Tube Dynamics and Low Energy Trajectory from the Earth to the Moon in the Coupled Three-Body System

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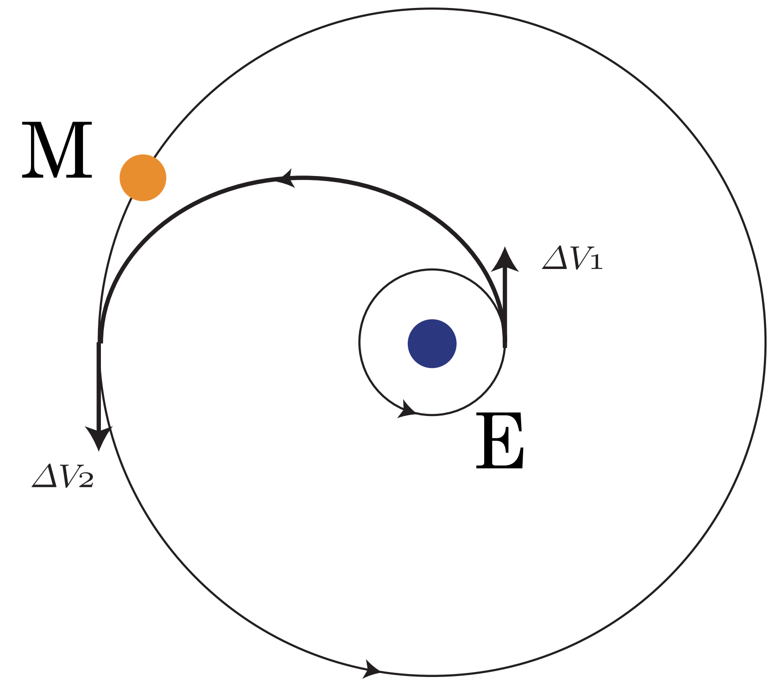
Hohmann transfer (2-body problem)

The elliptic orbit connecting with the low Earth orbit and the lunar orbit. The two impulsive maneuver are required.

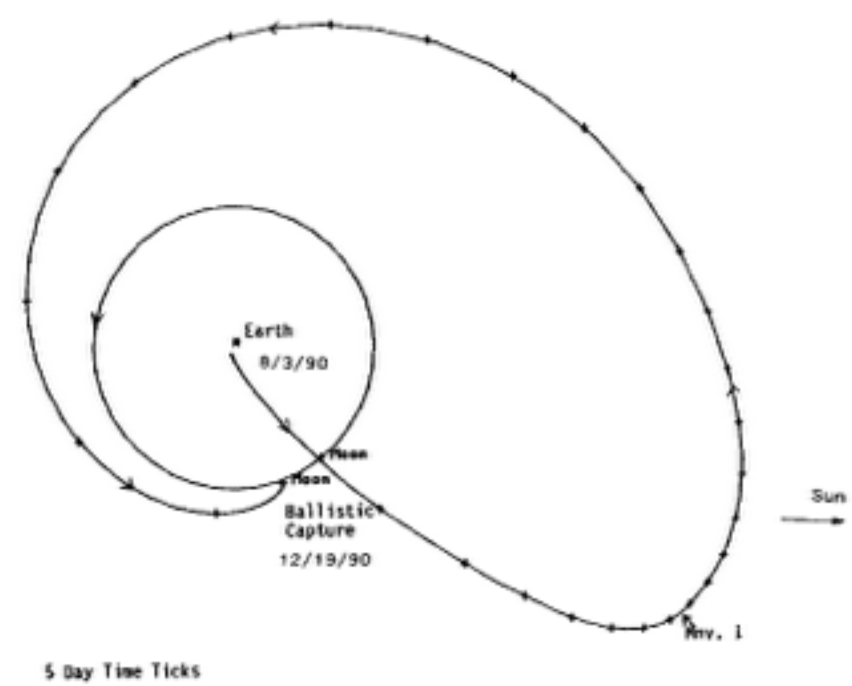
[Bate et al. (1971)]



Earth-Moon transfer with Sun-perturbation (4-body problem)

The Hiten transfer was established in the S-E-M-S/C 4-body problem by considering the Sun-perturbation and by employing the theory of Weak Stability Boundaries.

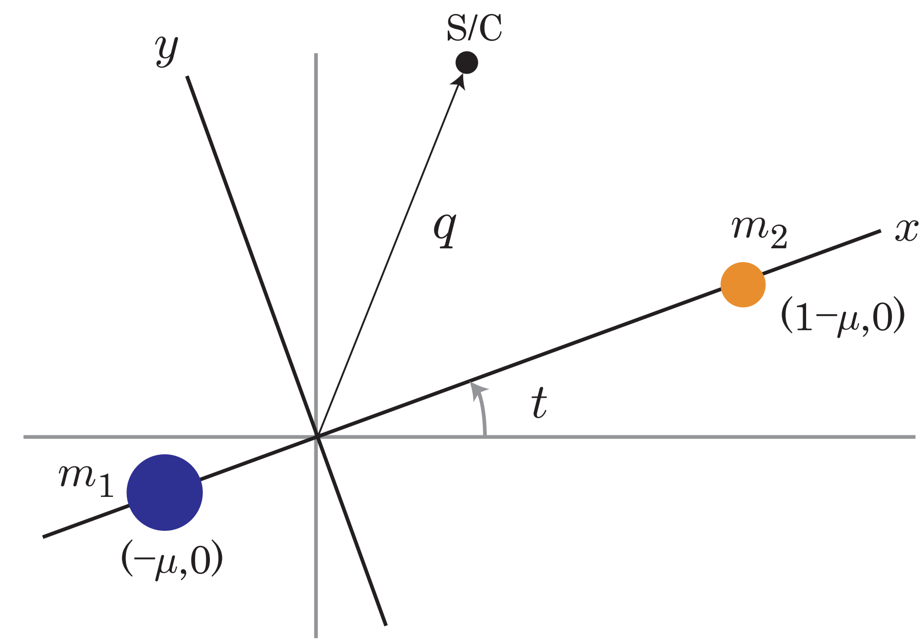
[Belbruno and Miller (1993)]



Hiten [Belbruno and Miller (1993)]

Coupled PRC3BS

The S-E-M-S/C 4-body problem is approximated to coupled two PRC3BS (S-E-S/C and E-M-S/C systems), and then the transfer is constructed based on the tube dynamics in the coupled system. [Koon et al. (2001)]



Mass ratio : $\mu = \dfrac{m_2}{m_1 + m_2}$

- **PRC3BP and Tube dynamics [Conley (1968)]**

- Equation of motion $q = (x, y)^T$

$$\ddot{q} - 2\,\tilde{\Omega}\,\dot{q} - q = -\frac{1-\mu}{|q-q_1|^3}(q-q_1) - \frac{\mu}{|q-q_2|^3}(q-q_2)$$

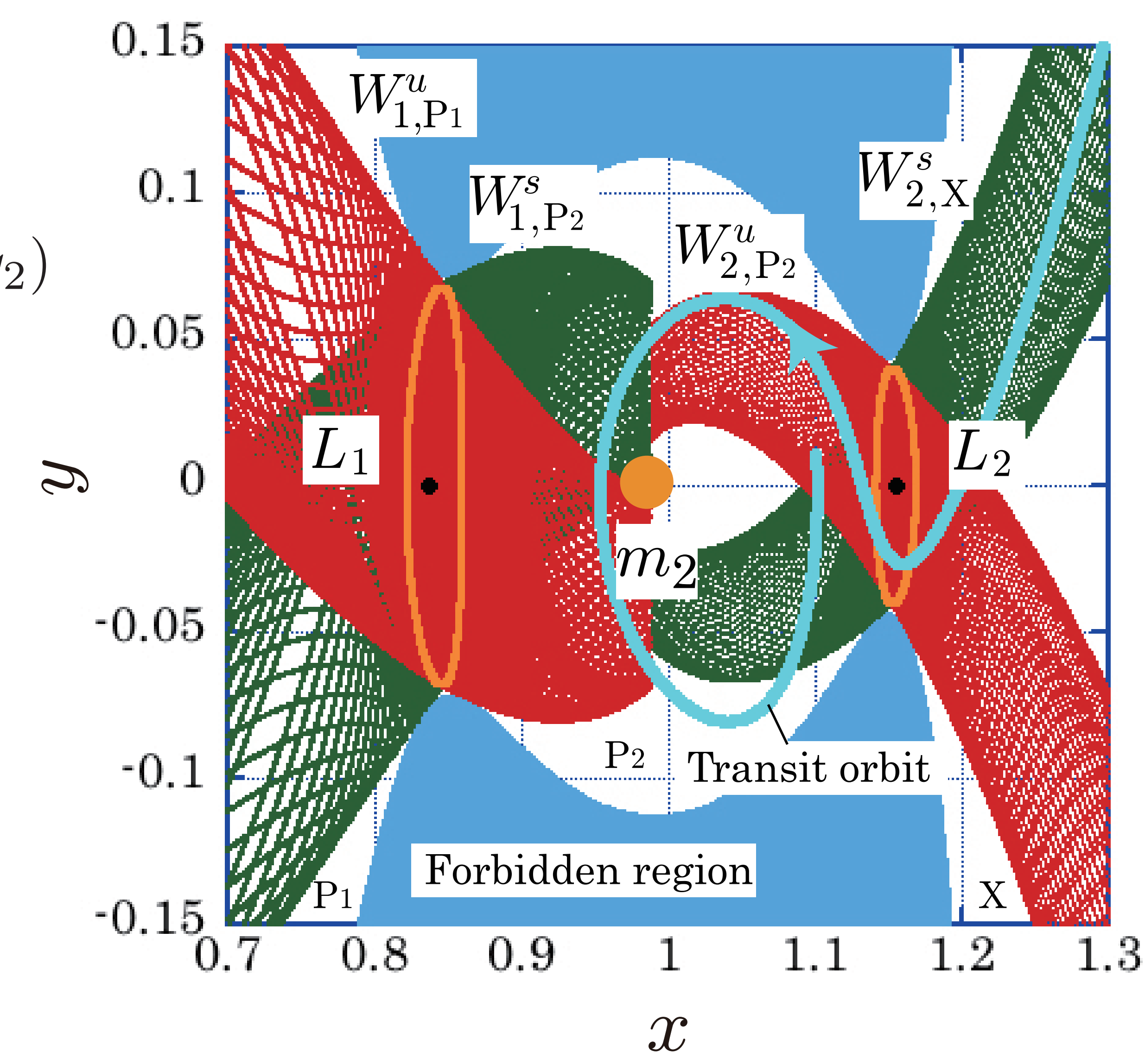
- Energy

$$E = \frac{1}{2}|\dot{q}|^2 - \frac{1}{2}|q|^2 - \frac{1-\mu}{|q-q_1|} - \frac{\mu}{|q-q_2|} = \text{const}$$

- Lagrangian points

L_1, L_2, L_3 **saddle × center**

L_4, L_5 Stable (S-E-S/C and E-M-S/C systems)



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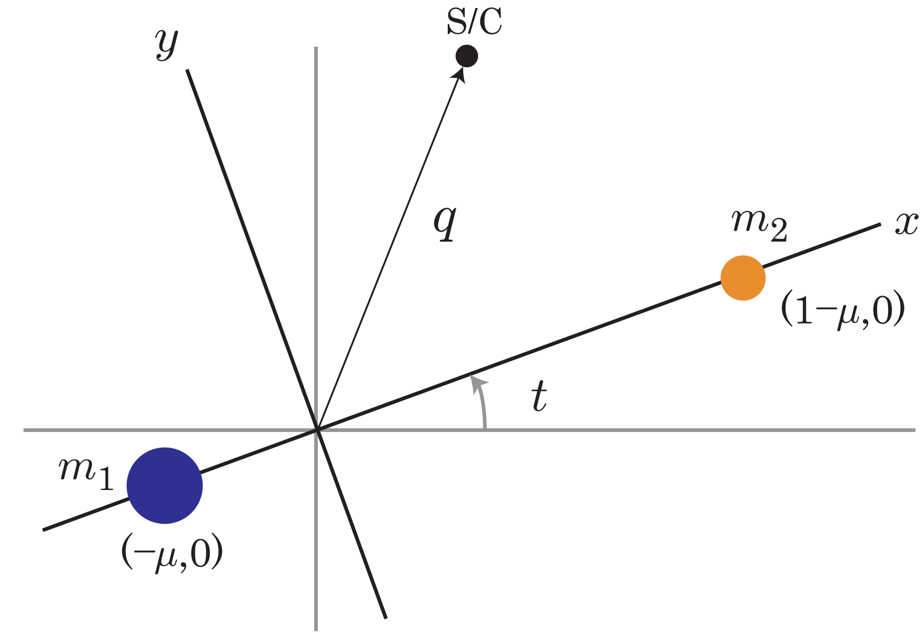

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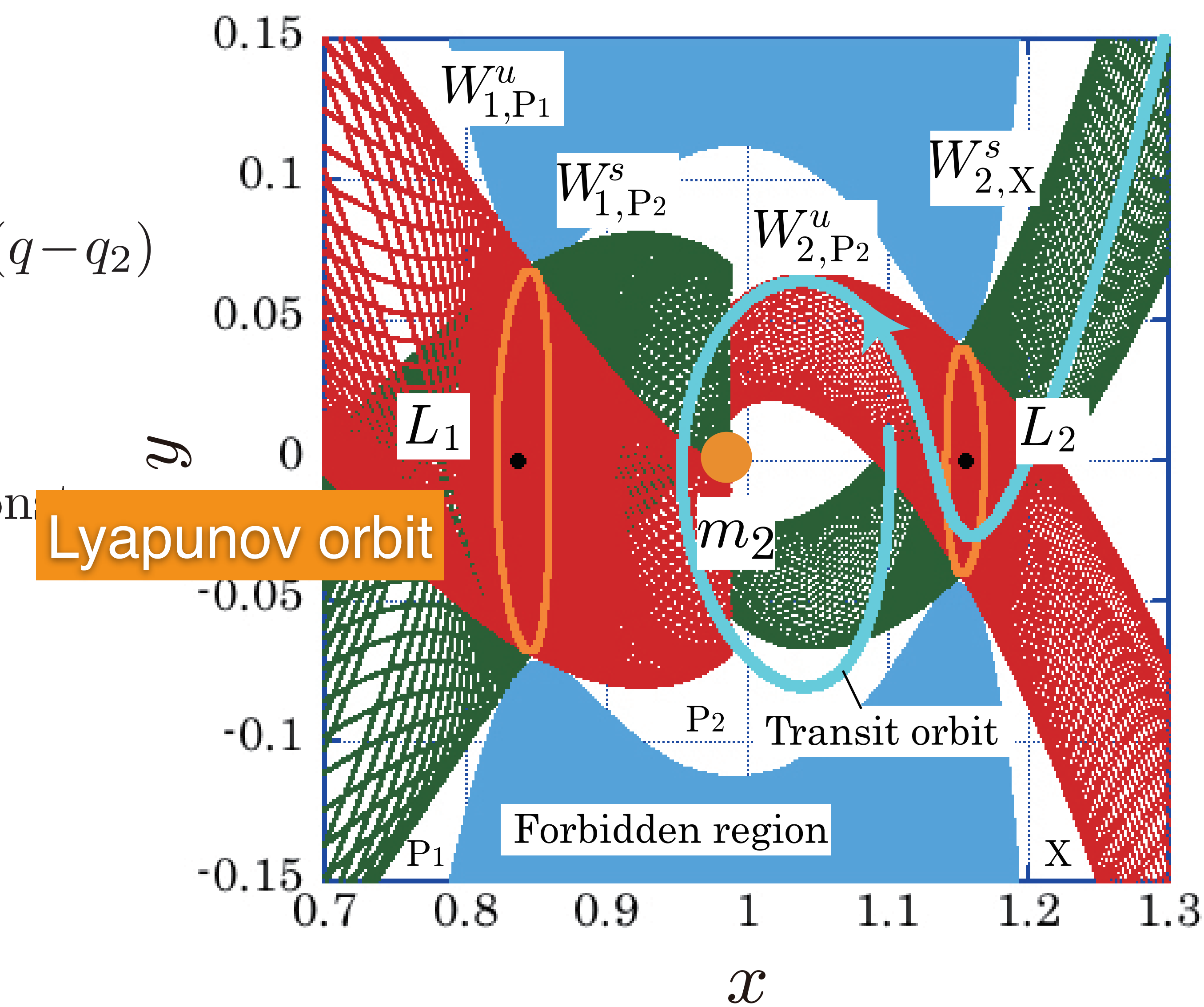

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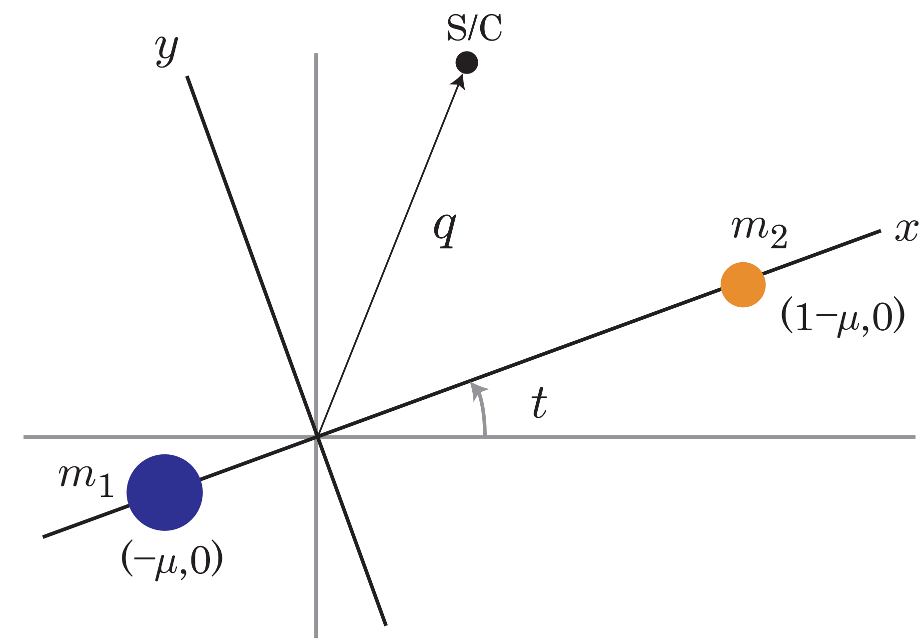

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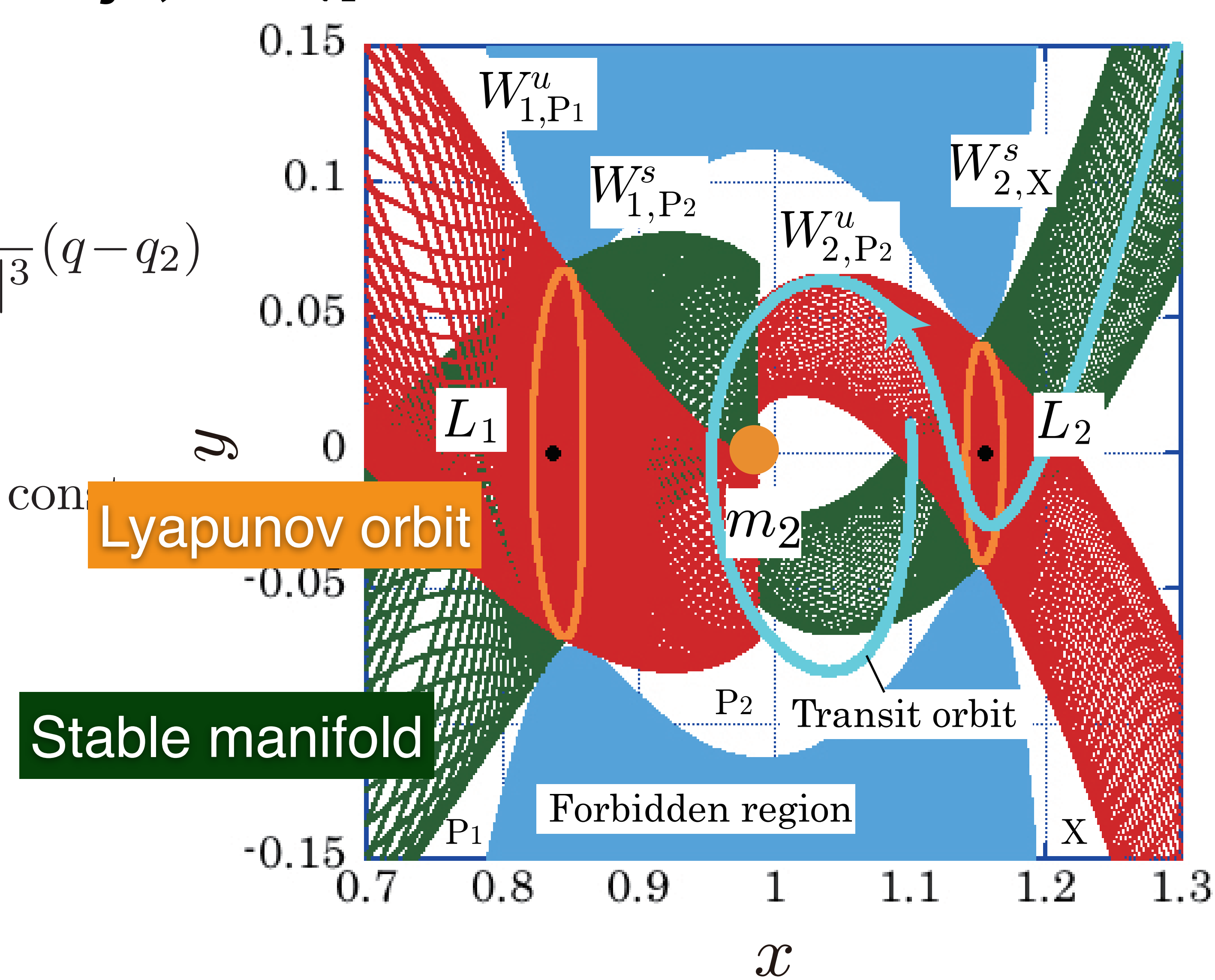

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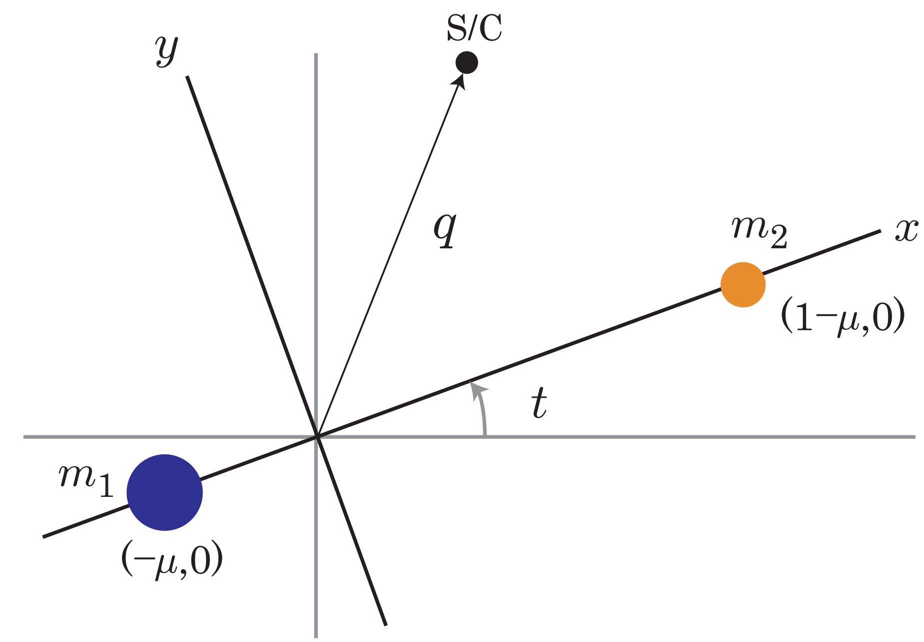

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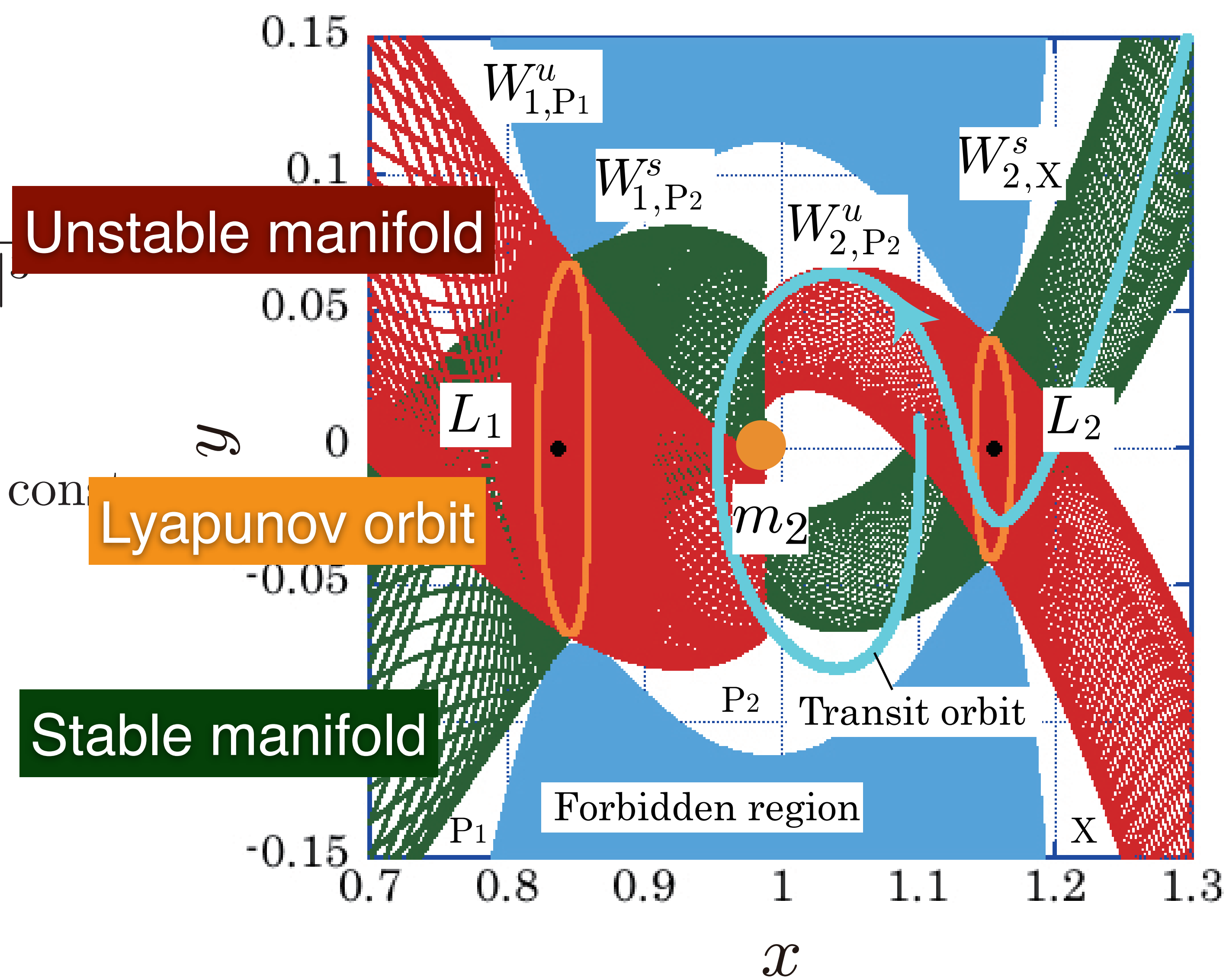

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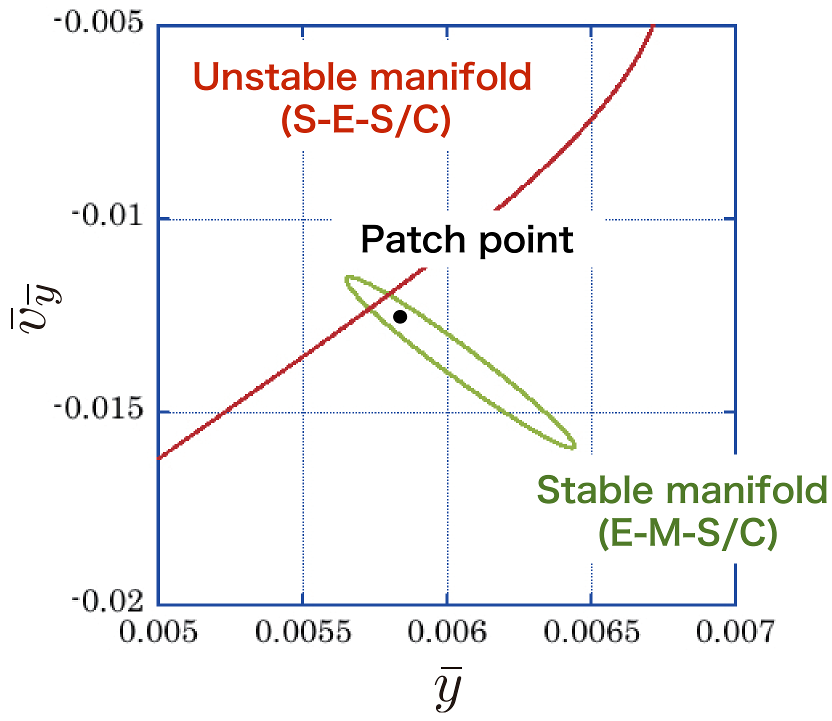
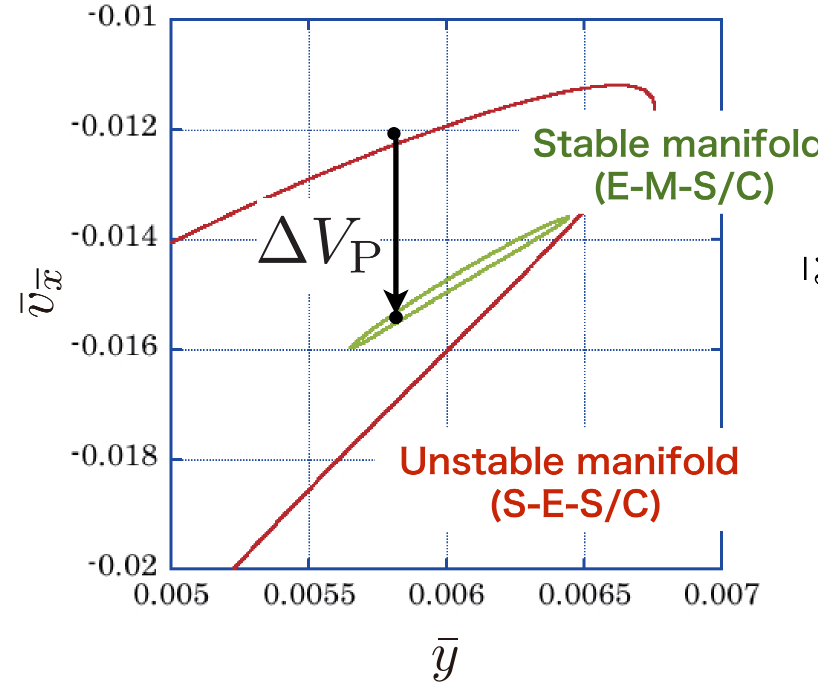


Invariant manifold on $\bar{\mathcal{U}}$



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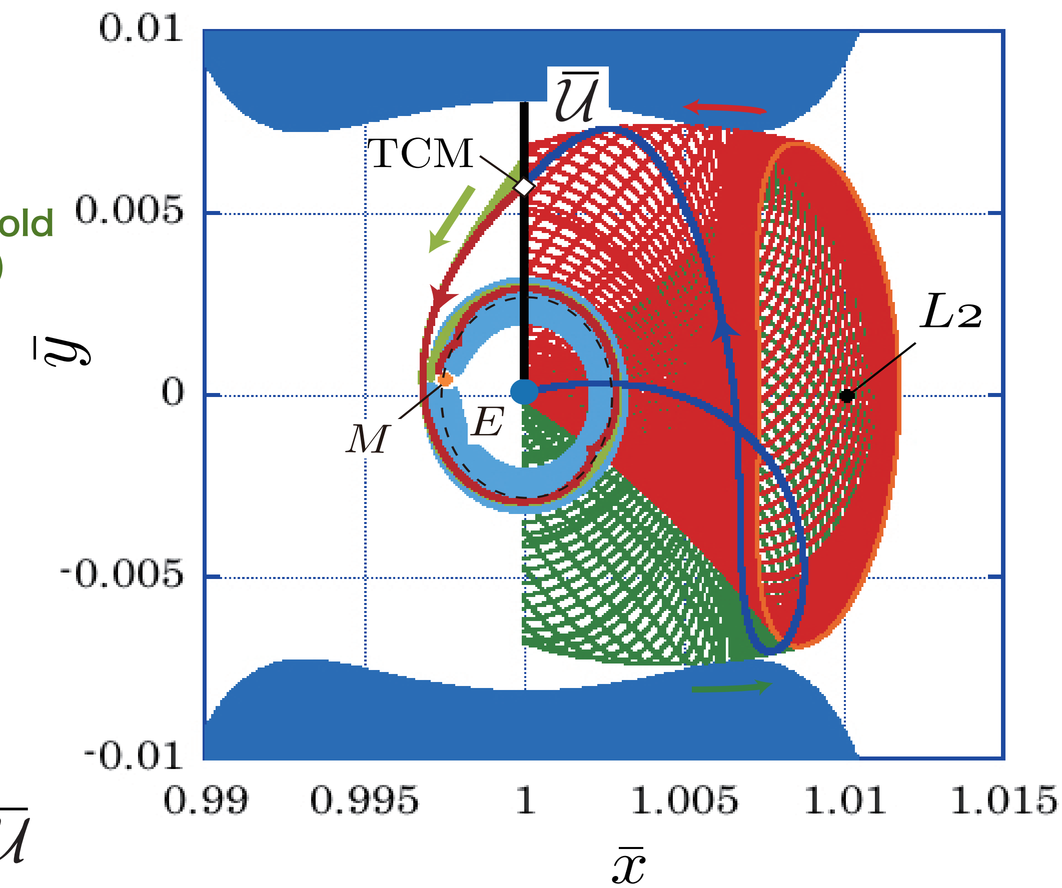
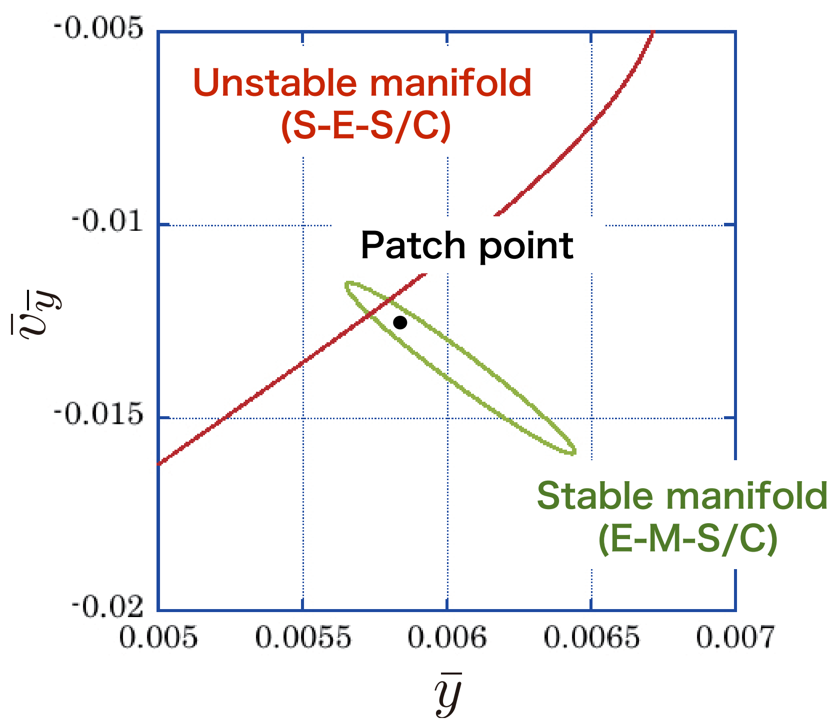


Transfer in the S-E rotating frame

Boundary condition
(departure：low Earth orbit 169km,　arrival：low lunar orbit 100km)

Transfer	ΔV_{E} [km/s]	ΔV_{M} [km/s]	ΔV_{P} [km/s]	$\Delta V_{\mathrm{Total}}$ [km/s]
Hohmann	3.141	0.838	—	3.979
Coupled PRC3BP [Koon et al. 2001]	3.537	1.989	0.098	5.624

Coupled PRC3BS



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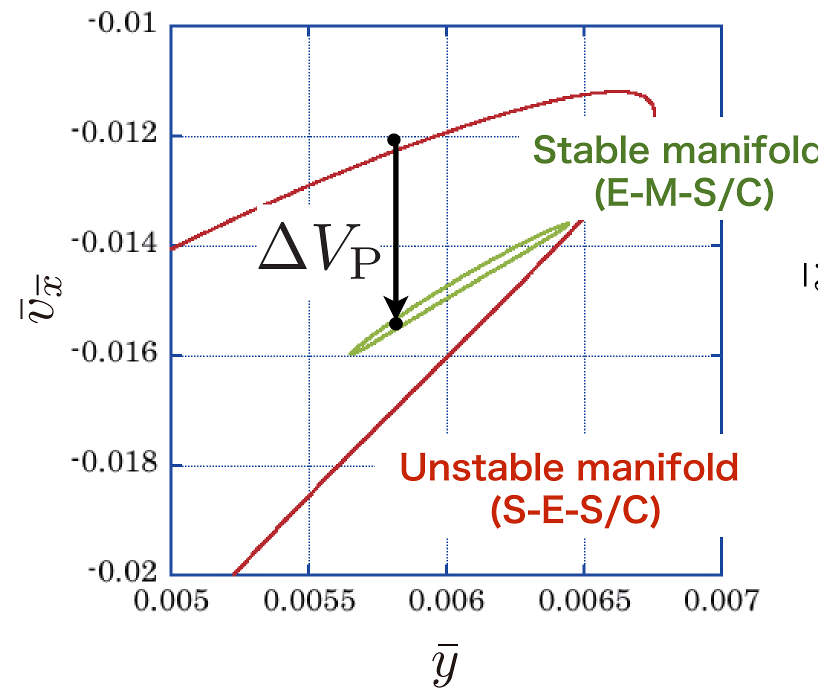

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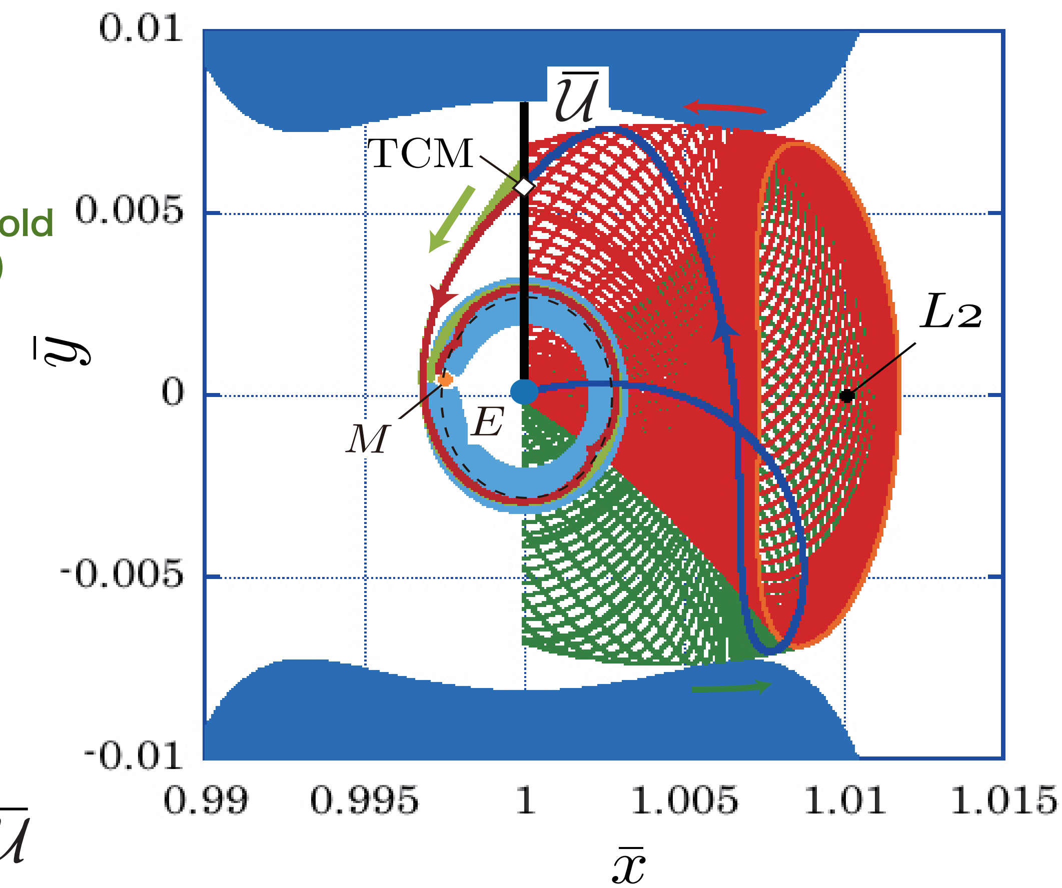

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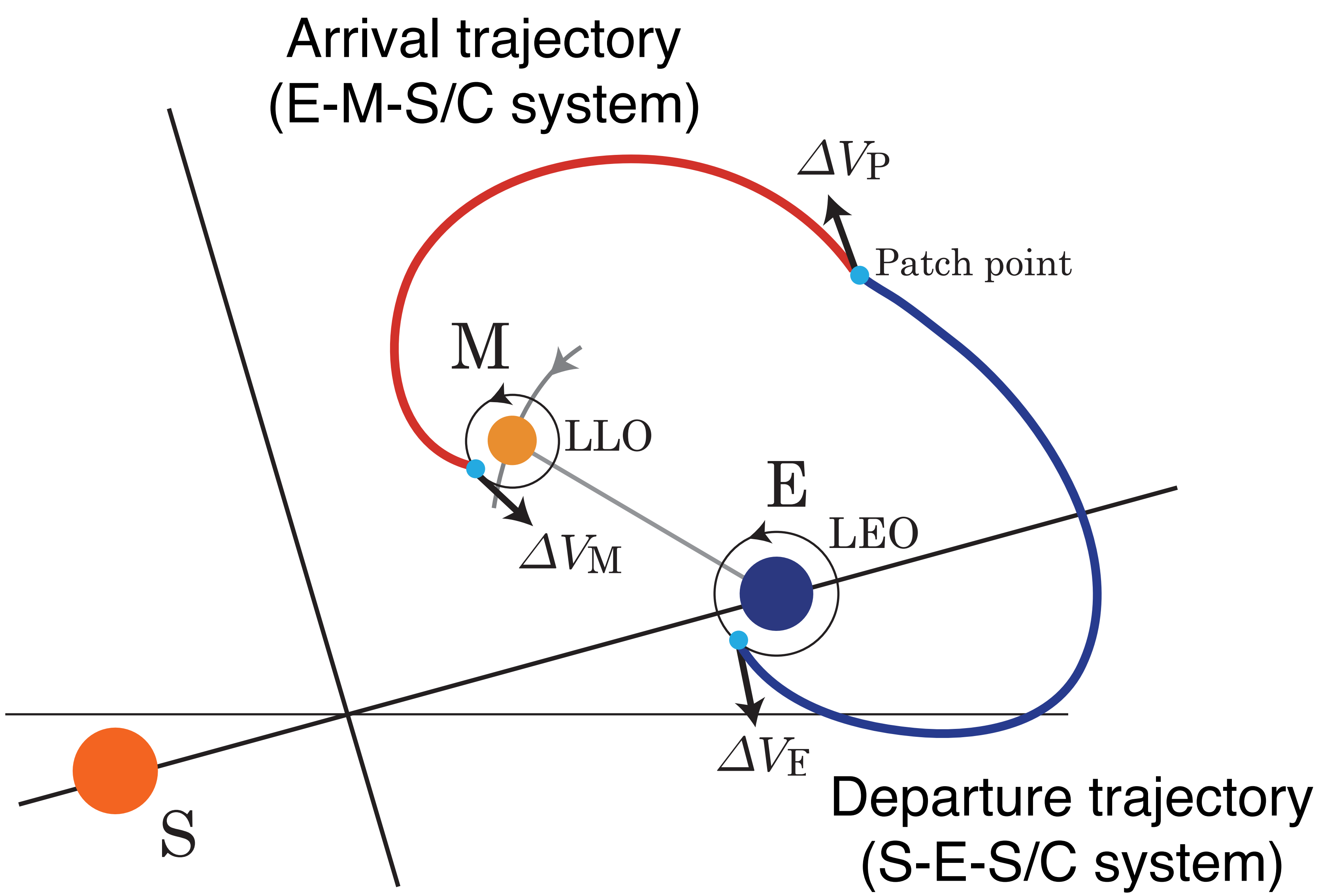
How do we find a low energy transfer in the coupled system ?

Approach to a problem

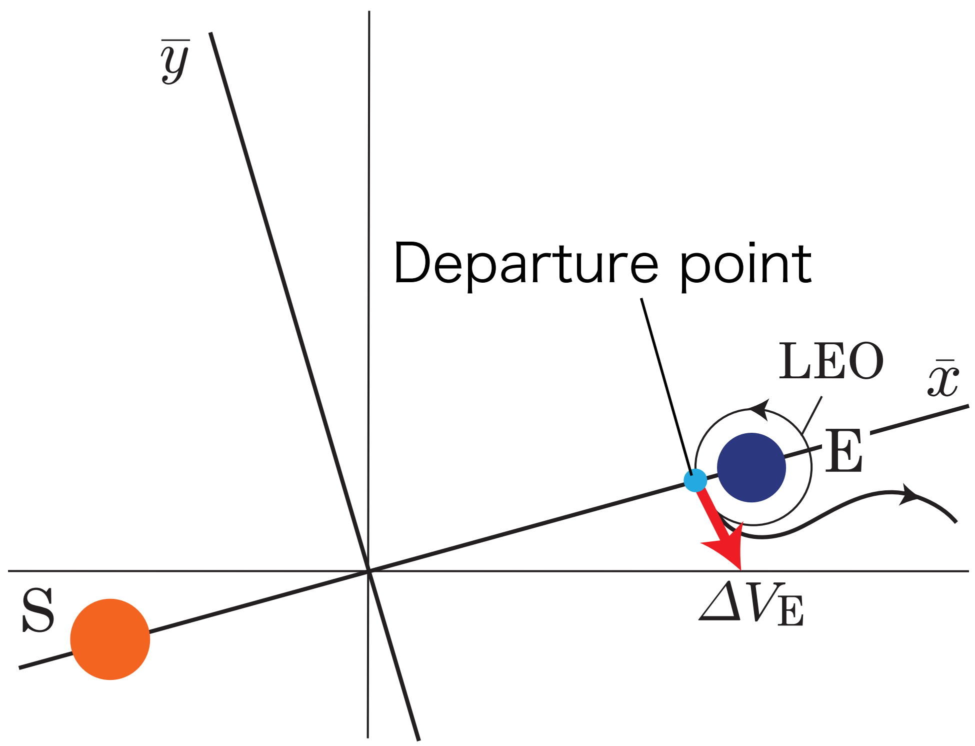
- Use optimization algorithm for a patch point to construct a low energy transfer [Peng et al. (2010)]
- **Utilize the tubes (invariant manifolds) near the LEO and LLO to obtain a low energy transfer**



LEO:
169 [km], 7.087 [km/s]

LLO:
100 [km], 1.633 [km/s]

Departure trajectory in the S-E-S/C system



LEO

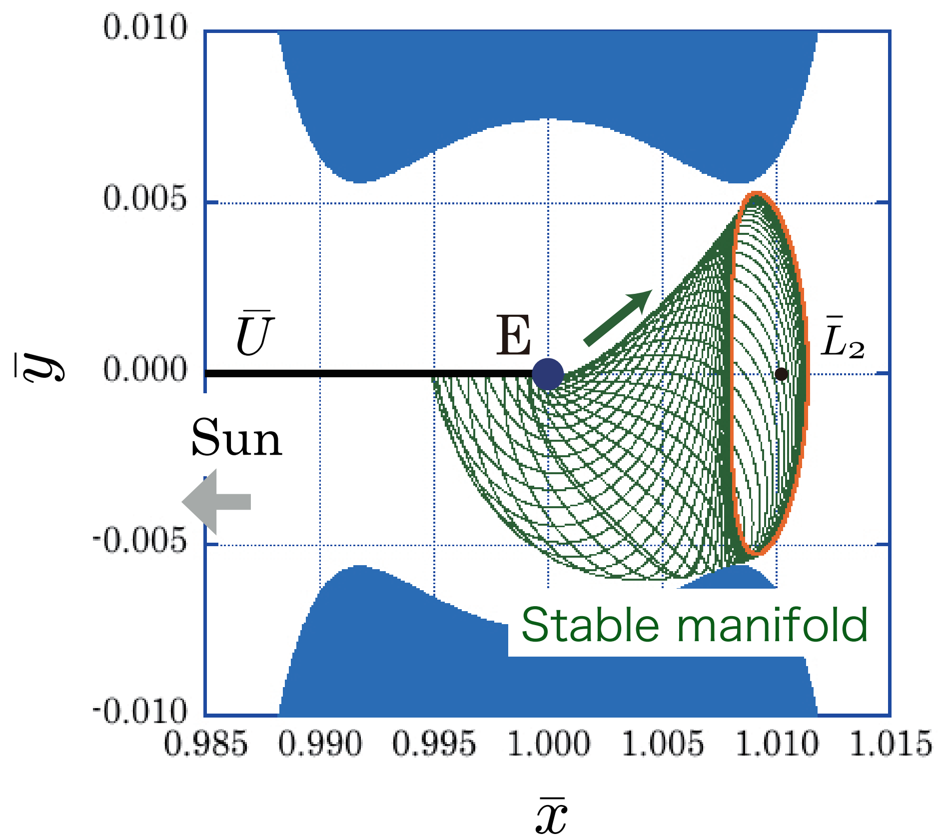
$$(\bar{x}, \bar{y}, \bar{v}_{\bar{x}}, \bar{v}_{\bar{y}}) = (1 - \mu_{\rm S} - \bar{r}_{\rm LEO}, 0, 0, -\bar{v}_{\rm LEO})$$

Velocity : increase

Departure trajectory

$$(\bar{x}, \bar{y}, \bar{v}_{\bar{x}}, \bar{v}_{\bar{y}}) = (1 - \mu_{\rm S} - \bar{r}_{\rm LEO}, 0, 0, -\bar{v}_{\rm LEO} - \Delta V_{\rm E})$$

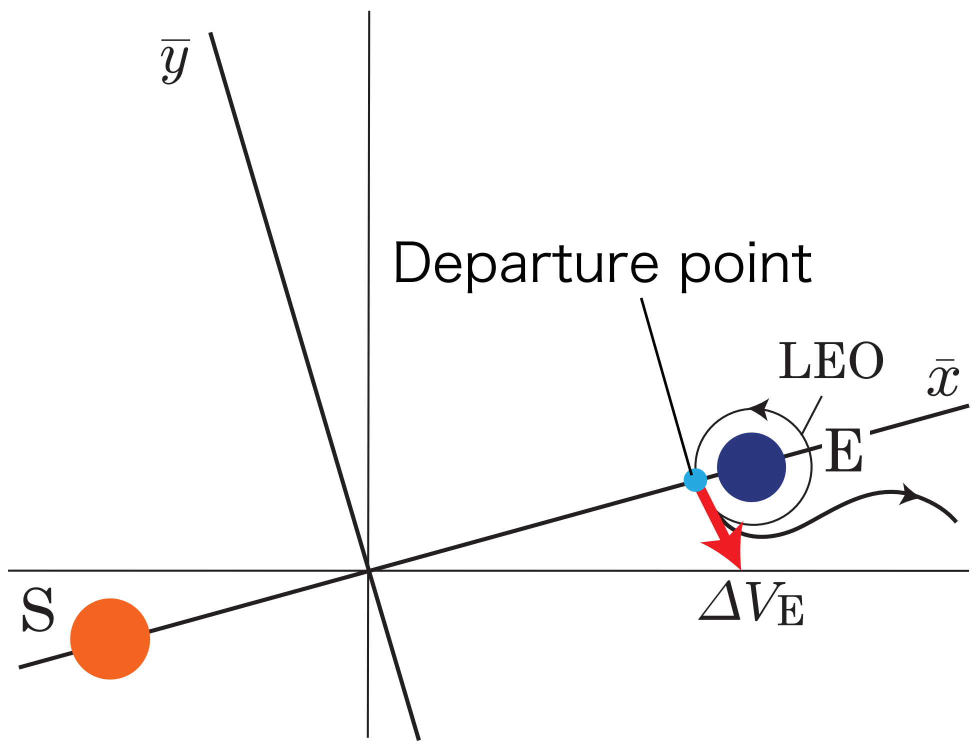
＊maneuver $\Delta V_{\rm E}$ uniquely gives $\bar{E}^{SE}$



$(\bar{E}^{SE} = -1.50039)$

Investigate **the energy range** ($\Delta V_{\rm E}$ range) such that an orbit is to be a **non-transit orbit**

Departure trajectory in the S-E-S/C system



LEO   Energy $\bar{E}^{SE}_{\mathrm{LEO}} = -1.53501$

$$(\bar{x}, \bar{y}, \bar{v}_{\bar{x}}, \bar{v}_{\bar{y}}) = (1 - \mu_{\mathrm{S}} - \bar{r}_{\mathrm{LEO}}, 0, 0, -\bar{v}_{\mathrm{LEO}})$$

Velocity : increase   Energy : increase

Departure trajectory   Energy $\bar{E}^{SE}$

$$(\bar{x}, \bar{y}, \bar{v}_{\bar{x}}, \bar{v}_{\bar{y}}) = (1 - \mu_{\mathrm{S}} - \bar{r}_{\mathrm{LEO}}, 0, 0, -\bar{v}_{\mathrm{LEO}} - \Delta V_{\mathrm{E}})$$

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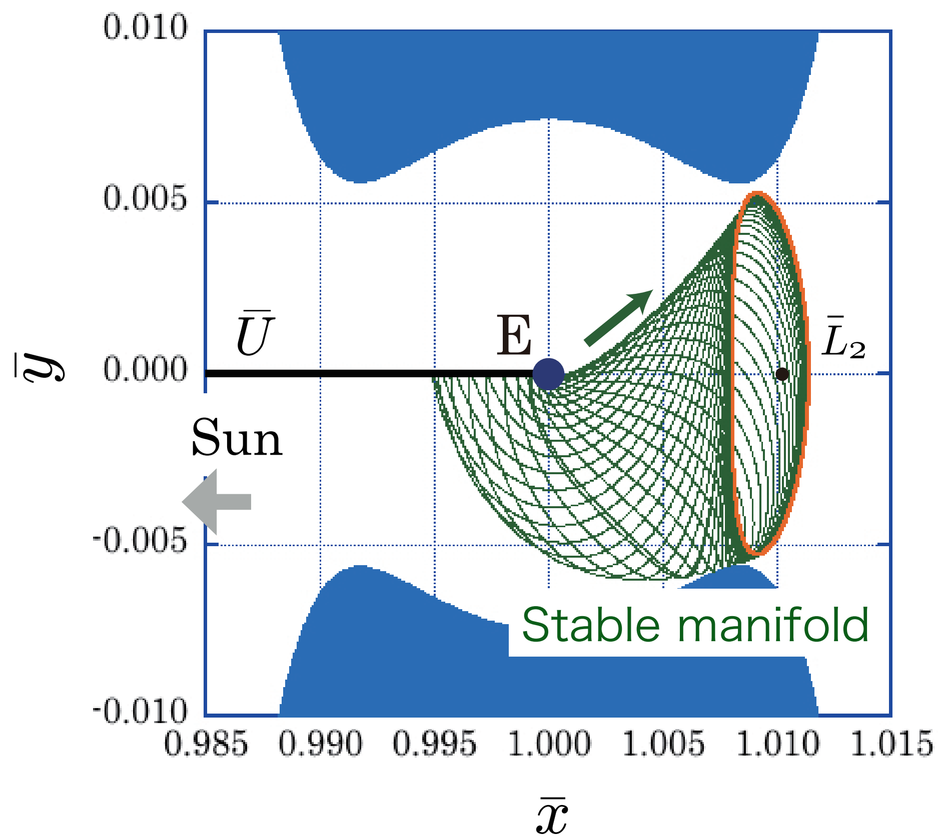

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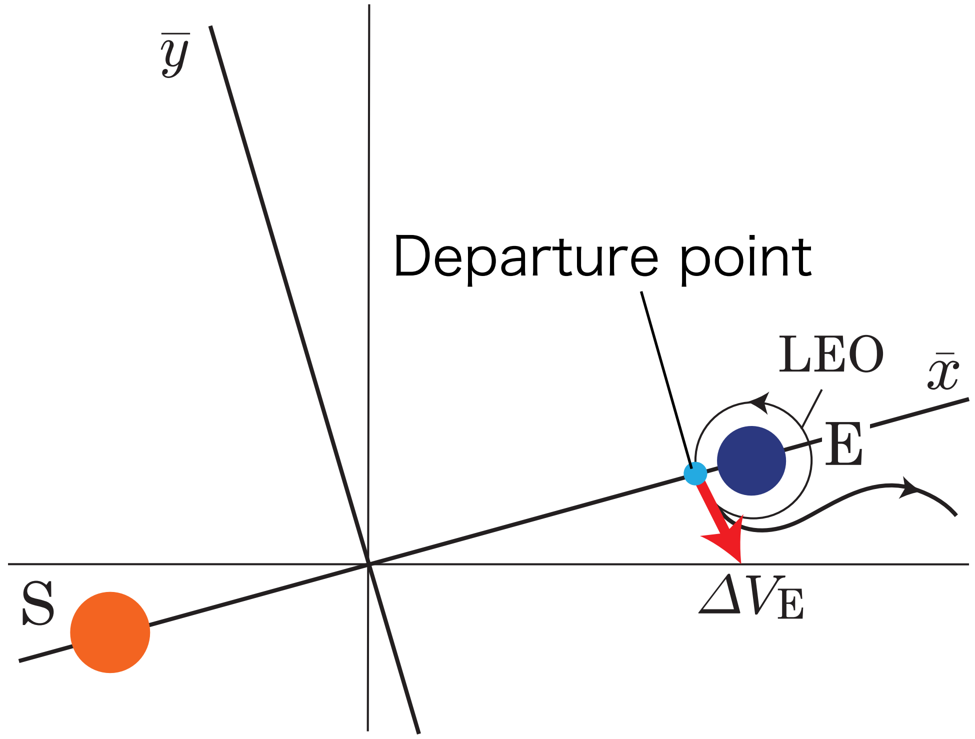

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Velocity : increase Energy : increase

Departure trajectory Energy $\bar{E}^{SE}$

$$(\bar{x}, \bar{y}, \bar{v}_{\bar{x}}, \bar{v}_{\bar{y}}) = (1 - \mu_\mathrm{S} - \bar{r}_\mathrm{LEO}, 0, 0, -\bar{v}_\mathrm{LEO} - \Delta V_\mathrm{E})$$

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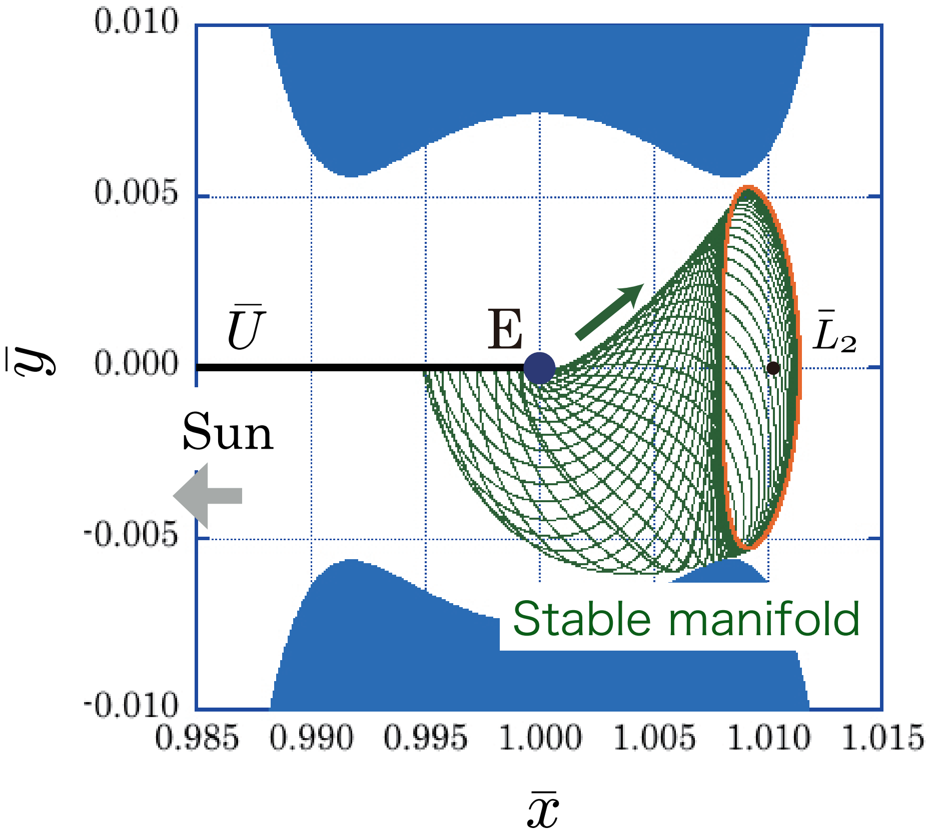
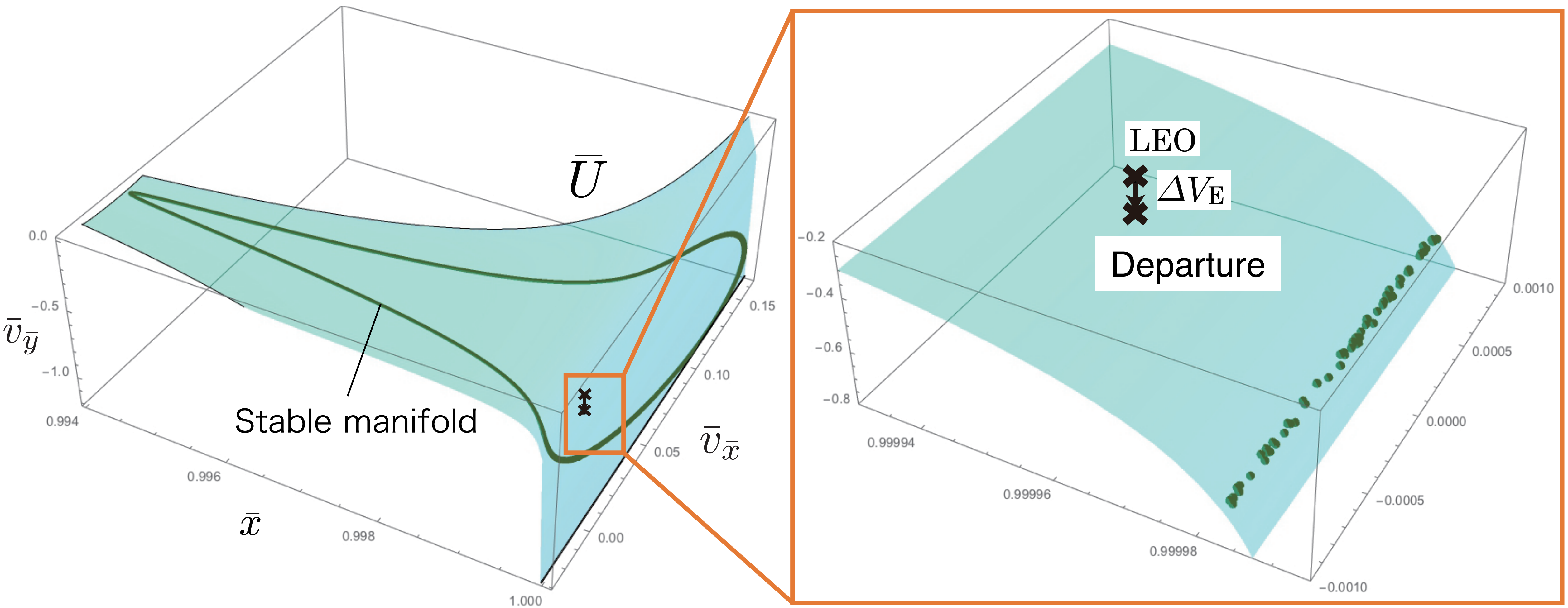


$(\bar{E}^{SE} = -1.50039)$

Investigate **the energy range** (ΔV_E range) such that an orbit is to be a **non-transit orbit**

Initial point of departure trajectory should be **outside** of the stable manifold

Departure trajectory in the S-E-S/C system

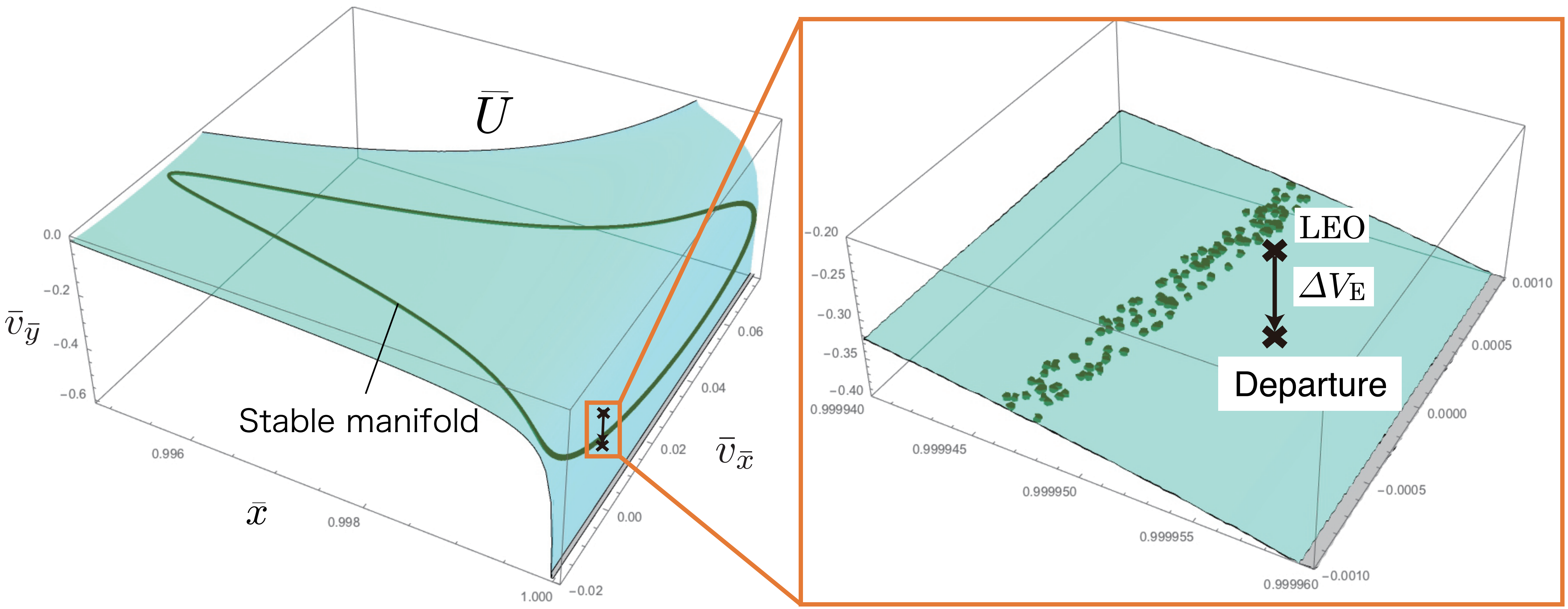


Inside of stable manifold

Stable manifold on $\bar{U}$

$$(\bar{E}^{SE} = -1.50039)$$

Departure trajectory in the S-E-S/C system



Outside of stable manifold

Stable manifold on $\bar{U}$

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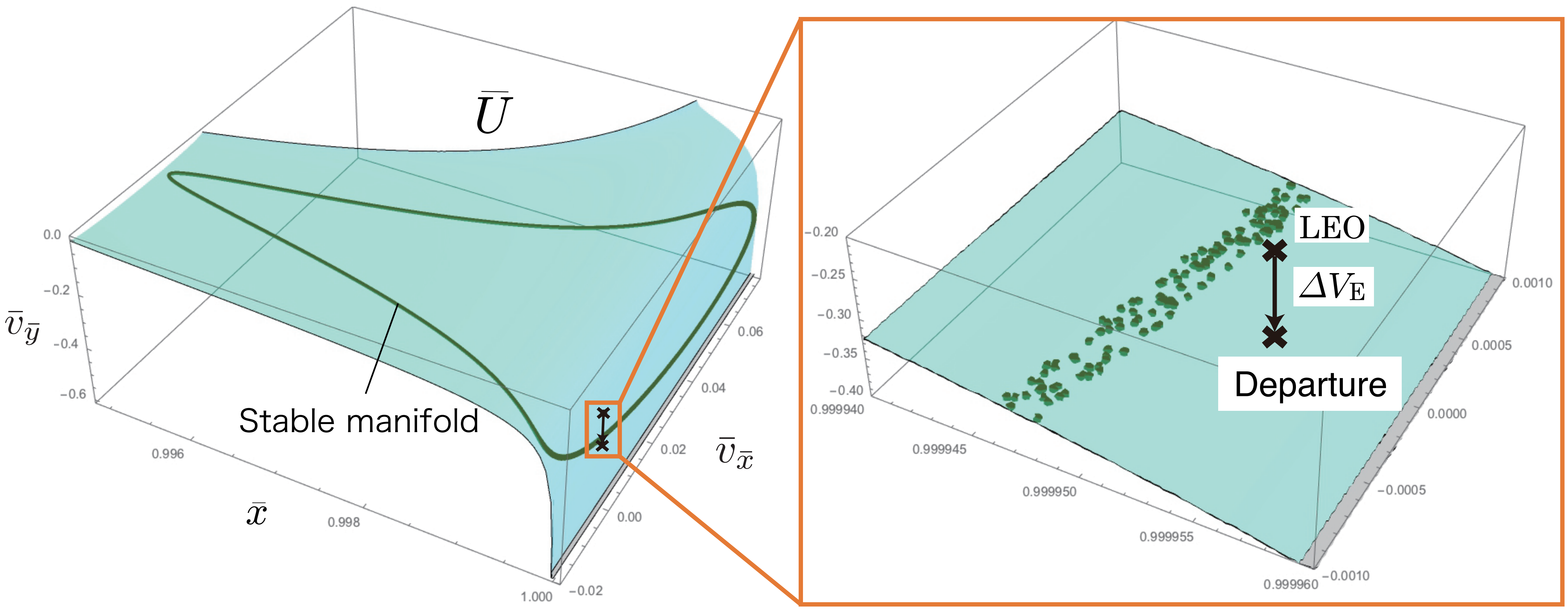
Departure trajectory in the S-E-S/C system



Outside of stable manifold

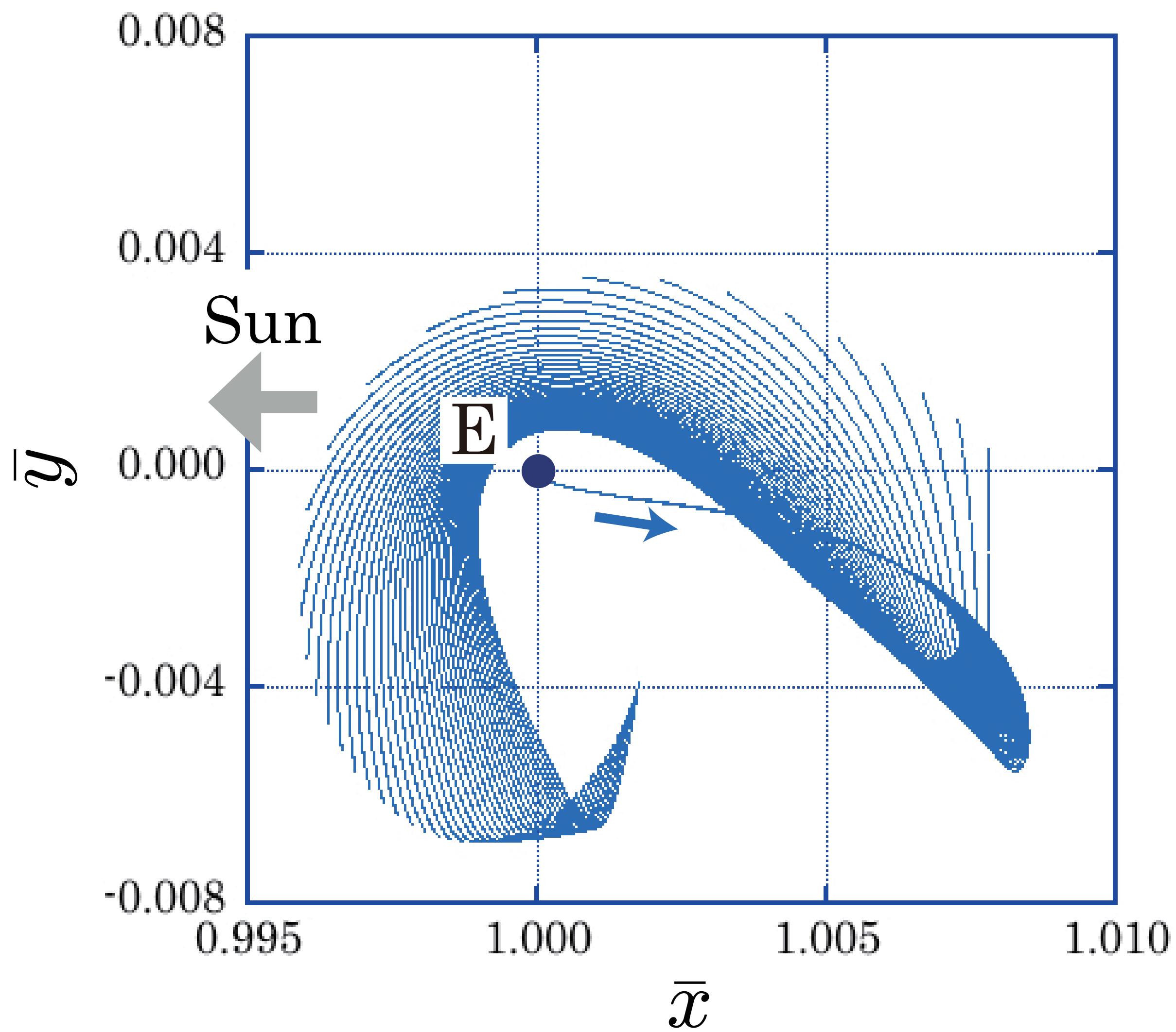
Stable manifold on $\bar{U}$

$$(\bar{E}^{SE} = -1.50040)$$

Upper limit of the energy of the departure trajectory (non-transit orbit)

$$\bar{E}^{SE}_{\mathrm{D_{max}}} = -1.50040$$

Departure trajectory in the S-E-S/C system



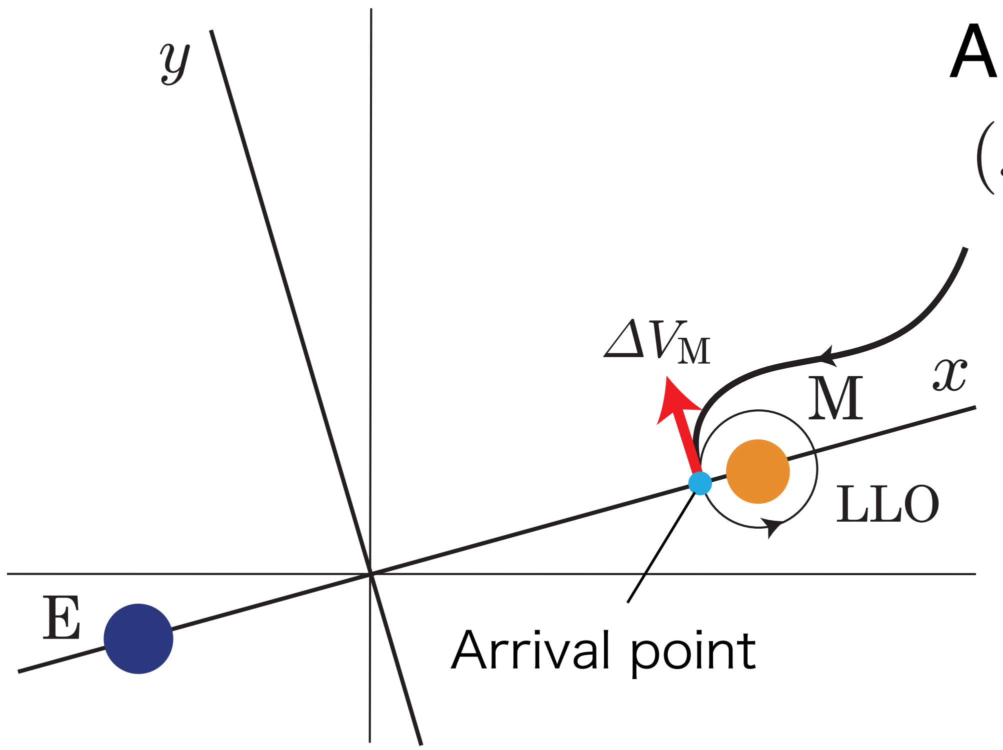
Family of the departure trajectories (non-transit orbits) parametrized by the energy

$$\bar{E}^{SE} \in [\bar{E}^{SE}_{\bar{L}_2}, \bar{E}^{SE}_{\mathrm{D_{max}}}]$$

Energy at the Lagrangian point $\bar{L}_2$: $\bar{E}^{SE}_{\bar{L}_2} = -1.50045$

Upper limit of the energy : $\bar{E}^{SE}_{\mathrm{D_{max}}} = -1.50040$

Arrival trajectory in the E-M-S/C system



Arrival trajectory   Energy E^{EM}

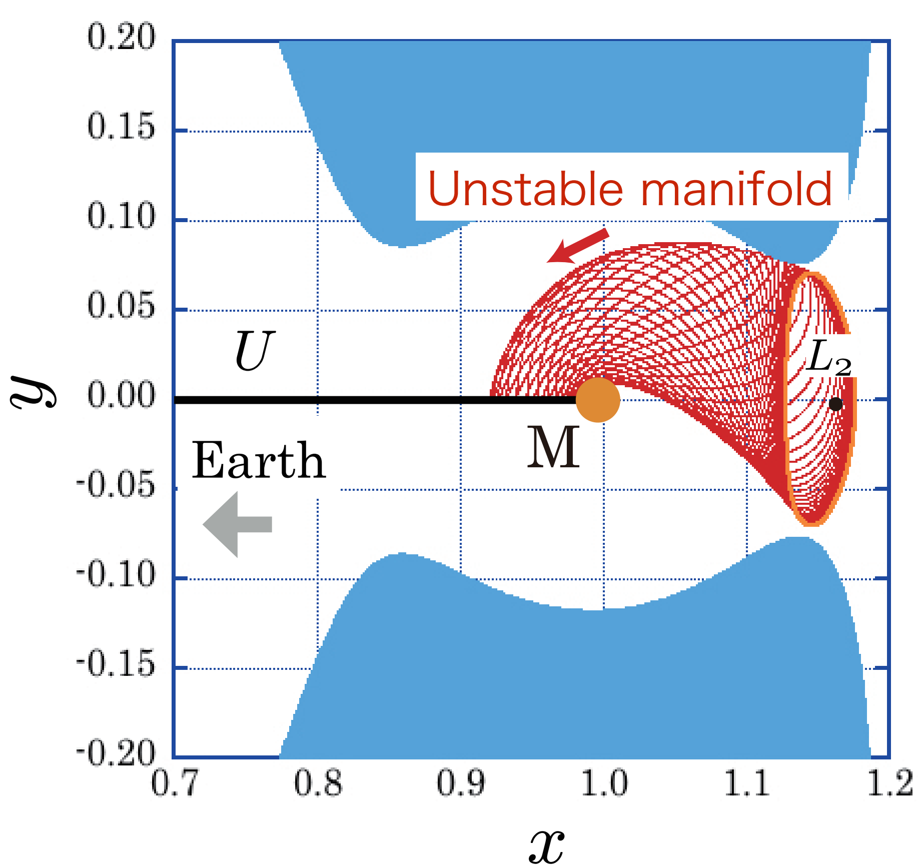
$$(x, y, v_x, v_y) = (1 - \mu_{\mathrm{M}} - \bar{r}_{\mathrm{LLO}}, 0, 0, -\bar{v}_{\mathrm{LLO}} - \Delta V_{\mathrm{M}})$$

Velocity and energy : decrease

LLO   Energy $E^{EM}_{\mathrm{LLO}} = -2.75466$

$$(x, y, v_x, v_y) = (1 - \mu_{\mathrm{M}} - \bar{r}_{\mathrm{LLO}}, 0, 0, -\bar{v}_{\mathrm{LLO}})$$

∗maneuver ΔV_{M} uniquely gives E^{EM}

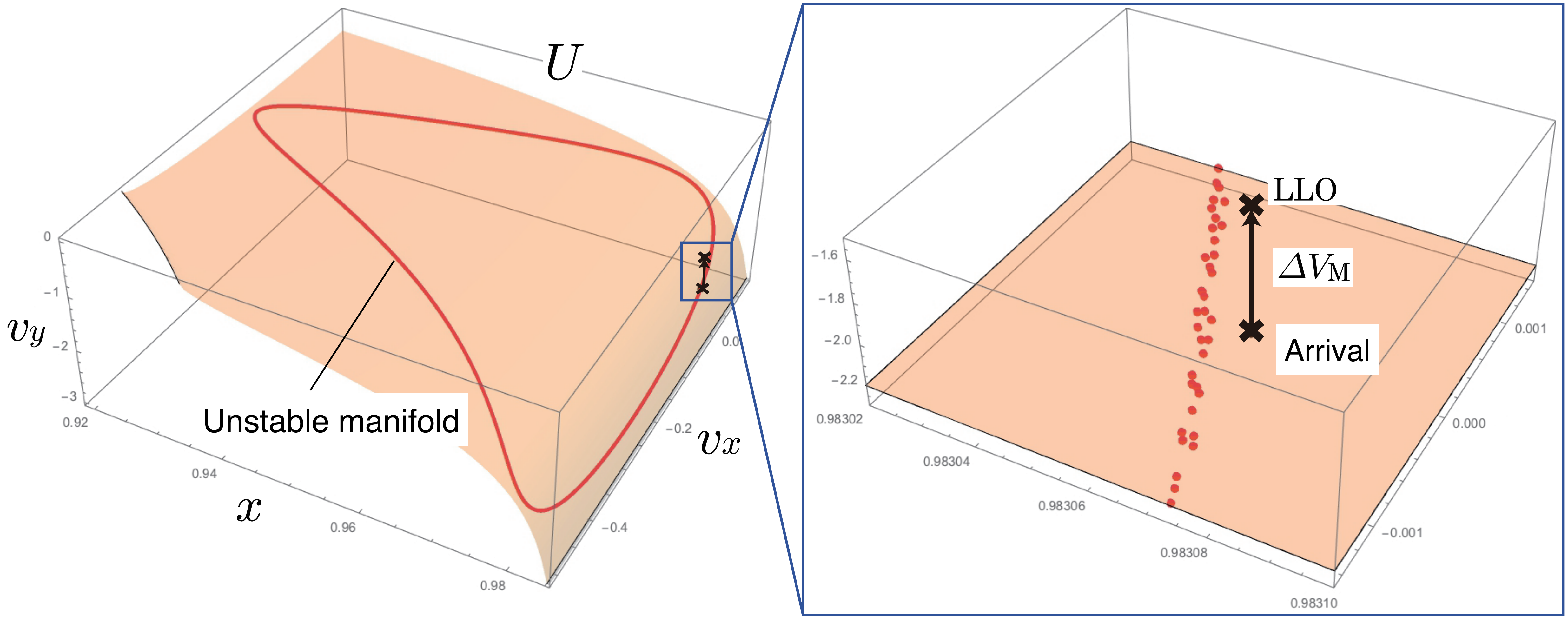


$(E^{EM} = -1.57962)$

Energy range (ΔV_{M} range) such that an orbit is to be a **transit orbit**

Final point is **inside** of the unstable manifold

Arrival trajectory in the E-M-S/C system

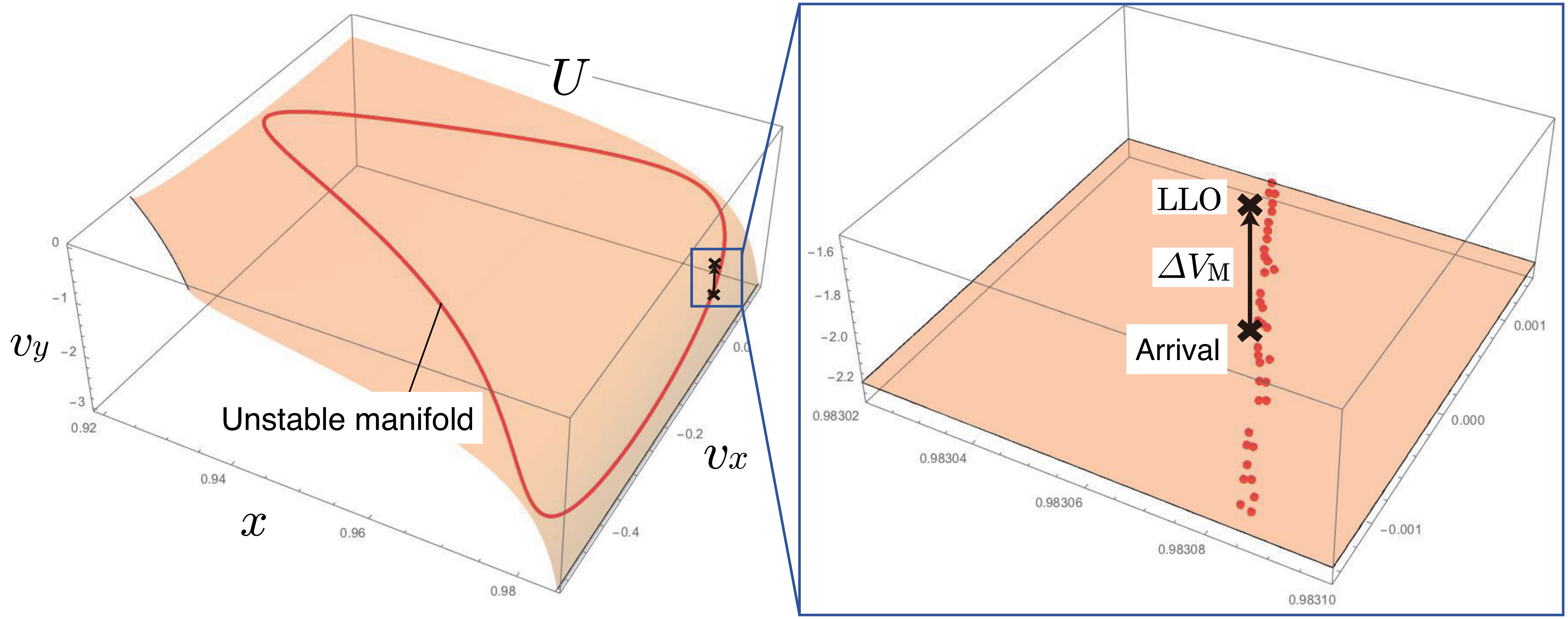


Outside of unstable manifold

Unstable manifold on U

$(E^{EM} = -1.57962)$

Arrival trajectory in the E-M-S/C system



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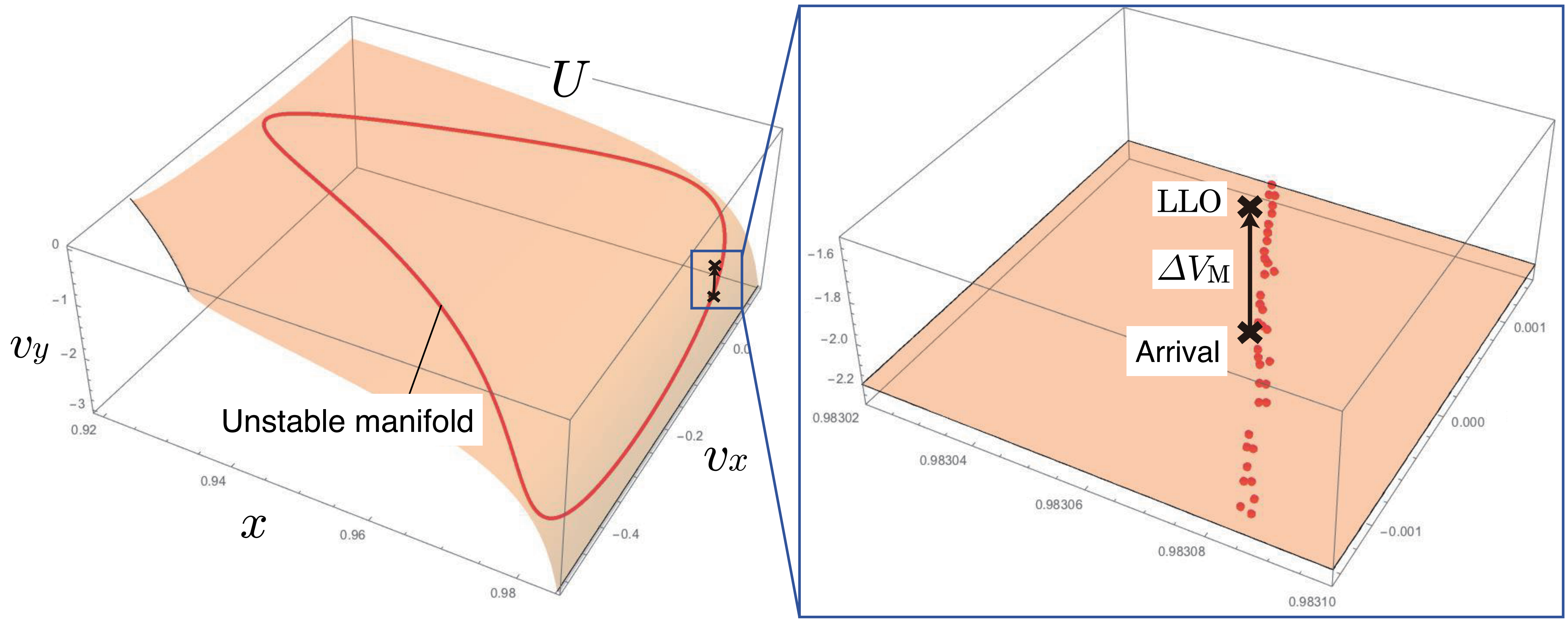


Inside of unstable manifold

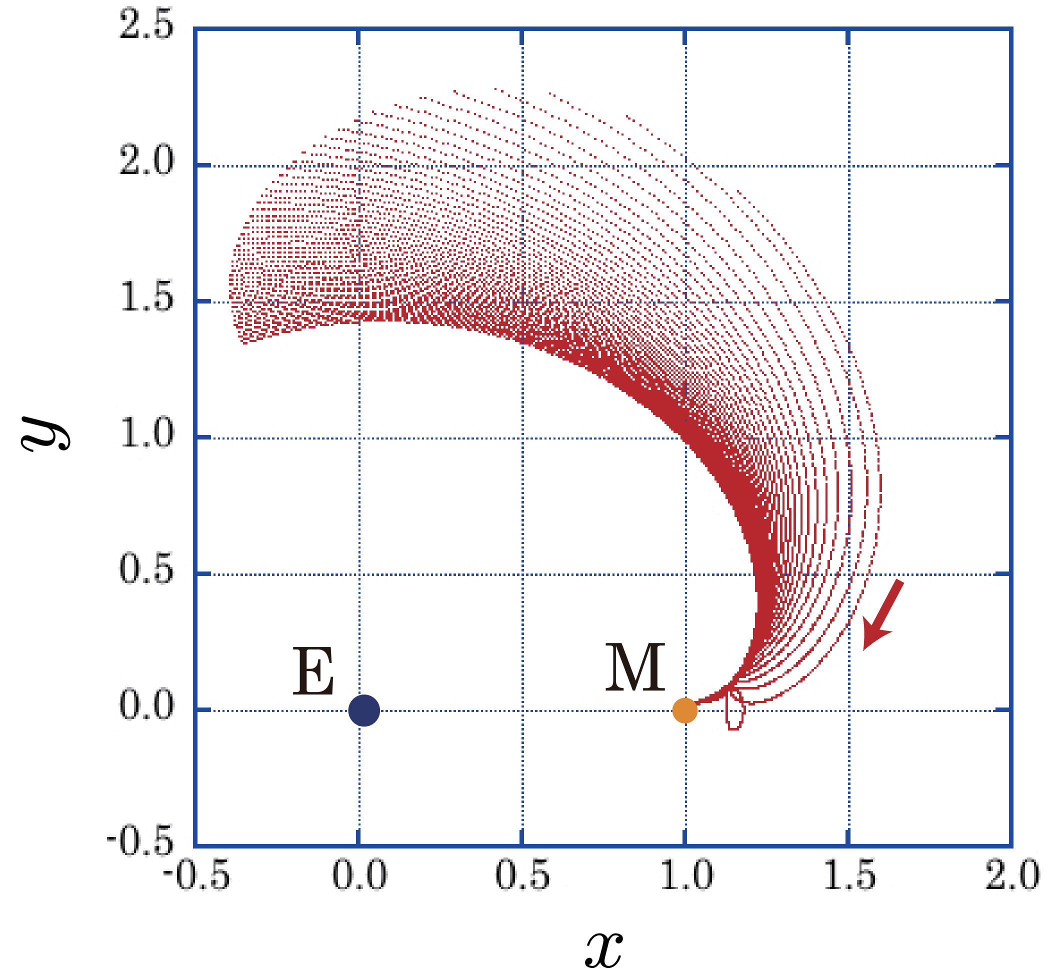
Unstable manifold on U

$$(E^{EM} = -1.57961)$$

Lower limit of the energy of the arrival trajectory (transit orbit)

$$E^{EM}_{\mathrm{A_{min}}} = -1.57961$$

Arrival trajectory in the E-M-S/C system



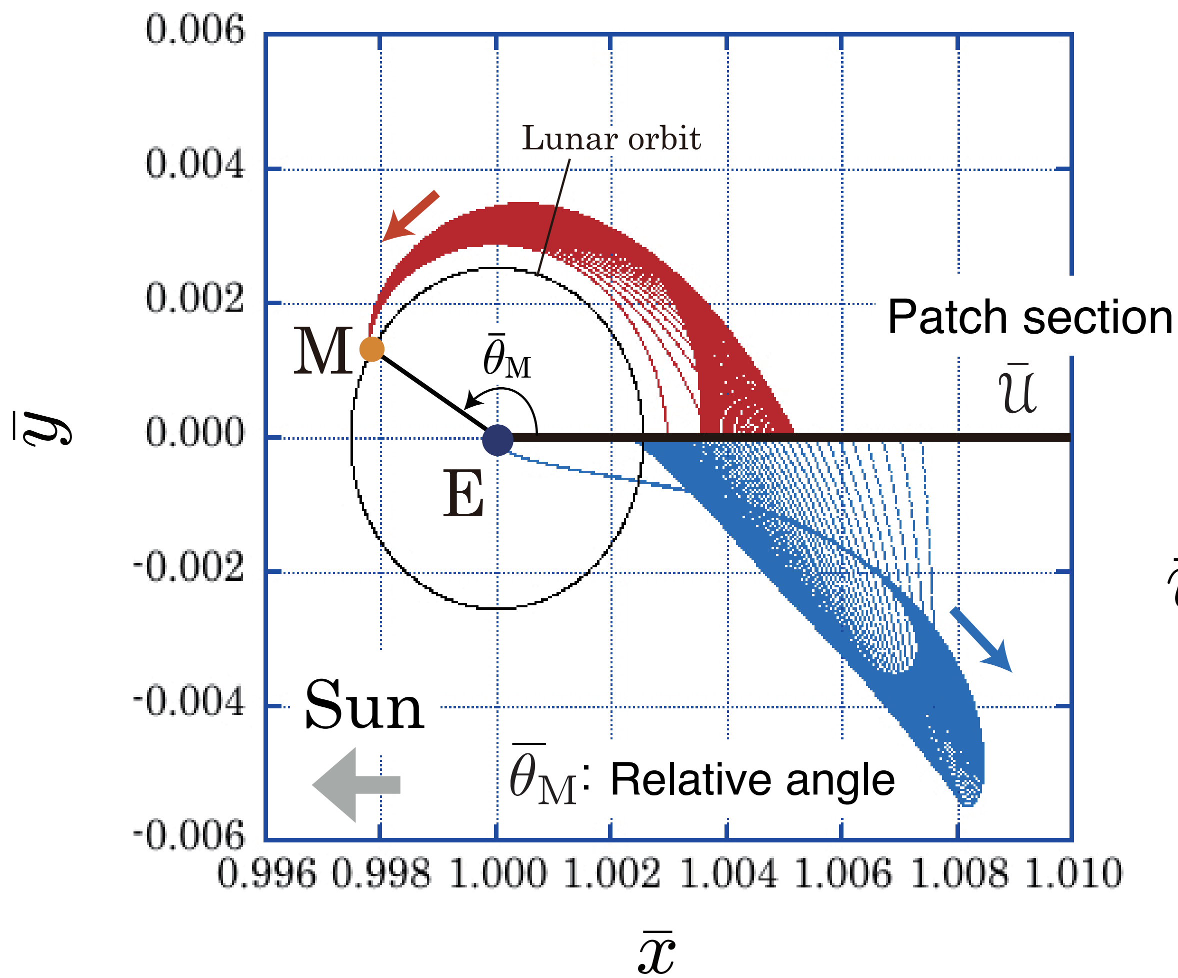
Family of the arrival trajectories (transit orbits) parametrized by the energy

$$E^{EM} \in [E^{EM}_{\mathrm{A_{min}}}, E^{EM}_{L_3}]$$

Lower limit of the energy : $E^{EM}_{\mathrm{A_{min}}} = -1.57961$

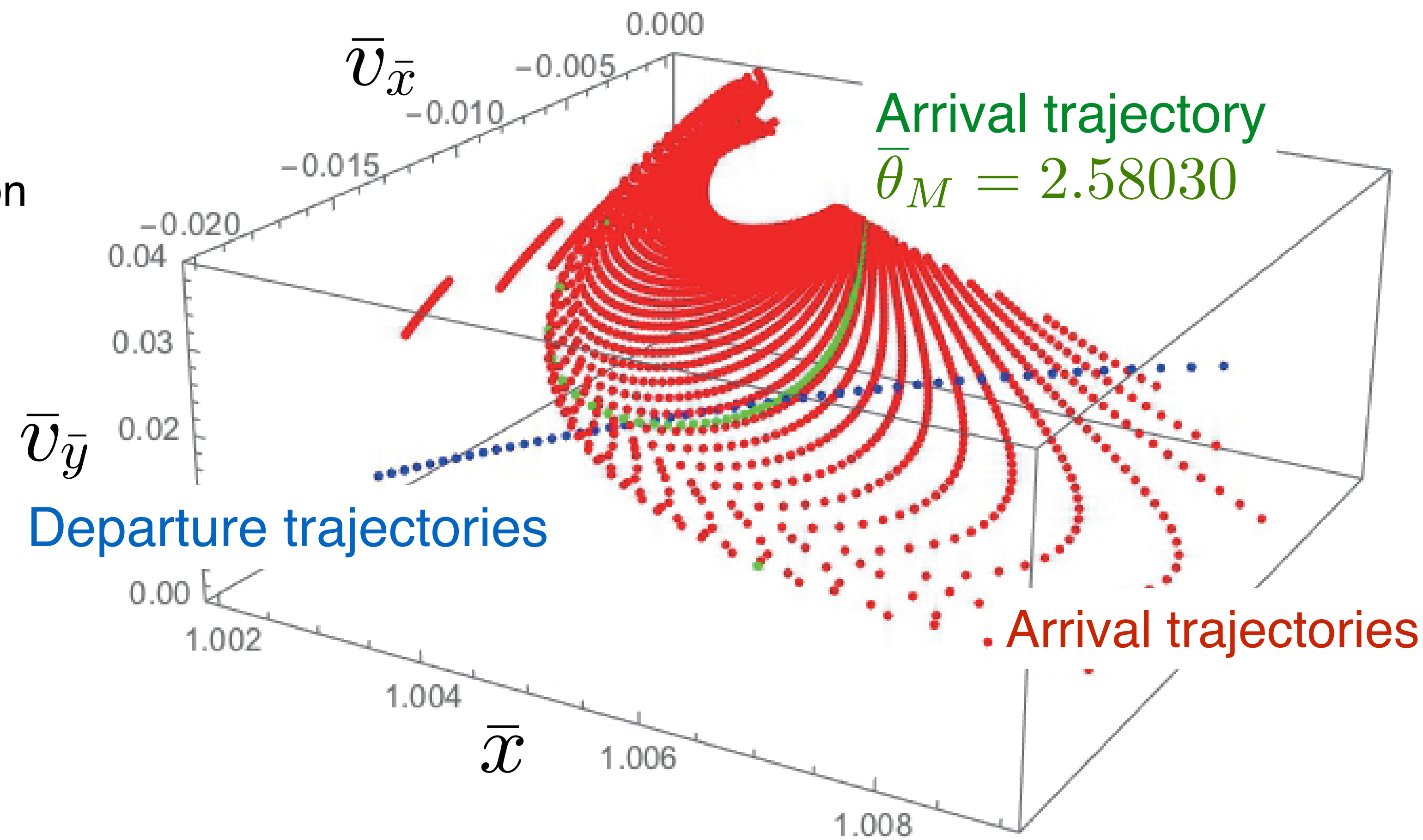
Energy at the Lagrangian point L_3 : $E^{EM}_{L_3} = -1.50608$

LEO-LLO transfer



Family of the departure and arrival trajectories in the S-E rotating frame

$$(\bar{\theta}_M = 2.58030)$$

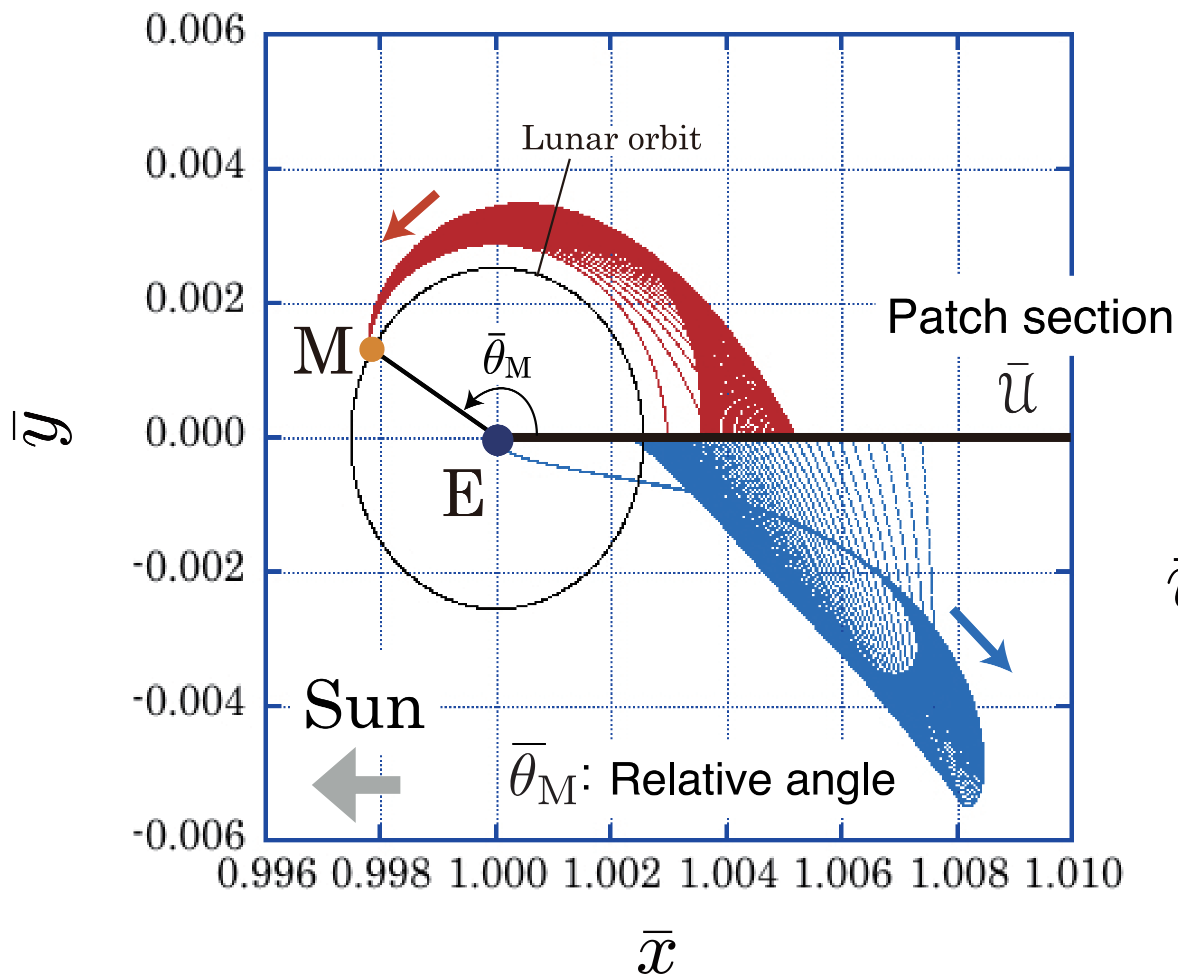


Family of the departure and arrival trajectories on $\bar{\mathbb{U}}$

$$\bar{\theta}_M \in [0, 2\pi)$$

∗E-M-S/C system is to be non-autonomous system depending on $\bar{\theta}_M$

LEO-LLO transfer



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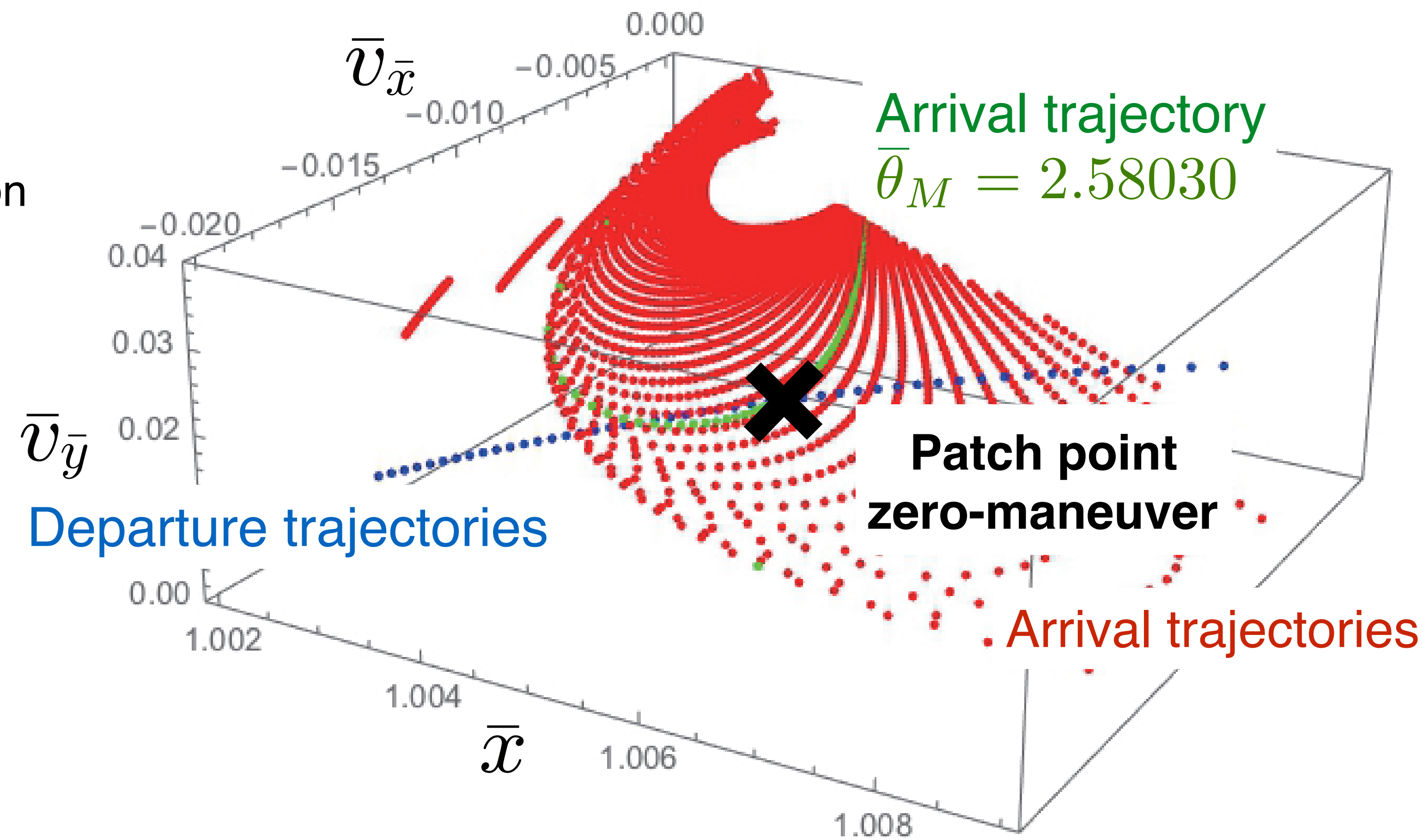

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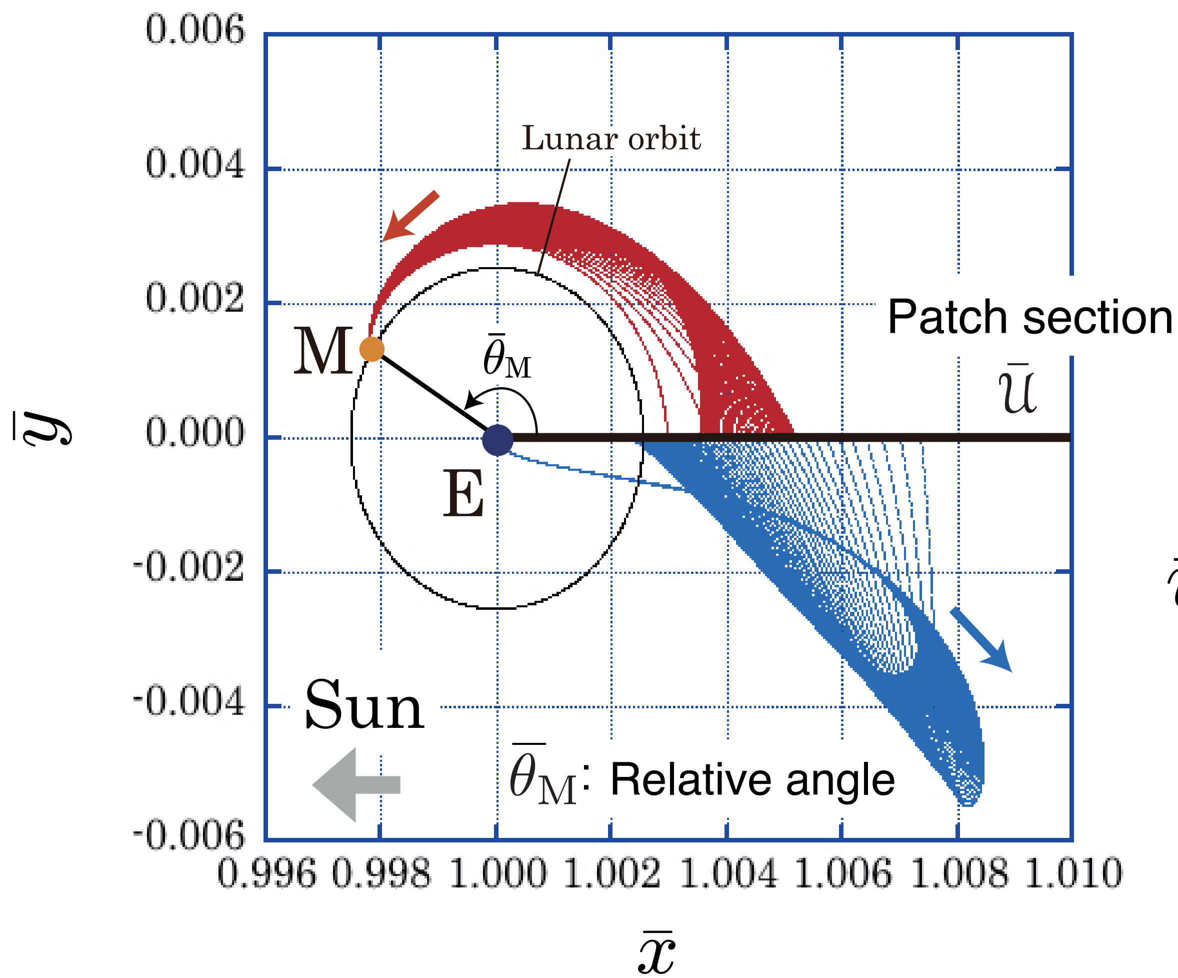

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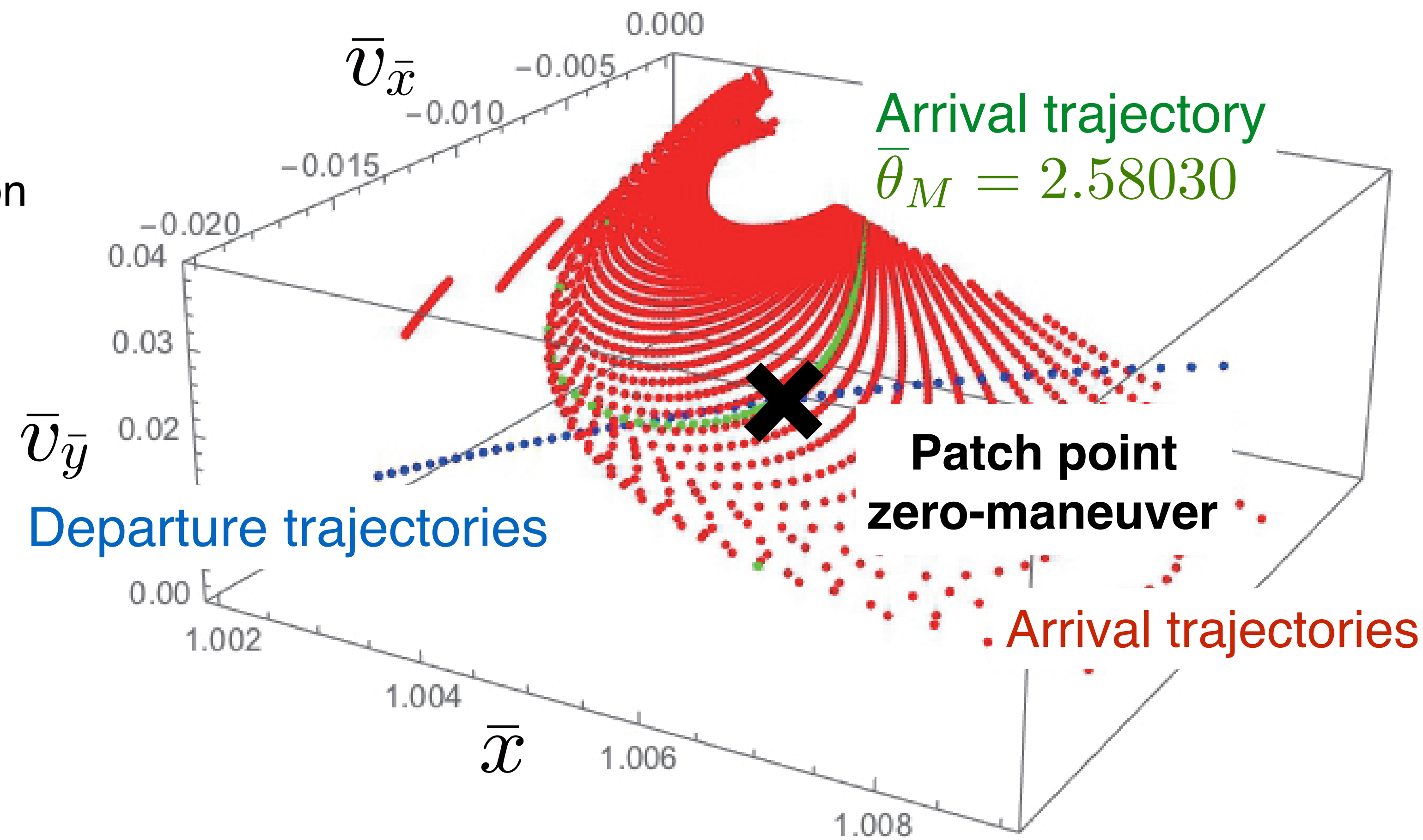

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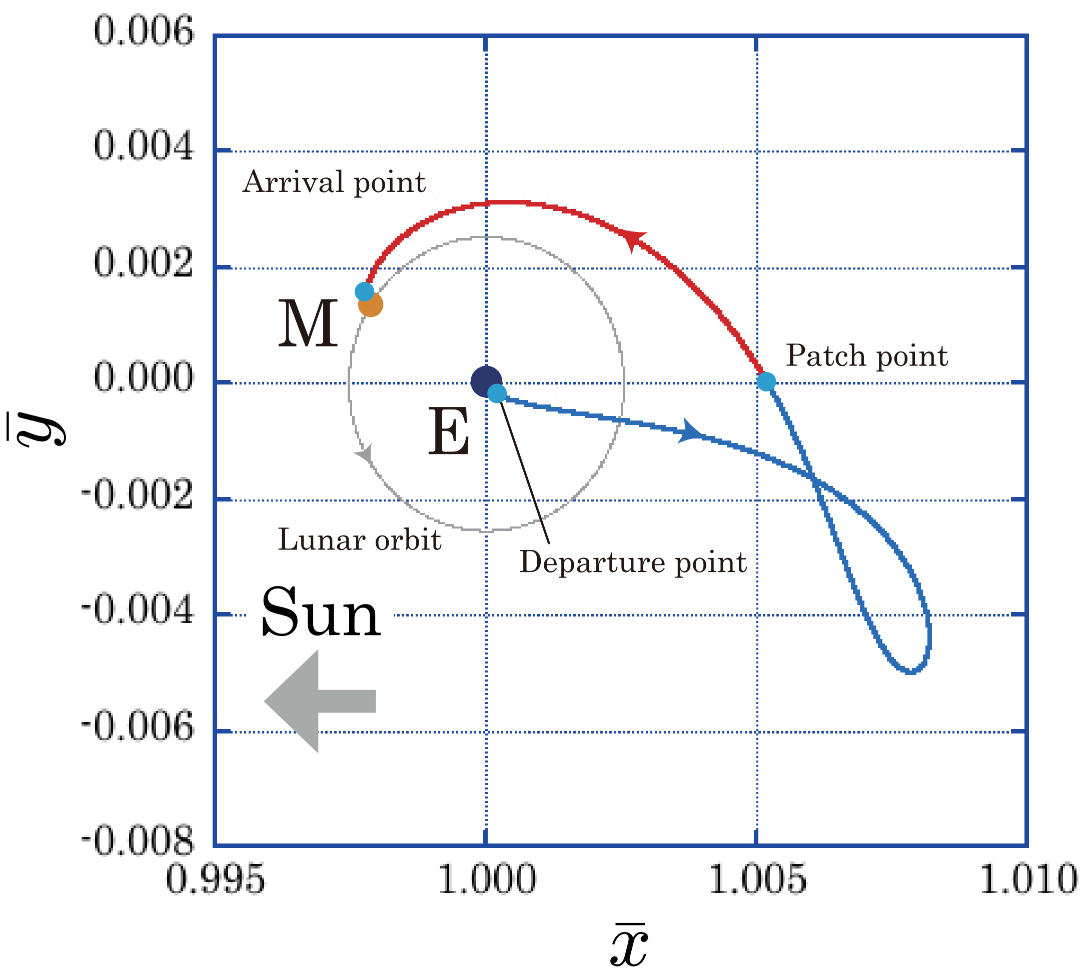
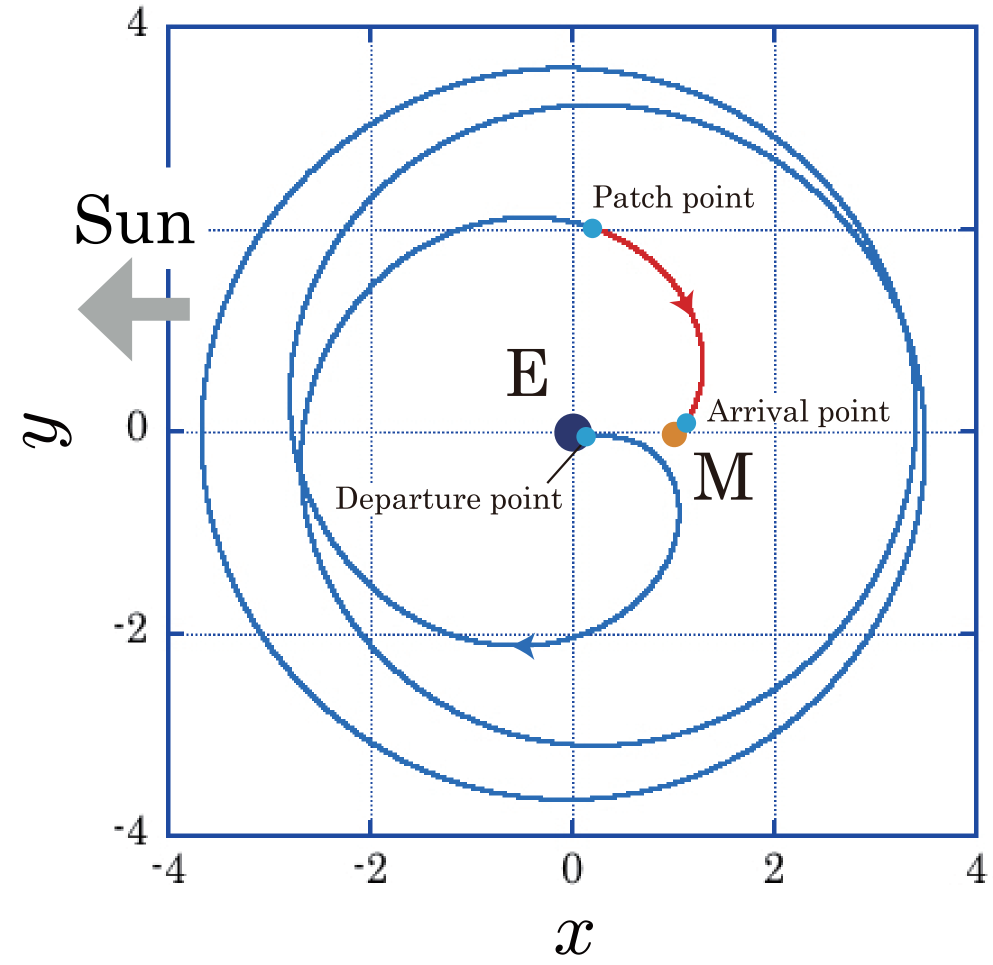
Optimal transfer for a patch point with zero-maneuver

LEO-LLO transfer



Transfer
in the S-E rotating frame



Transfer
in the E-M rotating frame

Transfer	ΔV_{E} [km/s]	ΔV_{M} [km/s]	ΔV_{P} [km/s]	$\Delta V_{\mathrm{Total}}$ [km/s]
Hohmann	3.141	0.838	—	3.979
Coupled PRC3BP	3.537	1.989	0.098	5.624
Proposed approach	3.270	0.642	0	3.912

Conclusions

- We designed the transfer from **the low Earth orbit (LEO)** to **the low lunar orbit (LLO)** in the context of **the coupled planar restricted 3-body system**, namely, the Sun-Earth-spacecraft and Earth-Moon-spacecraft systems.
- We constructed **the family of the departure trajectories (non-transit orbits) parametrized by the energy** by investigating the tube near the Earth. On the other hand, **the family of the arrival trajectories (transit orbits)** was obtained.
- We chosen the patch point so that **the families of the departure and arrival trajectories are intersected** on set section, and then we designed the low energy LEO-LLO transfer. The patch point required the **zero maneuver**, and thus we optimized the maneuver in patching. Further, the total maneuver is **0.068 [km/s] fewer** than the Hohmann transfer.

Thanks for your attention !