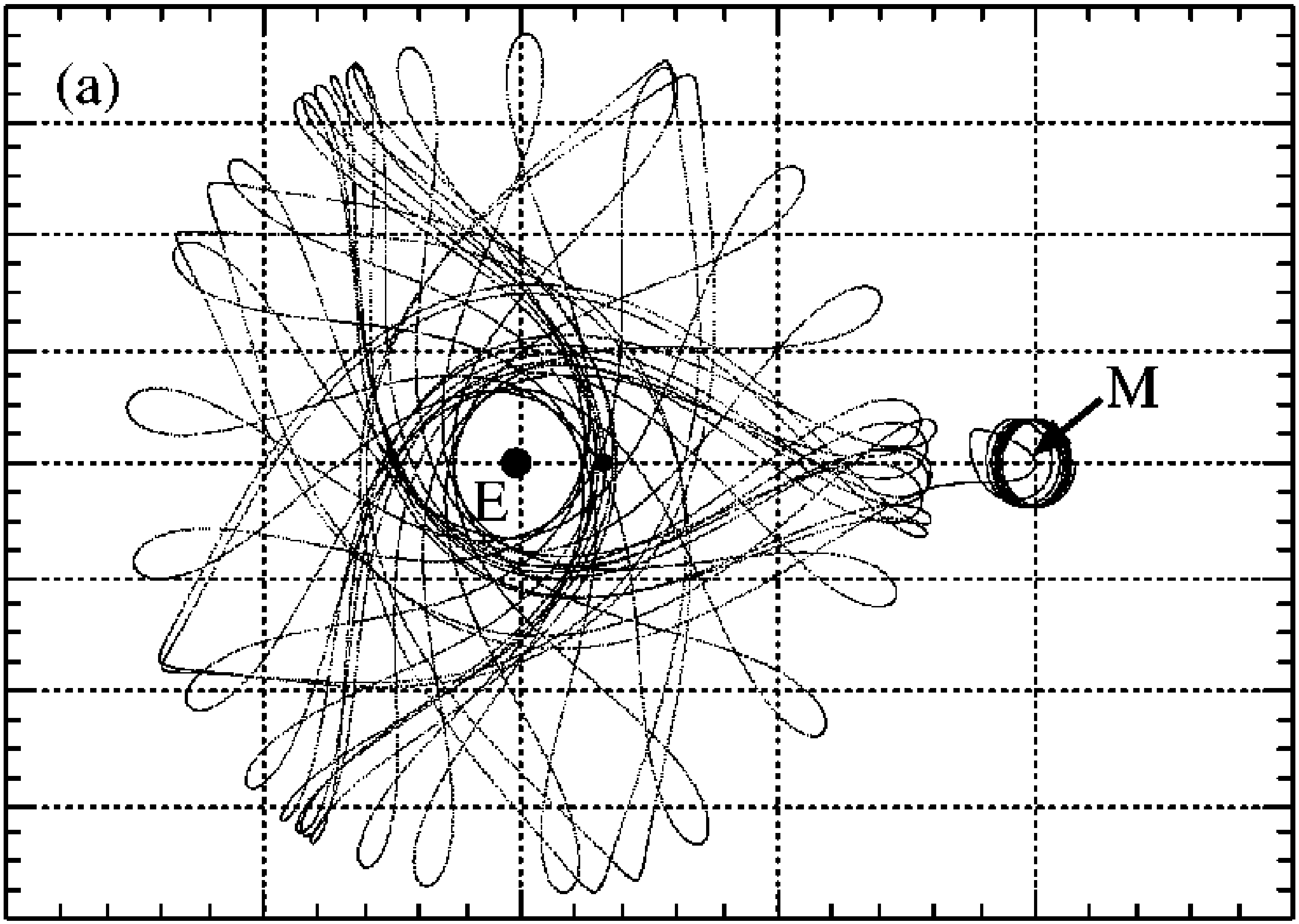
- Schroer & Ott [1997] : target passes btwn resonances
  - Leap-frog between a series of resonances using very small ΔV's until trajectory reaches the moon



Resonances: target the passes between them with small controls.

Some Background

- Trajectory found : 749 m/s, 378 days



Spacecraft trajectory in the rotating frame

Our Approach

Our approach

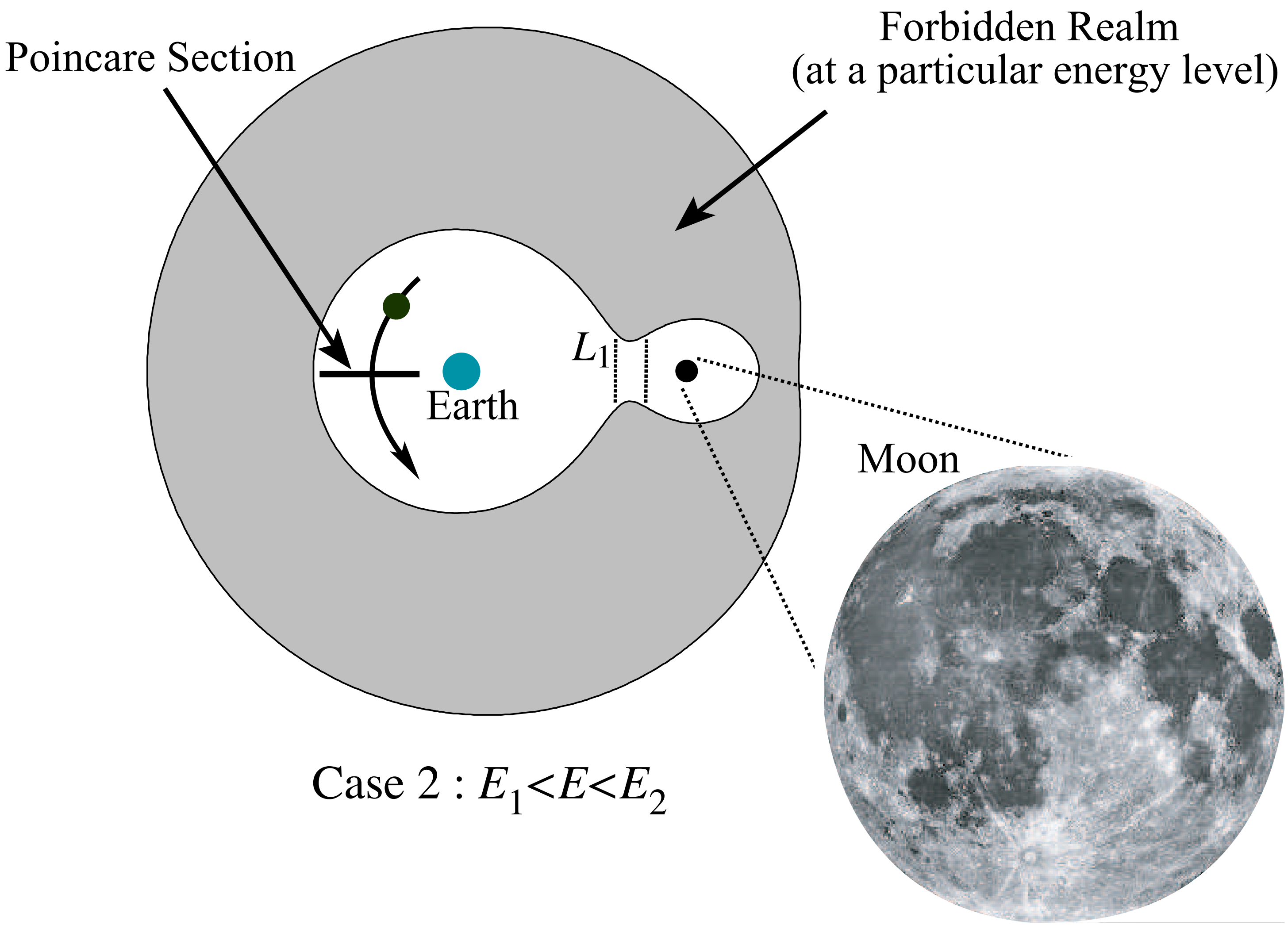
- Take full advantage of all known phase space structures: seek intersections between **resonances** and **tubes** leading to ballistic capture by the Moon

Building blocks

- **Appropriate energy:** transfer at three-body energy where resonances and tube dynamics are important
- **Poincare section:** reduces problem to 2D
- **Resonant gravity assists:** maximize change in orbit during every lunar encounter
- **Tube dynamics:** get captured by the Moon

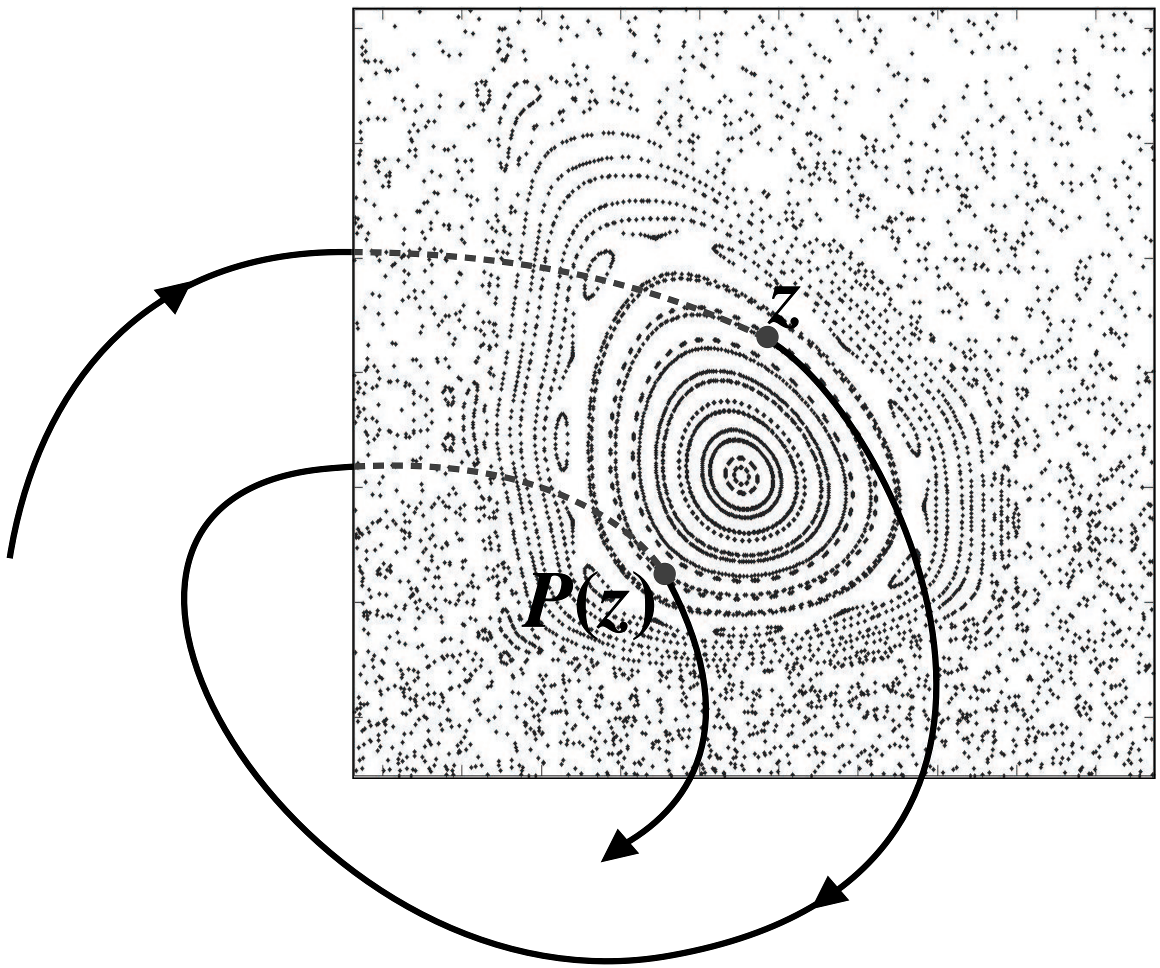
Appropriate Energy

- Transfer to occur on a single 3D energy surface
- Poincaré surface-of-section: motion on 2D map



Poincaré Surface of Section

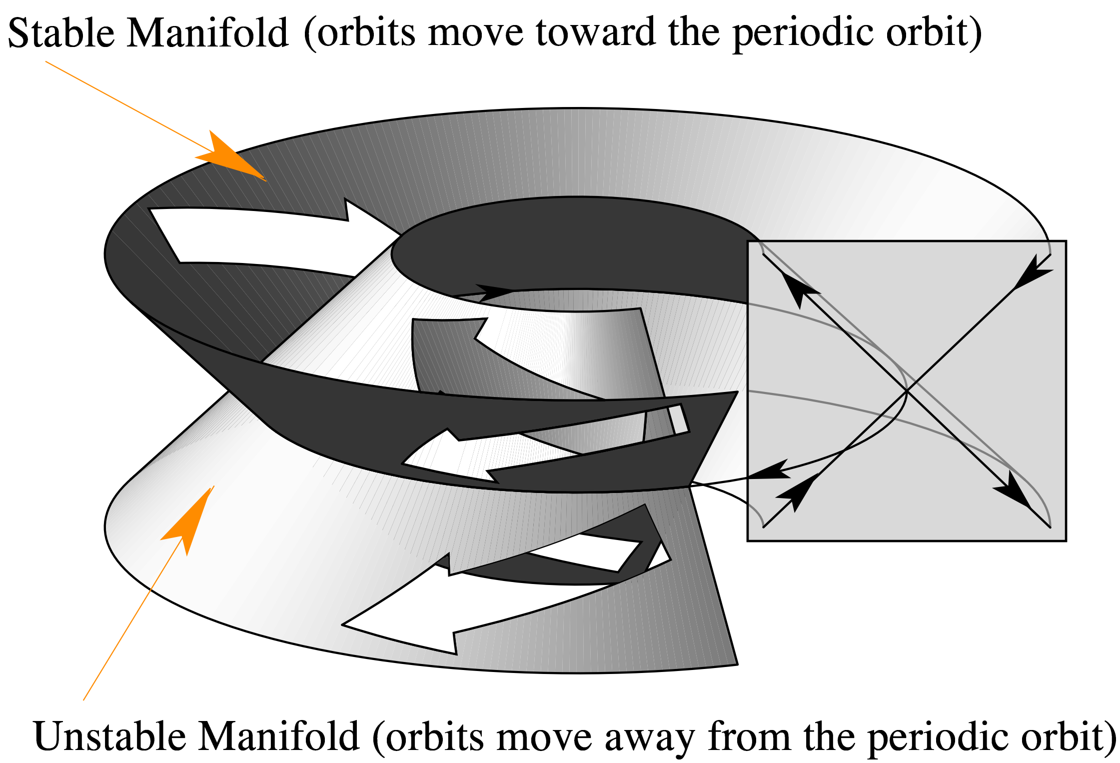
- Study Poincaré surface of section at fixed energy E, reducing system to a 2-dimensional area preserving map.



Poincaré surface of section

Resonant Gravity Assists

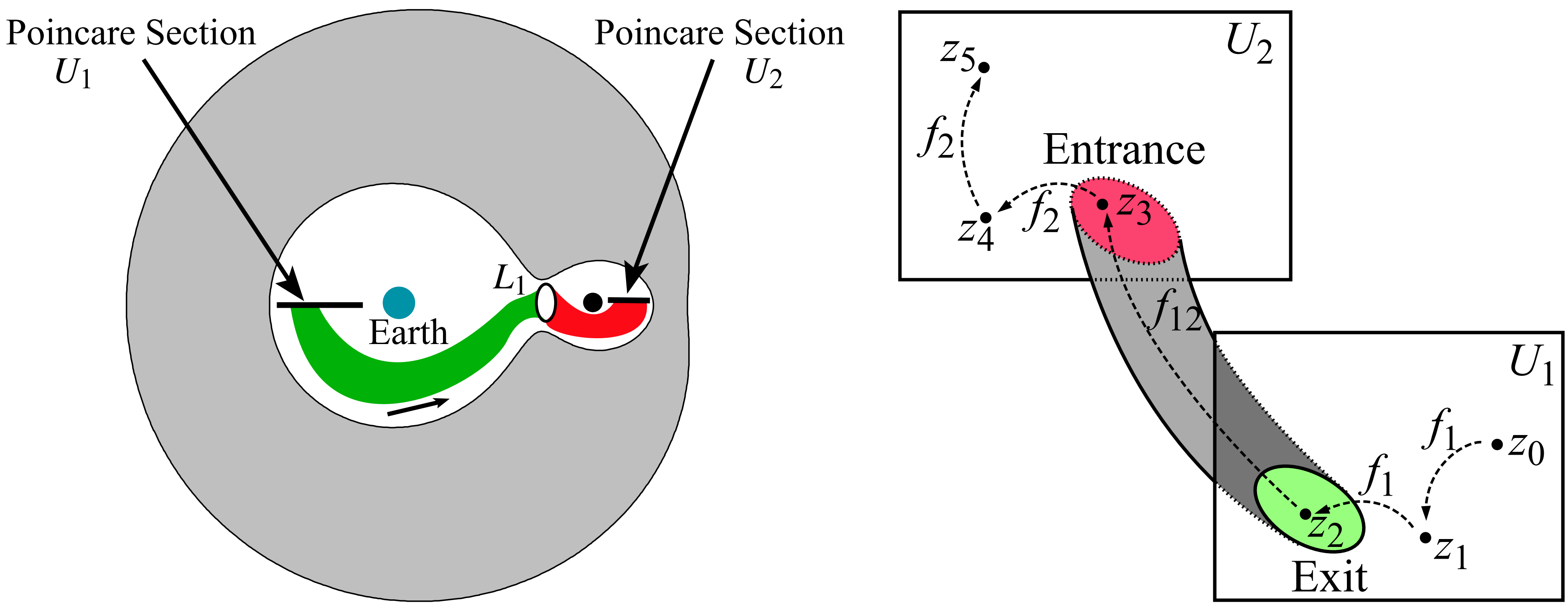
- Unstable resonances: Periodic orbits forming a dynamical "back-bone," via their stable/unstable manifolds.
- Physically, these manifolds correspond to orbits undergoing repeated close encounters with the Moon.



Unstable resonances and their manifolds.

Tube Dynamics

- Jump to the Moon's vicinity via invariant manifold tubes associated to a periodic orbit about L_1.
- Track orbits via **exits/entrances** on Poincaré sections.



Tube dynamics: going from one Poincaré section to another.

Resonances and Tubes

Resonances and tubes are linked

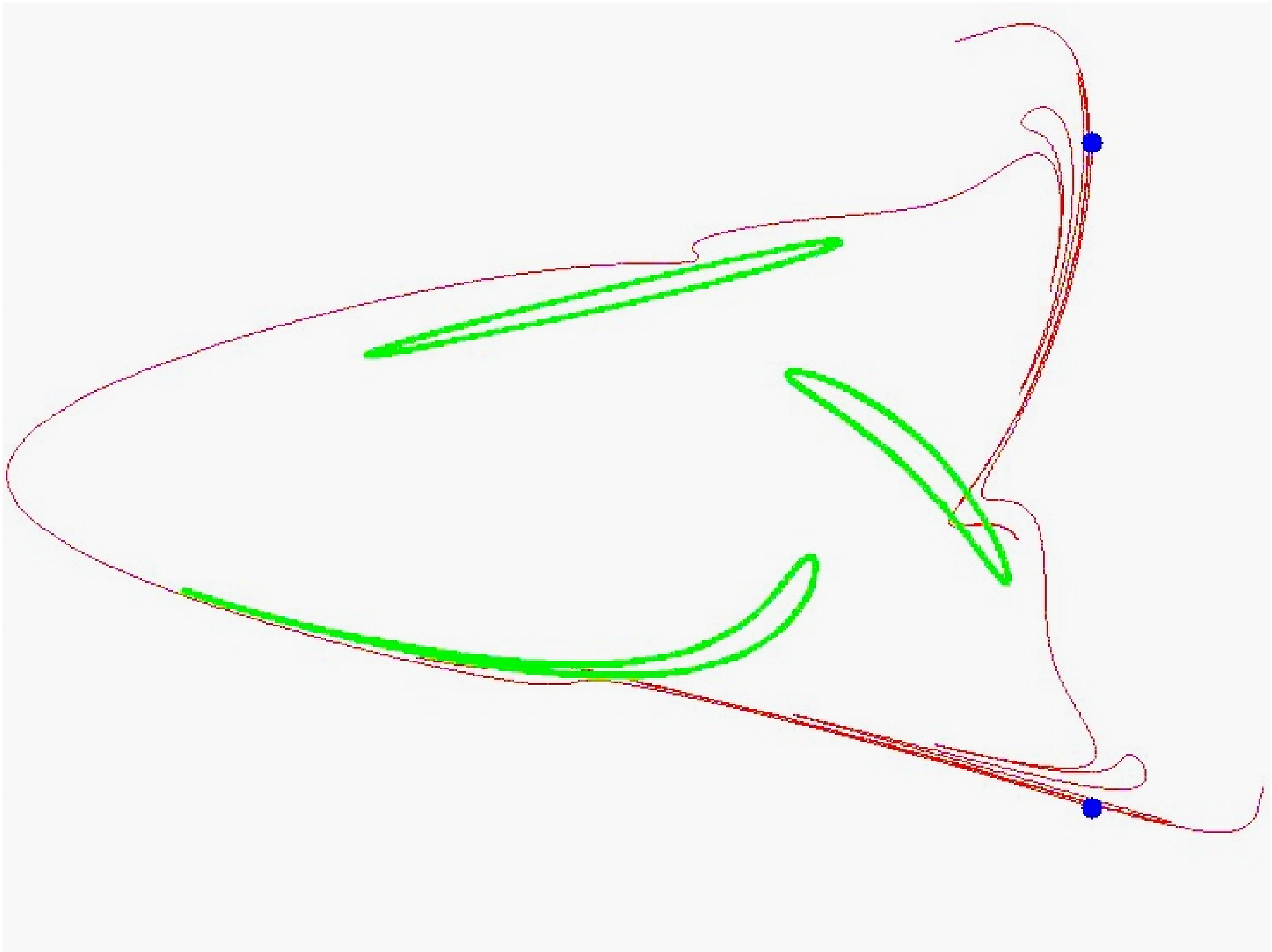
- It has been observed that the tubes of capture orbits are coming from certain resonances.
  - Koon, Lo, Marsden, Ross [2001]

Designing an efficient transfer

- First, from the starting Earth orbit, perform a ΔV placing the spacecraft on a trajectory near one of the resonances which is linked to capture tubes.
- Then perform small maneuvers to steer into a capture tube – none may be needed!

Resonances and Tubes

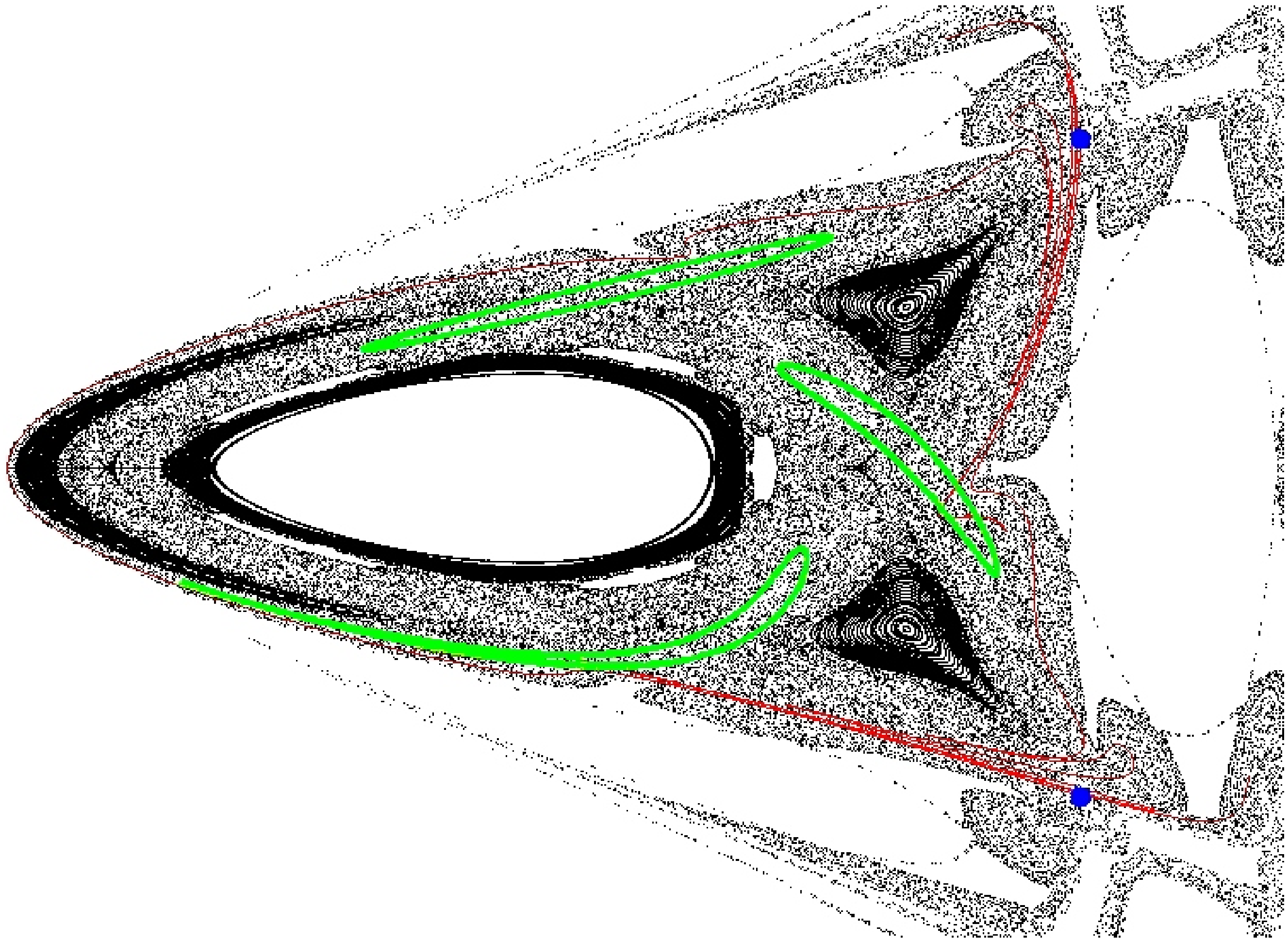
- Poincaré section: tube cross-sections are closed curves and resonance manifolds are windy curves.



Poincaré section showing tubes and resonances.

Resonances and Tubes

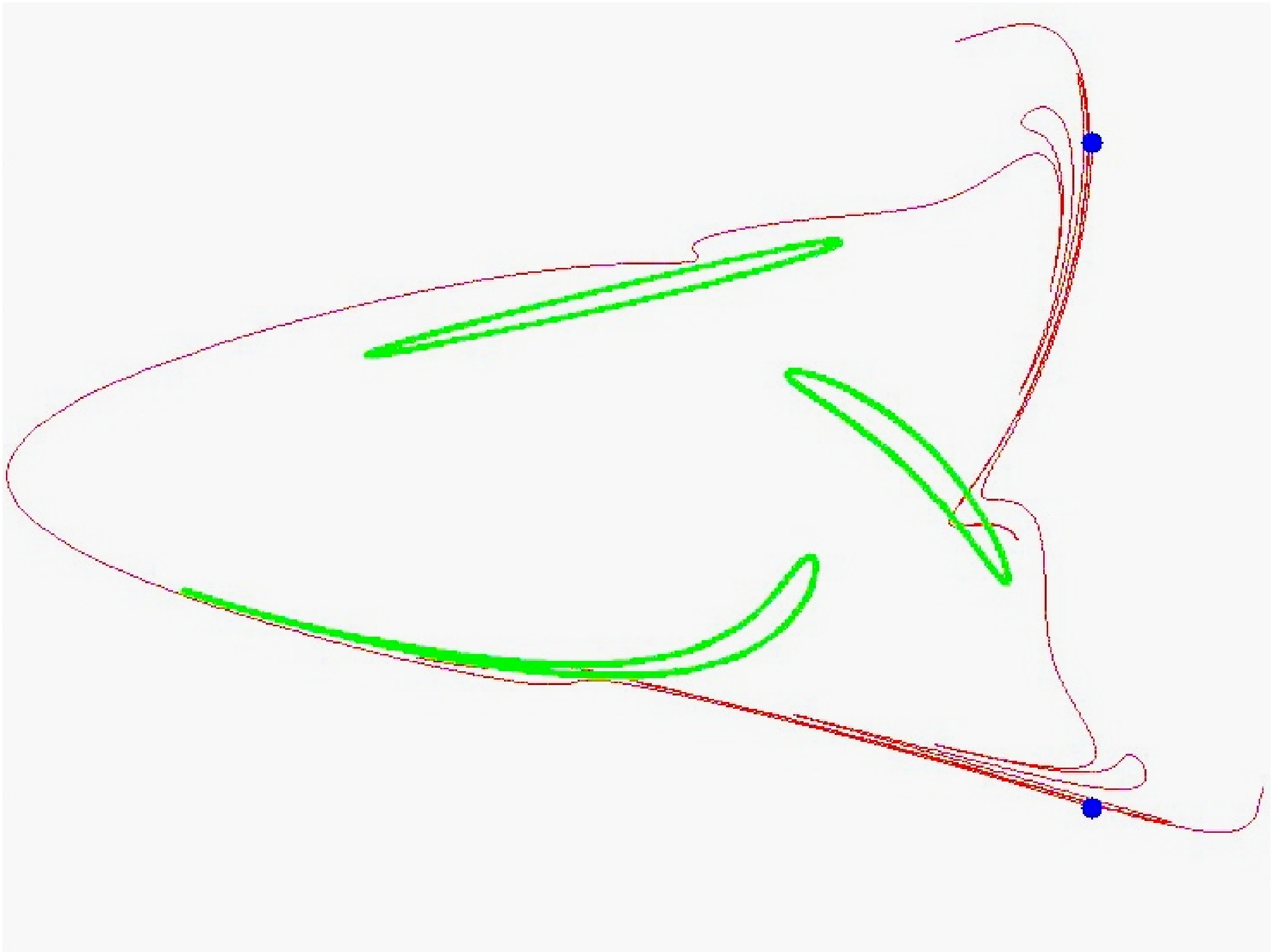
- Invisible structure: both of them reveal the structure in the "chaotic" part of phase space.



Poincaré section showing tubes and resonances and background points.

Resonances and Tubes

- Tubes and resonance manifolds intersect, i.e., there is a free transfer from the resonance to the Moon's vicinity.



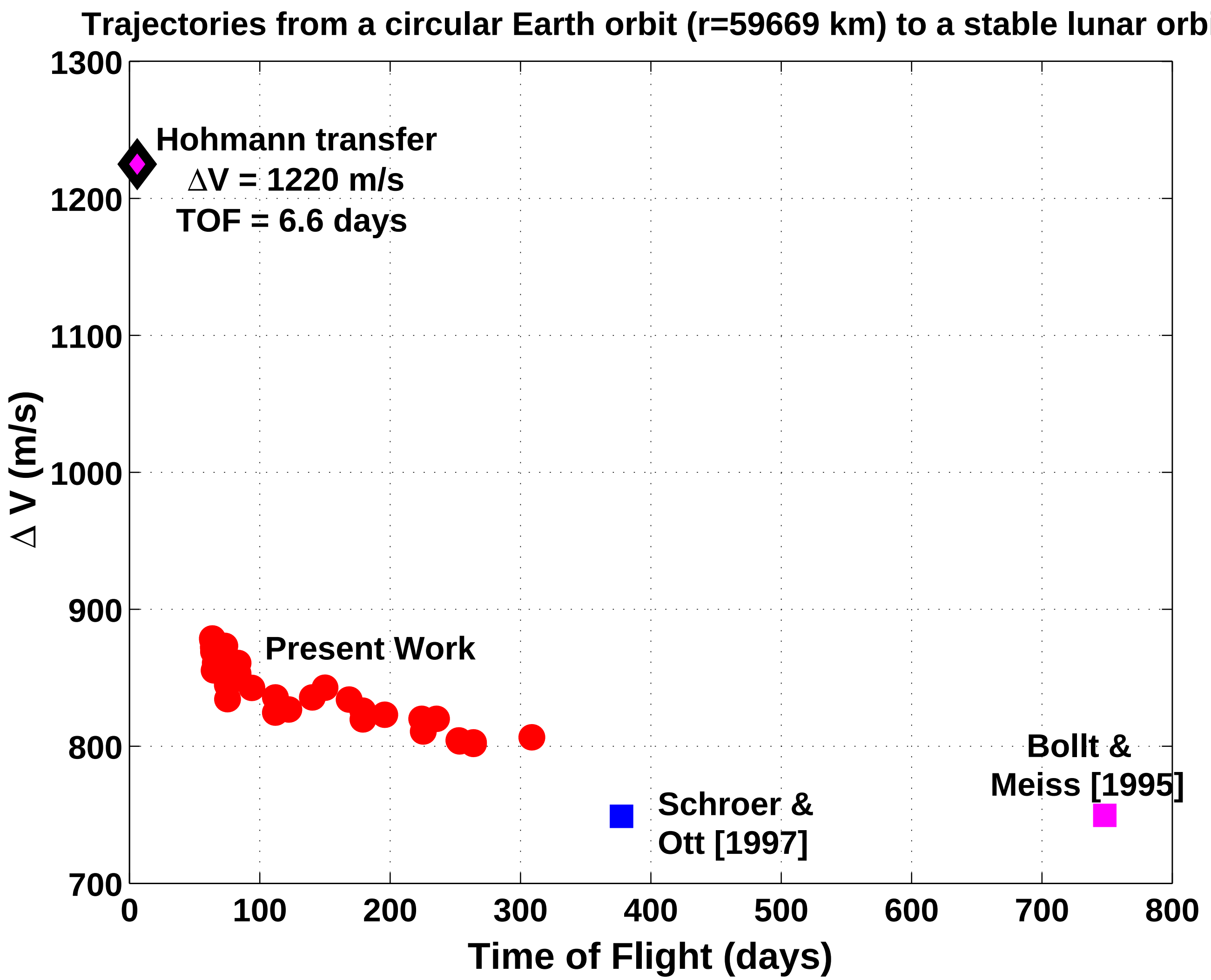
Poincaré section showing tubes and resonances.

Resonances and Tubes

- The free transfer corresponds to a flight time of over 250 days, but we can do better.
- The tubes and resonance manifold come “close” to intersecting in several places, i.e., a small ΔV can drastically cut out unnecessary flight time.
- By searching for ΔV’s of a particular size, one can systematically get a curve of ΔV vs. time of flight.

Resonances and Tubes

- **Results**: much shorter transfer times than previous authors for only slightly more ΔV



Resonances and Tubes

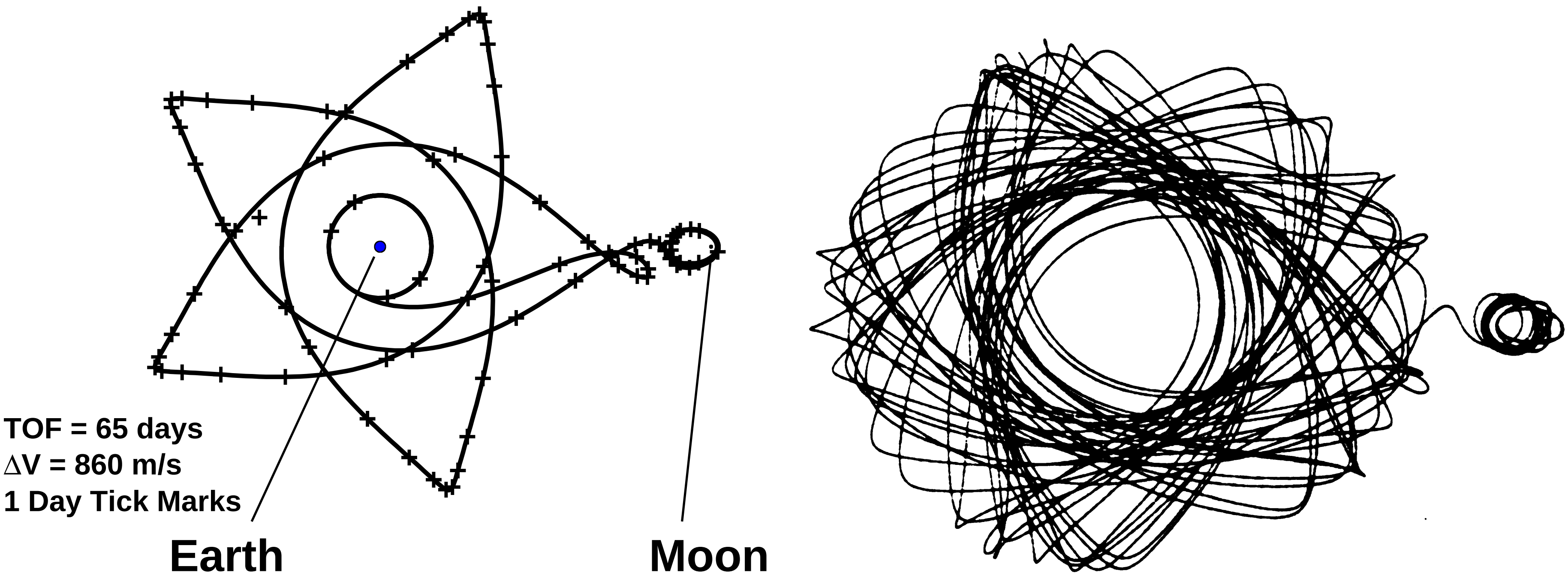
- Compare with Bollt and Meiss [1995]
  - A tenth of the time for only 100 m/s more

Current Result

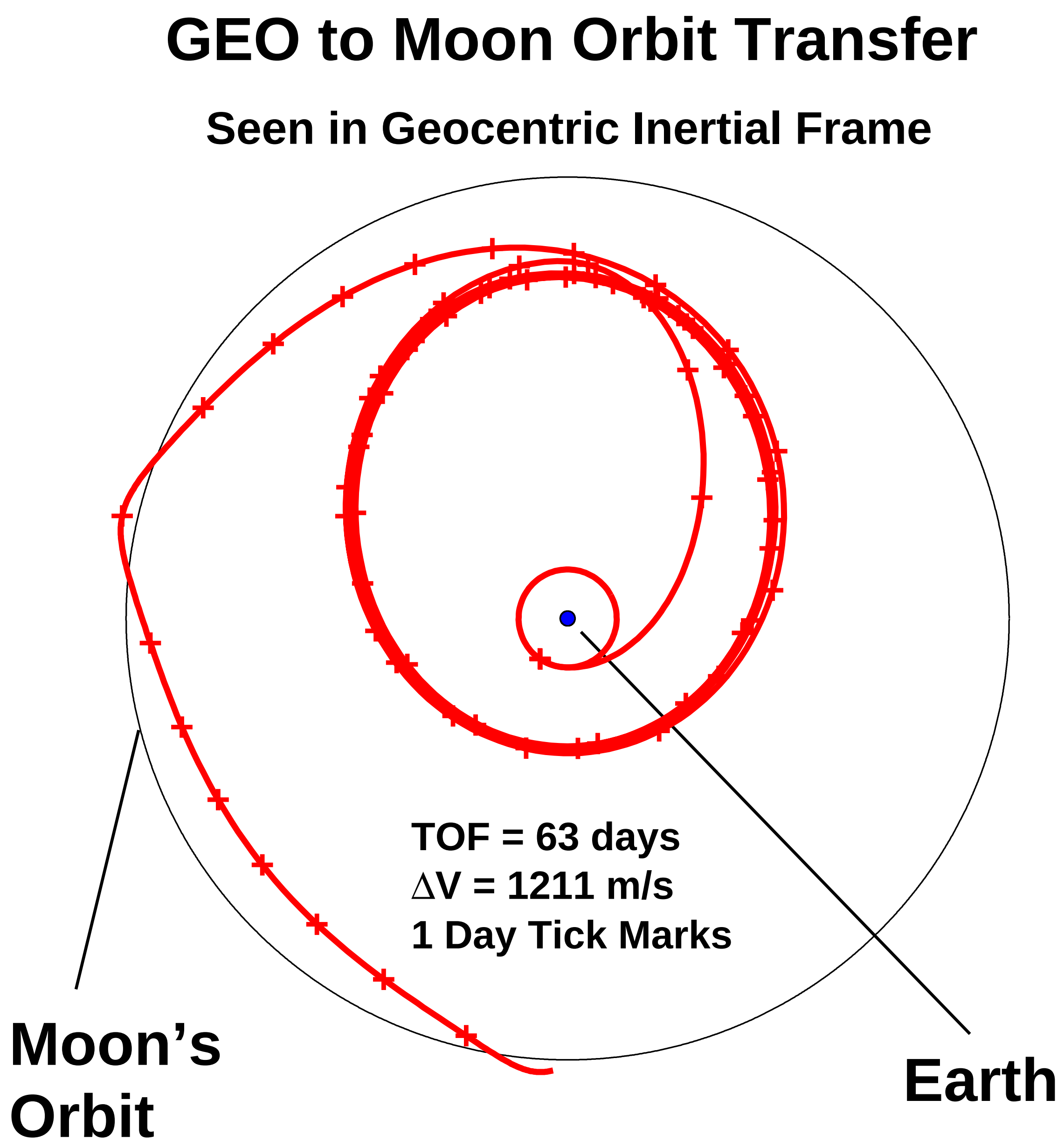
65 days, $\Delta V = 860$ m/s

Bollt and Meiss [1995]

748 days, $\Delta V = 750$ m/s



e.g., GEO to Lunar Orbit



e.g., GEO to Lunar Orbit

GEO to Moon - rotating frame

e.g., GEO to Lunar Orbit

GEO to Moon - inertial frame

References

- ***Trade-Off Between Fuel and Time Optimization***, in preparation.
- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2002] ***Constructing a low energy transfer between Jovian moons.*** *Contemporary Mathematics* 292, 129–145.
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- Koon, W.S., M.W. Lo, J.E. Marsden & S.D. Ross [2001] ***Resonance and capture of Jupiter comets.*** *Celestial Mechanics & Dynamical Astronomy* 81(1-2), 27–38.
- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2000] ***Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics.*** *Chaos* 10(2), 427–469.

For papers, movies, etc., visit the websites:

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