Dynamical Systems and Space Mission Design

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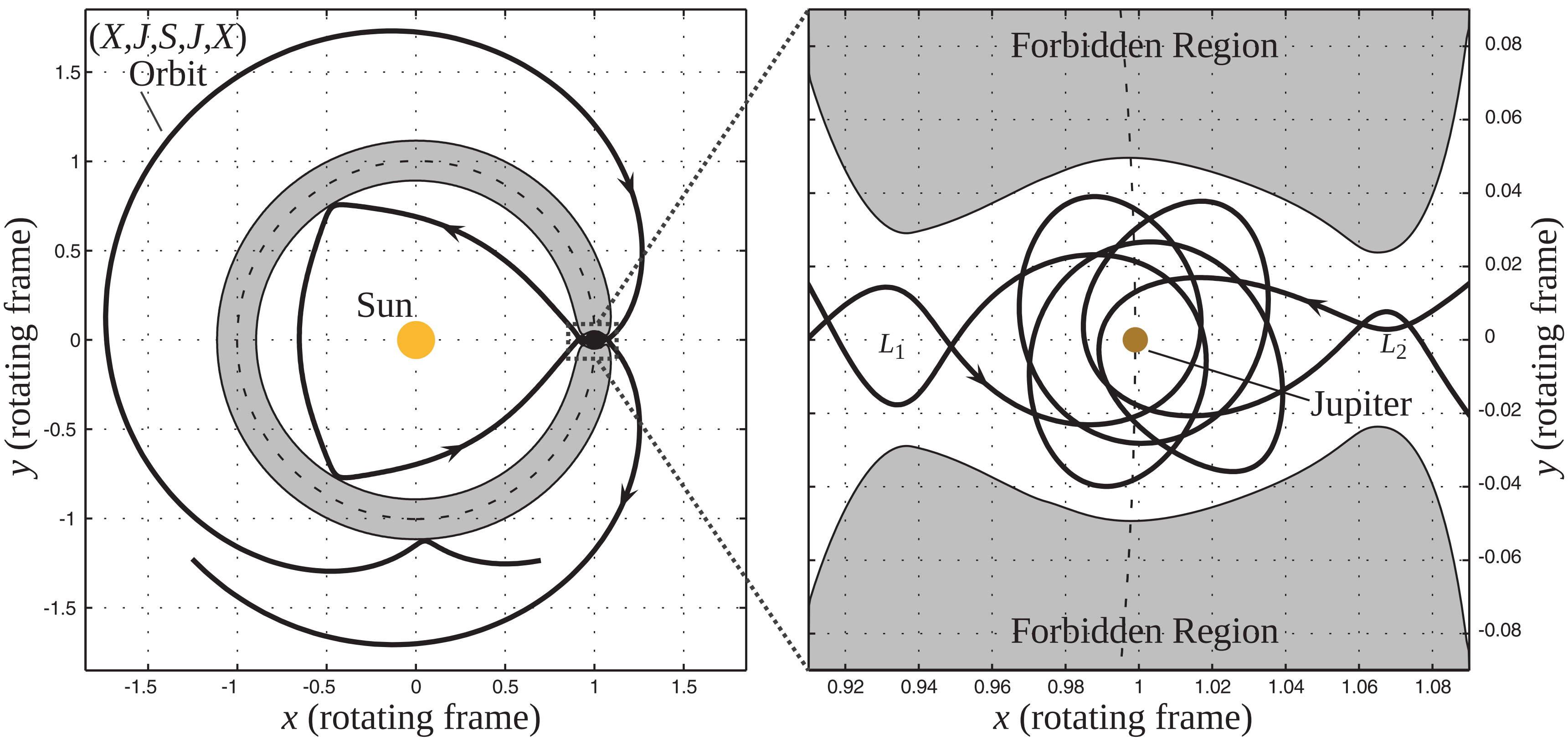
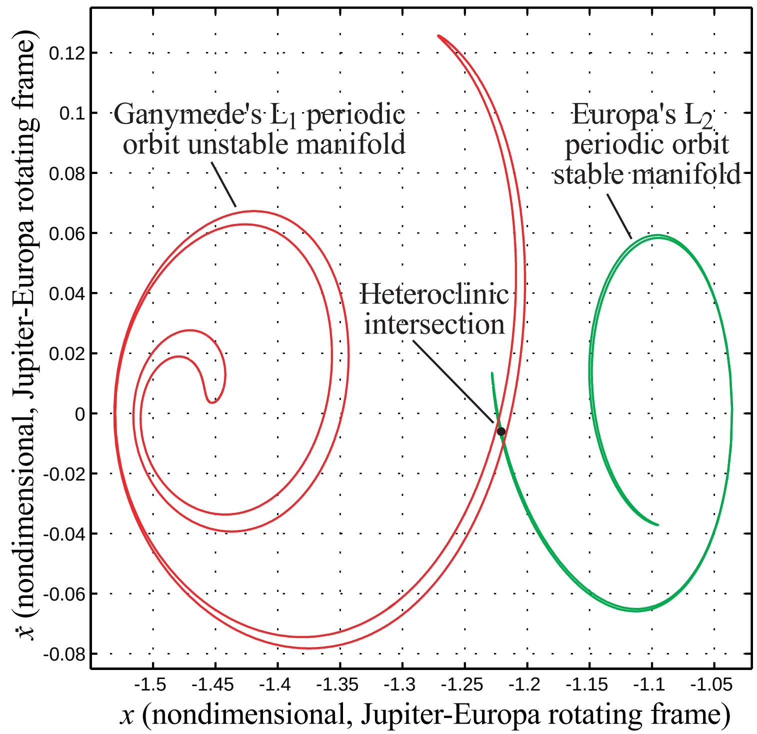
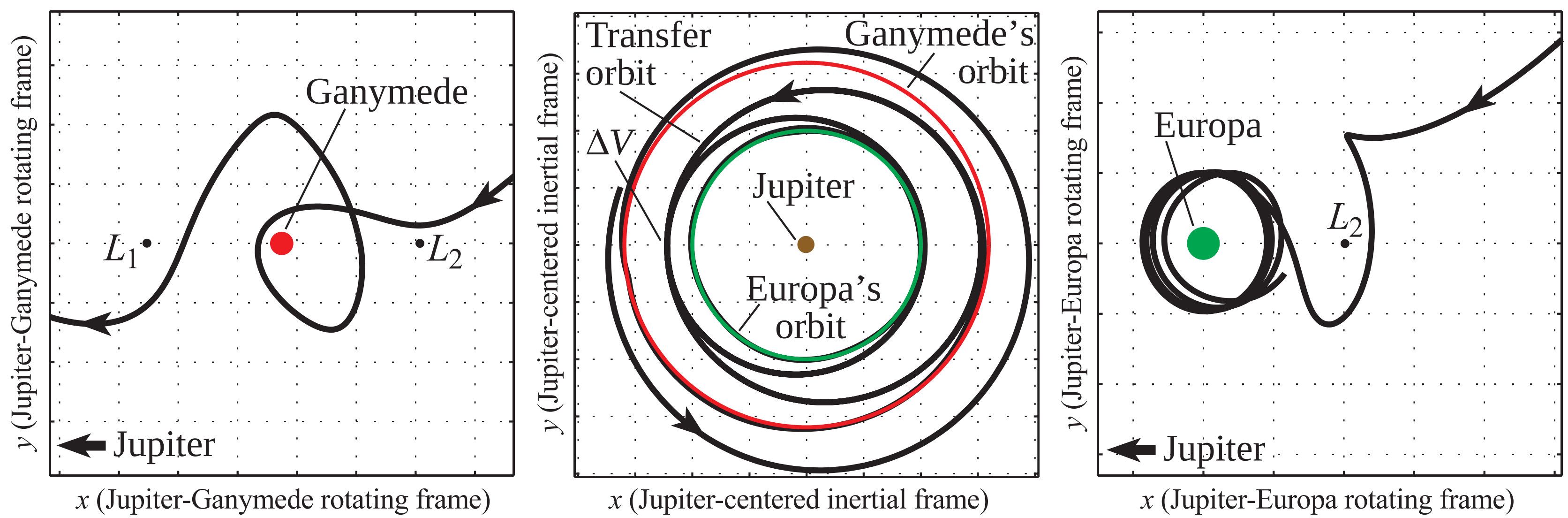
Constructing Orbits of Prescribed Itineraries: Outline

- Outline of Lecture 4A:
  - Using the proof of the **Main Theorem** as the guide, we develop a procedure to construct orbits with **prescribed itineraries**.
  - Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$.
  - Application: **Petit Grand Tour** of Jovian moons.



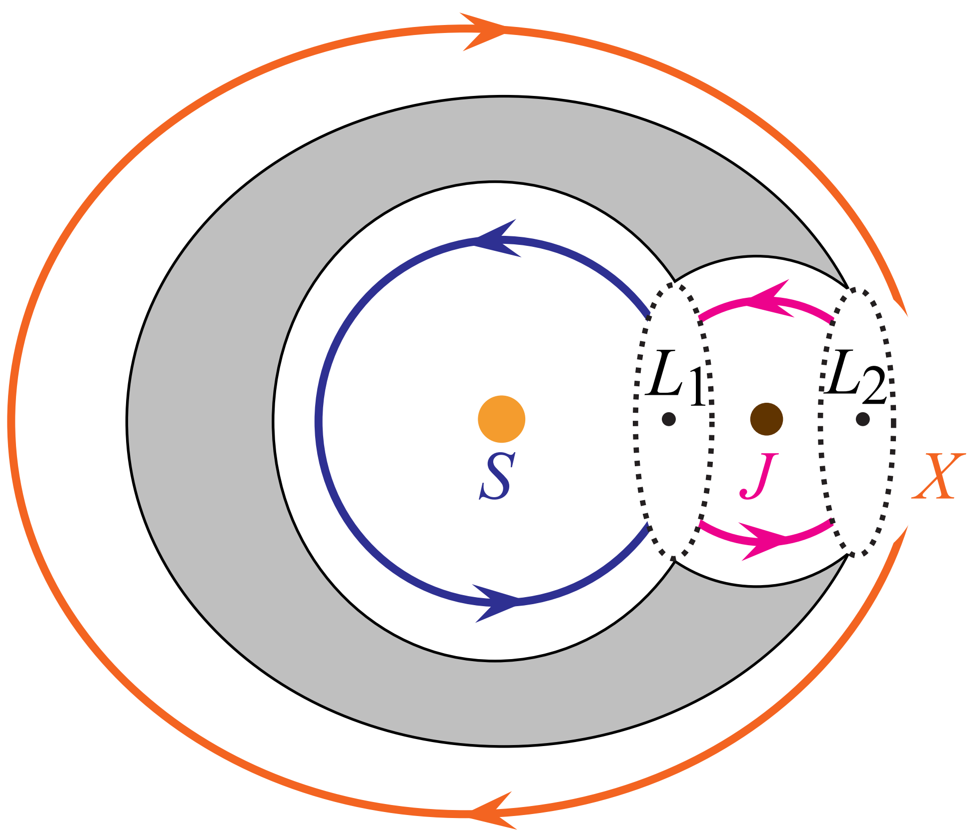
The Petit Grand Tour of the Jovian Moons





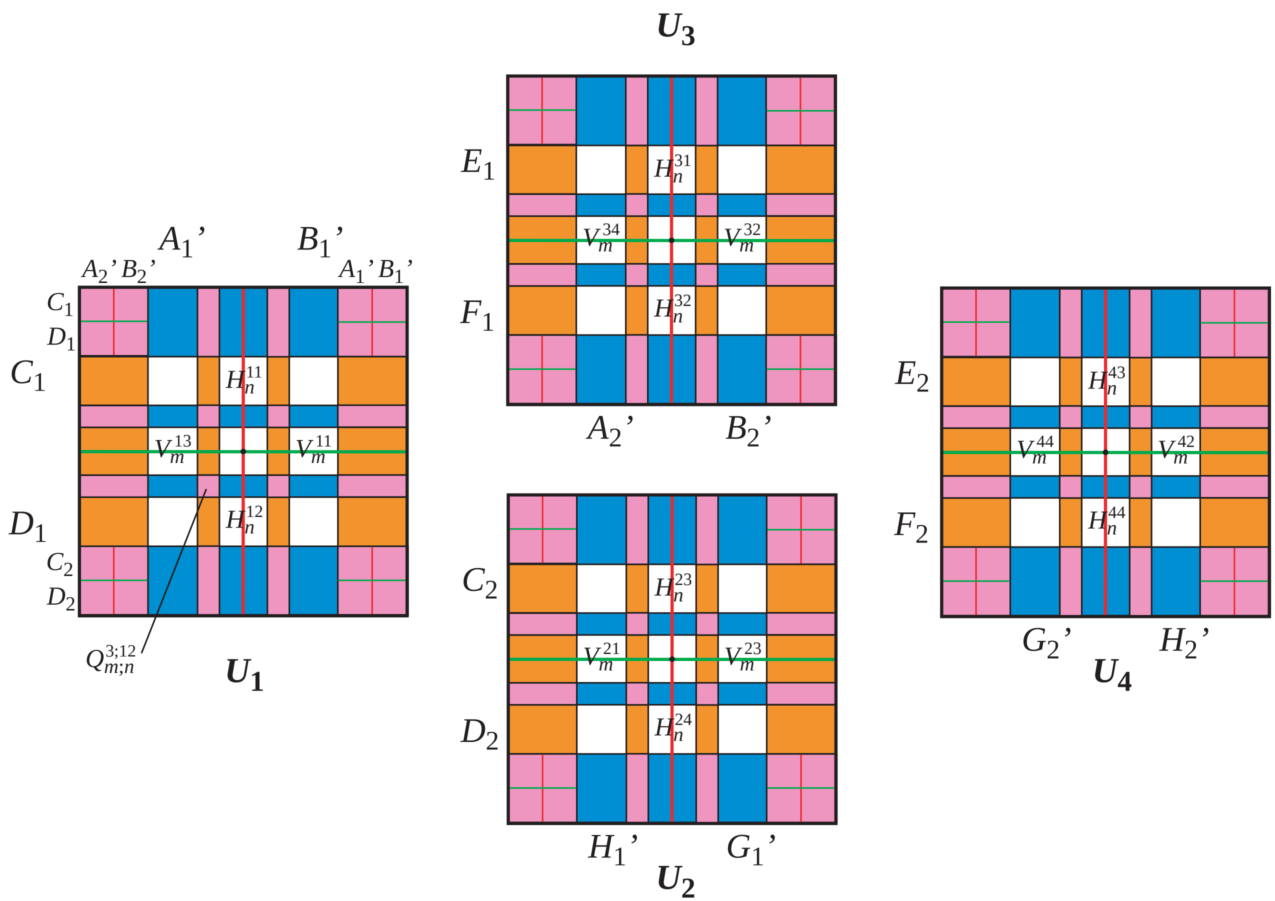
Global Orbit Structure: Review

- **Symbolic sequence** used to label itinerary of each orbit.
- ***Main Theorem***: For any admissible **itinerary**, e.g., $(\ldots, \mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X}, \ldots)$ in the informal notation, there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet (or spacecraft) makes around L_1 & L_2 as well as Sun & Jupiter.



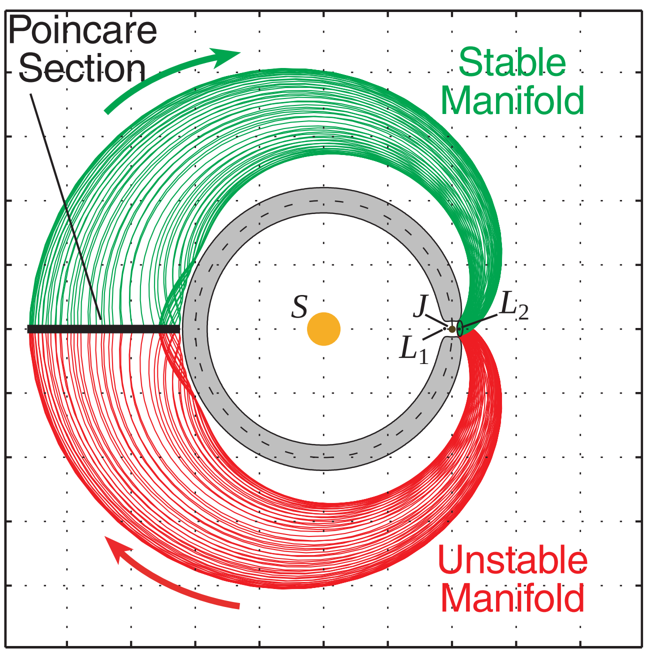
Construction of Orbits with Prescribed Itineraries

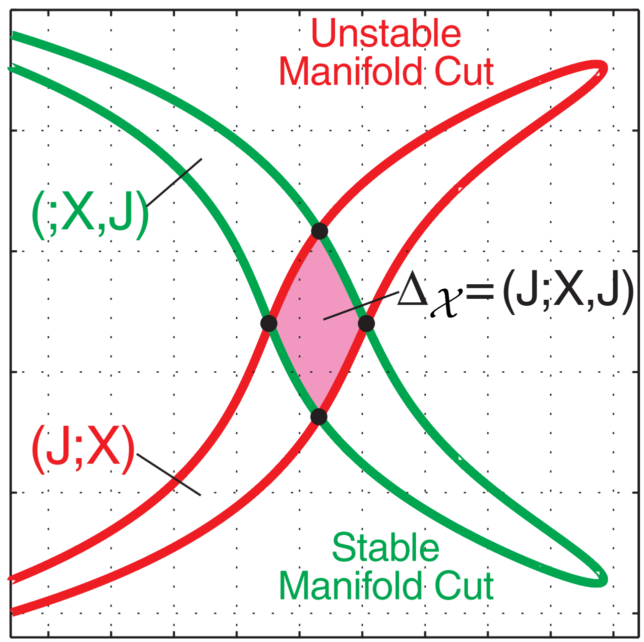
- Recall from Lecture 3B, that the Main Theorem concerning **bi-infinite itineraries** (e.g., $(\dots, \mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X}, \dots)$) relied on the construction of the **invariant set**, a "cloud of points."
- To generate usable trajectories, **truncate** construction of invariant set at a finite number of iterations of the Poincaré map P.



Construction of Orbits with Prescribed Itineraries

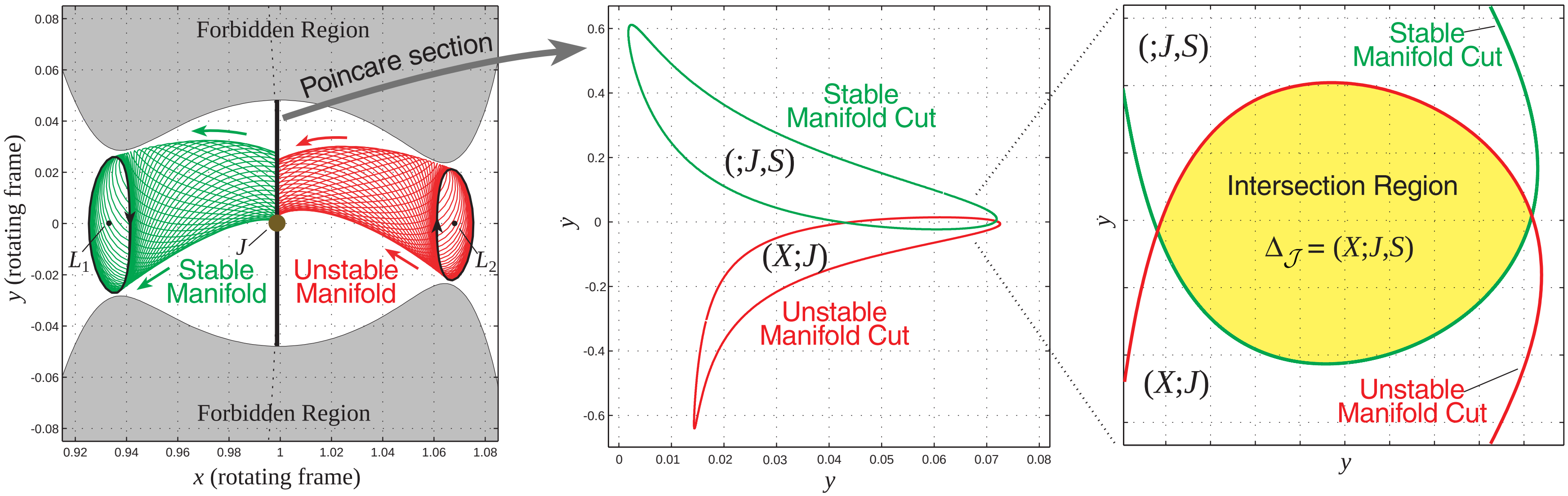
- The sets of orbits with different itineraries are easily visualizable on our chosen Poincaré section as areas in which all the orbits have the same finite itinerary.
- We will also no longer be limited to a small neighborhood of a homoclinic-heteroclinic chain, but can obtain more global results.
- Example: An orbit with itinerary $(\mathbf{J}, \mathbf{X}, \mathbf{J})$.





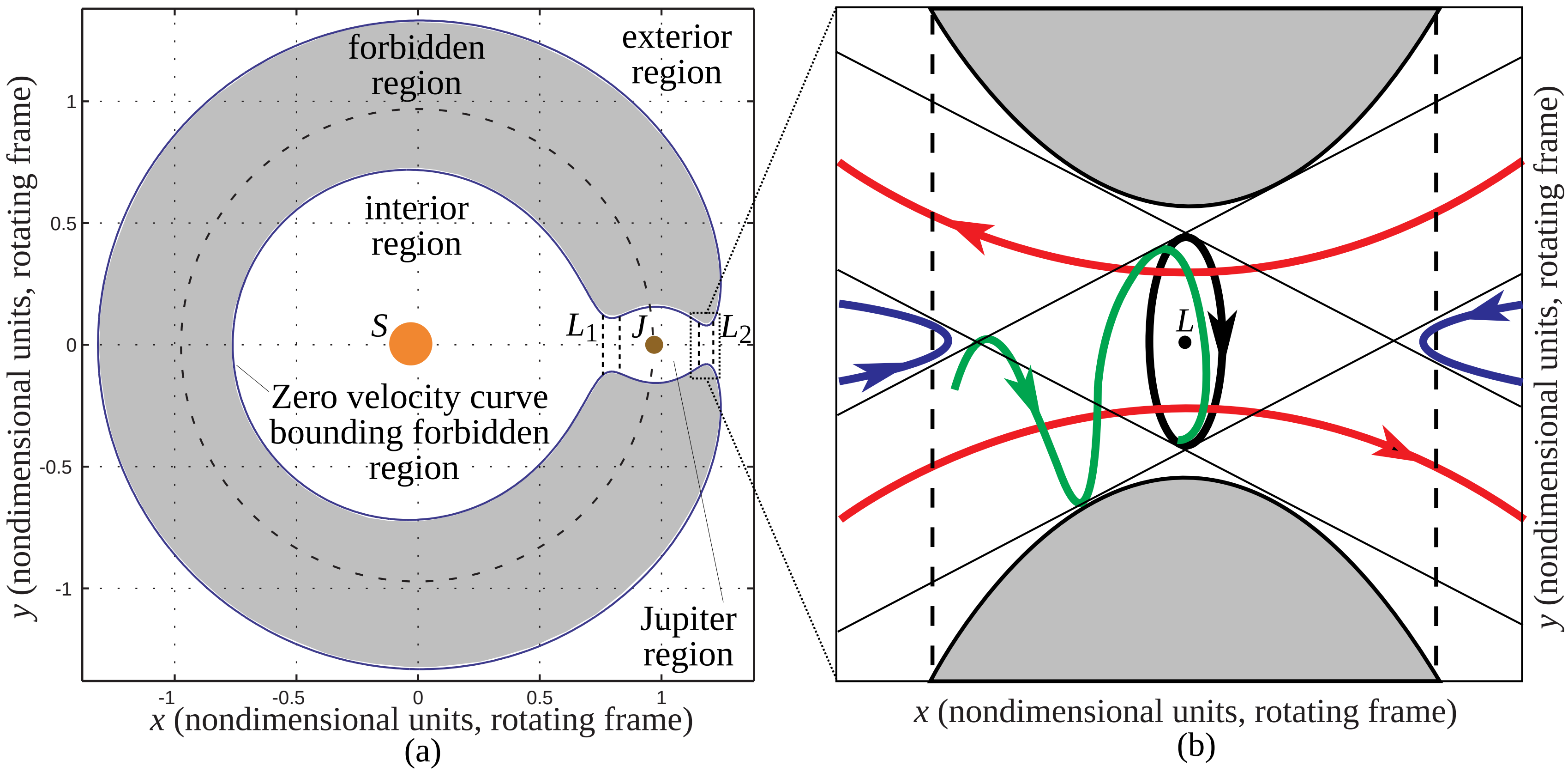
Construction of Orbits with Prescribed Itineraries

- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}, \mathbf{S})$, passing from the exterior region to the interior region via the Jupiter region.
- Look at Poincaré section where L_2 periodic orbit **unstable manifold** intersects L_1 periodic orbit **stable manifold**.
- Intersection region contains orbits with itinerary $(\mathbf{X}, \mathbf{J}, \mathbf{S})$.



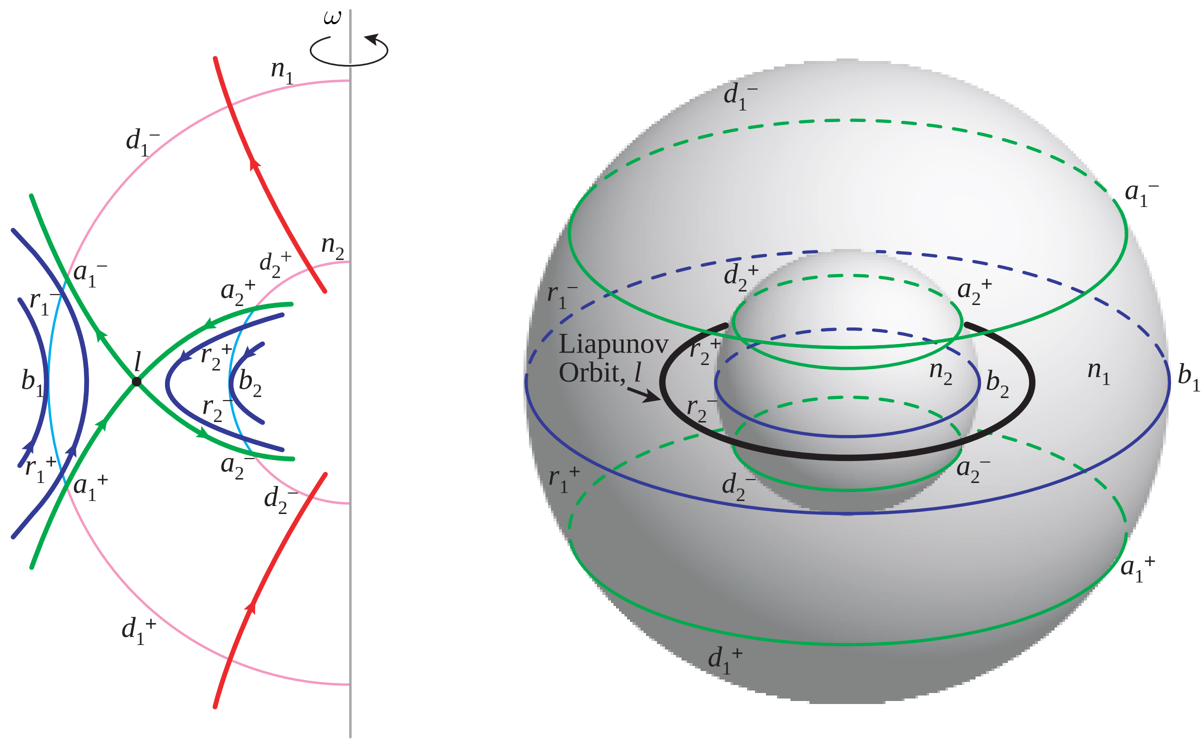
The Flow near L_1 and L_2: Review

- Recall from Lecture 2B, for energy values just above that of L_2,
  - **Hill's region** contains a "**neck**" about L_1 and L_2.
- [Conley] The flow in the **equilibrium region** $\mathcal{R}$ has **4 types** of orbits: **periodic**, **asymptotic**, **transit** and **non-transit** orbits.



McGehee Representation: Equilibrium Region $\mathcal{R}$

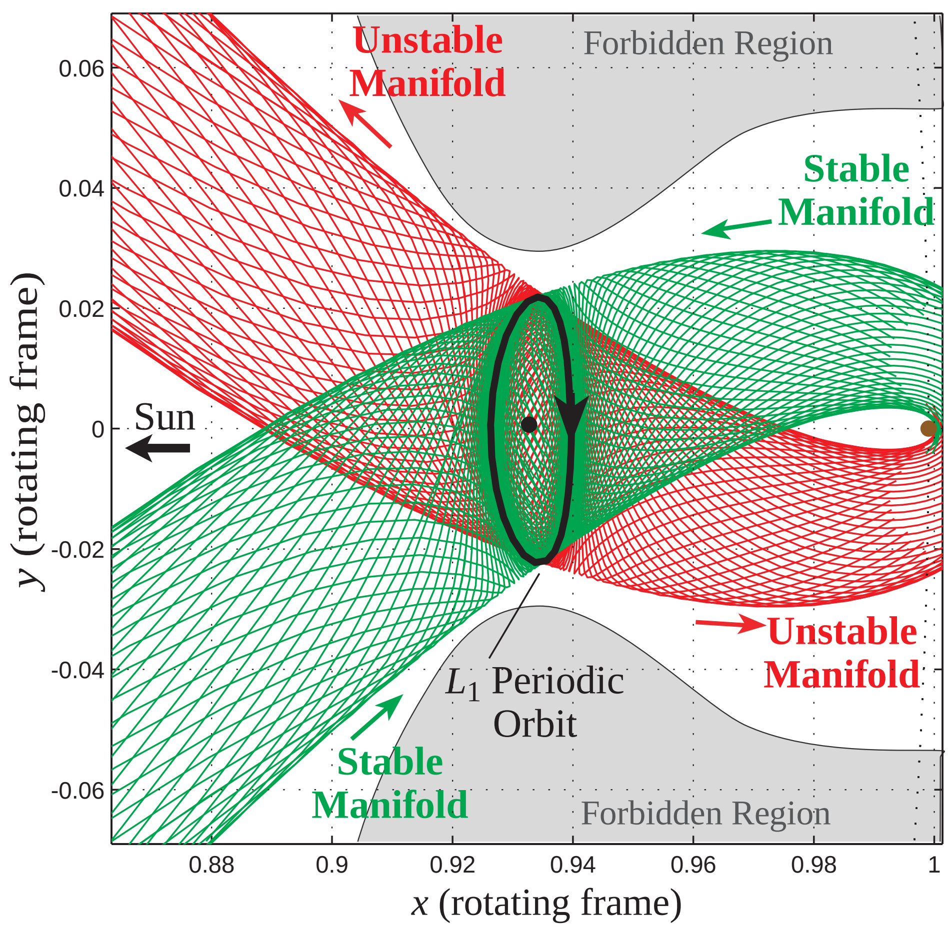
- **4 types** of orbits in equilibrium region $\mathcal{R}$:
  - **Black** circle l is the unstable **periodic** orbit.
  - 4 cylinders of **asymptotic** orbits form pieces of stable and unstable manifolds. They intersect the bounding spheres at asymptotic circles, separating **spherical caps**, which contain **transit** orbits, from spherical zones, which contain non-transit orbits.



(a) (b)

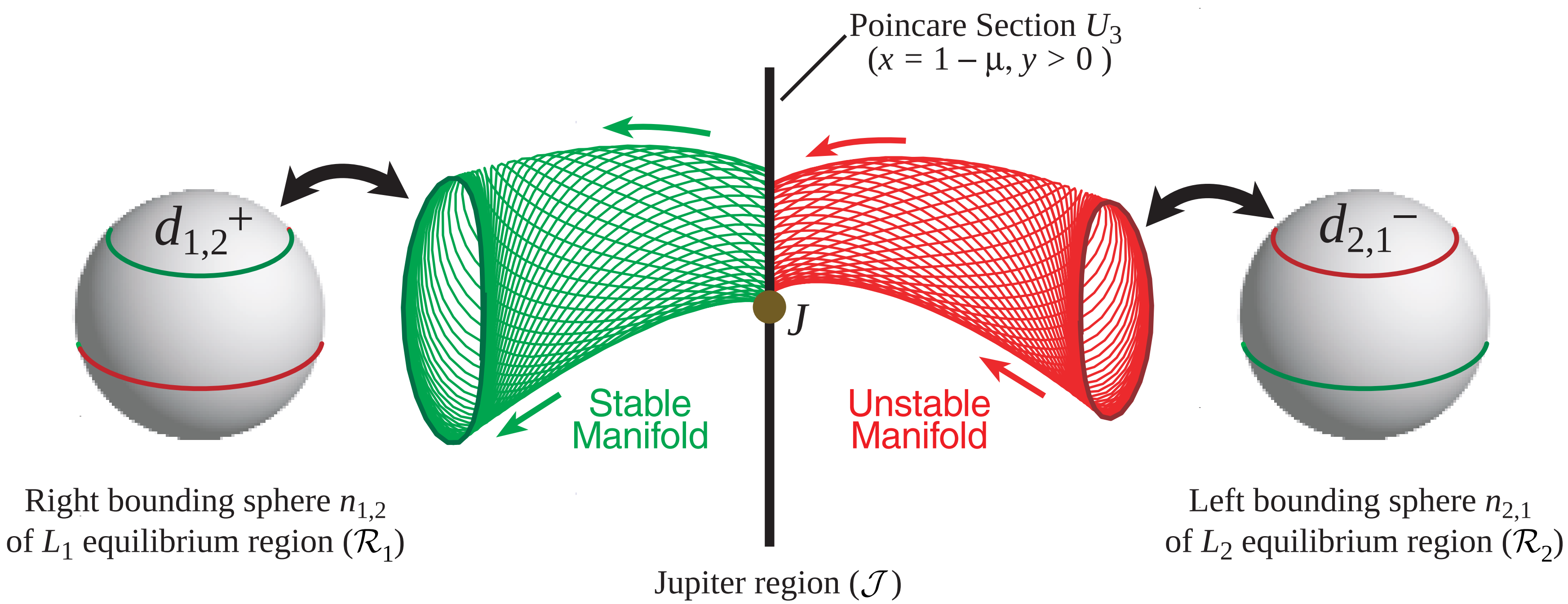
Tubes Partition the Energy Surface

- Cylinders of asymptotic orbits form **“tubes”** in the energy surface.
  - Periodic orbit **stable** and **unstable** manifold tubes.
  - To transit from one region to another, must be inside the tubes.
  - e.g., transit from Jupiter to Sun region via L_1 periodic orbit tubes.



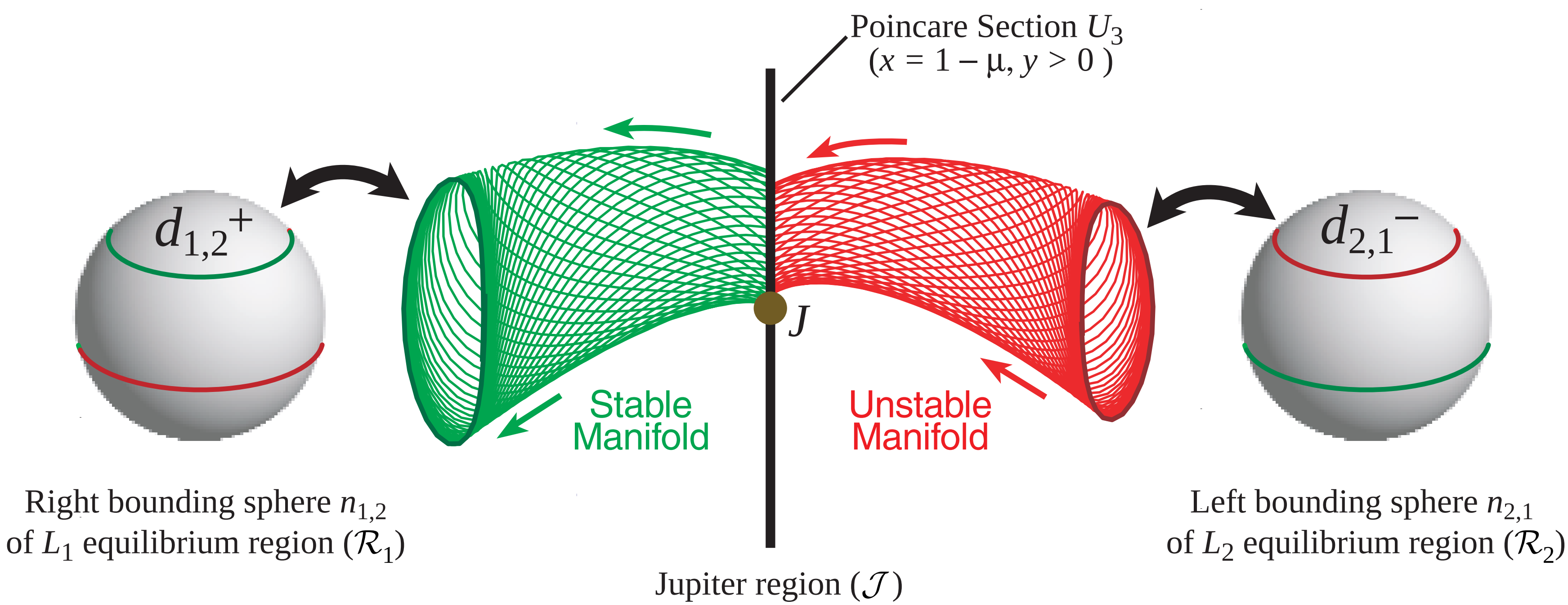
Spherical Caps Containing Transit Orbits

- We will be focusing on the **spherical caps** of **transit orbits**. In particular, we will look at their images and pre-images on a suitable Poincaré section, since these spherical caps will be building blocks from which we construct orbits of prescribed itineraries.
  - The images and pre-images of the spherical caps form the tubes that partition the energy surface.

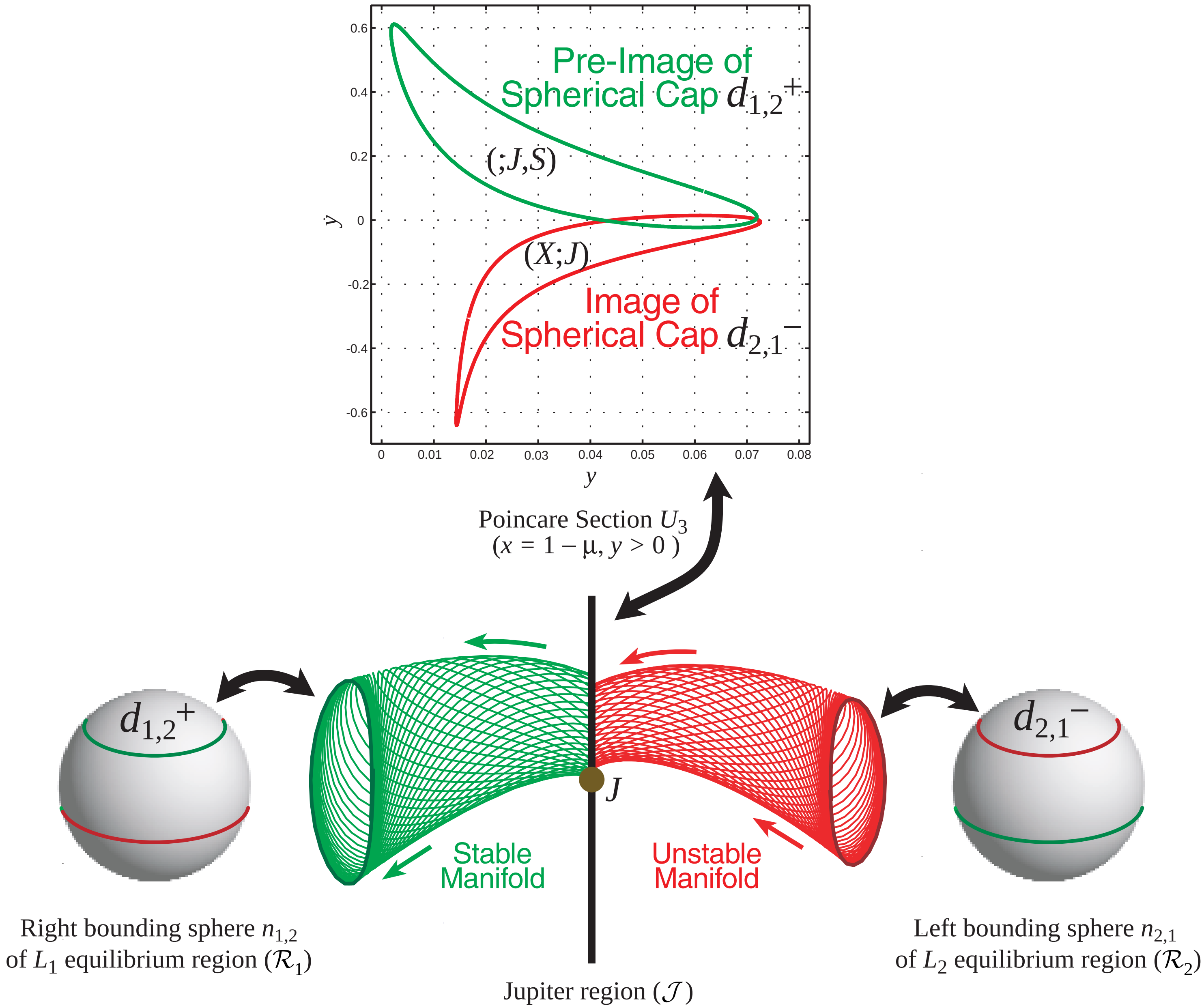


Poincaré Section: Images of Bounding Sphere Caps

- For instance, on a Poincaré section between L_1 and L_2,
  - We look at the **image of the cap** on the left bounding sphere of the L_2 equilibrium region $\mathcal{R}_2$ containing **orbits leaving** $\mathcal{R}_2$.
  - We also look at the **pre-image of the cap** on the right bounding sphere of $\mathcal{R}_1$ containing **orbits entering** $\mathcal{R}_1$.

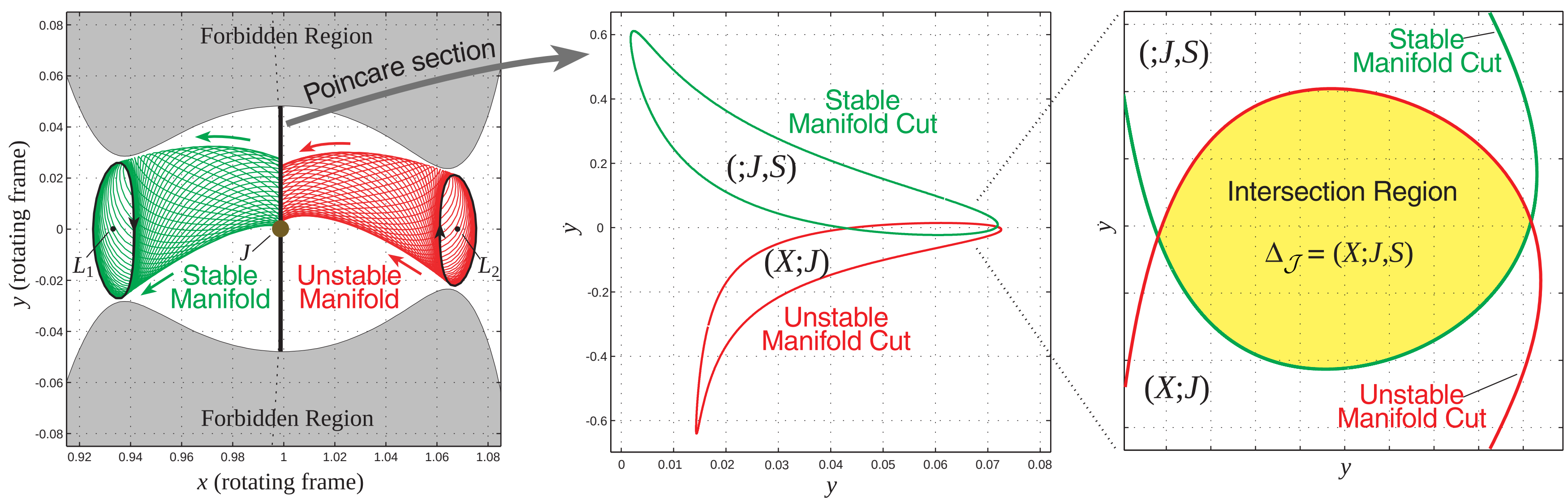


Poincaré Section: Images of Bounding Sphere Caps



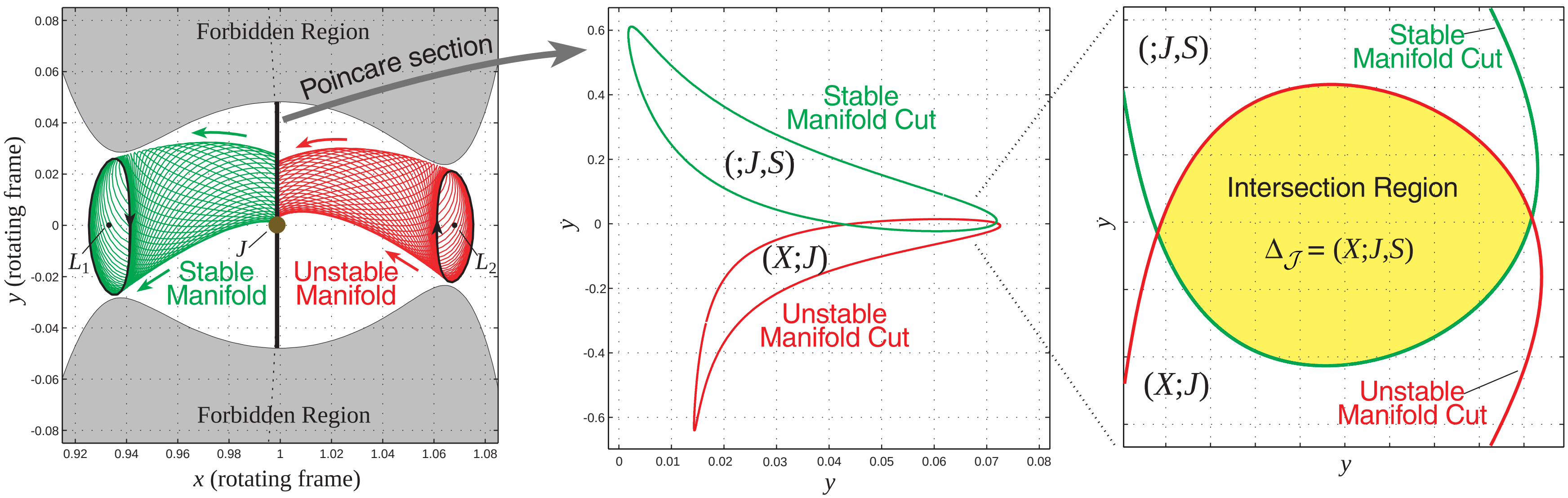
Construction of Orbits with Prescribed Itineraries

- The Poincaré cut of the **unstable manifold** of the L_2 periodic orbit forms the boundary of the **image of the cap** containing transit orbits leaving $\mathcal{R}_2$.
  - All of these orbits came from the exterior region and are now in the Jupiter region, so we label this region $(\mathbf{X}; \mathbf{J})$.



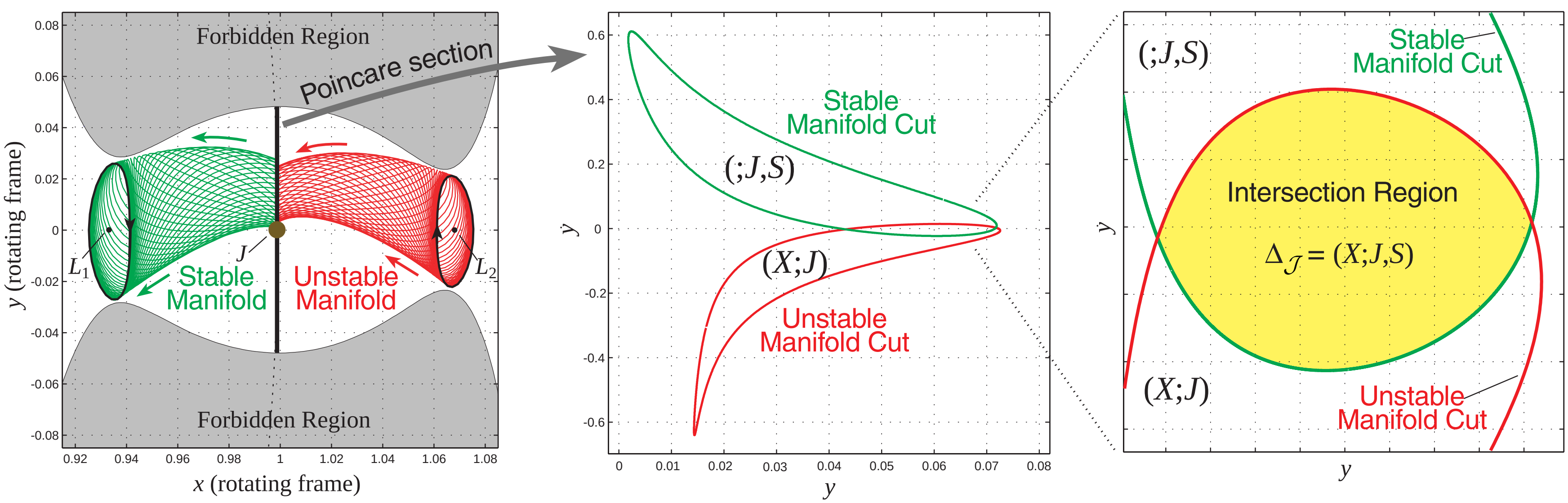
Construction of Orbits with Prescribed Itineraries

- Similarly, the Poincaré cut of the **stable manifold** of the L_1 periodic orbit forms the boundary of the **pre-image of the cap** of transit orbits entering $\mathcal{R}_1$.
  - All of these orbits are now in the Jupiter region and are headed for the interior (Sun) region, so we label this region $(;\mathbf{J},\mathbf{S})$.



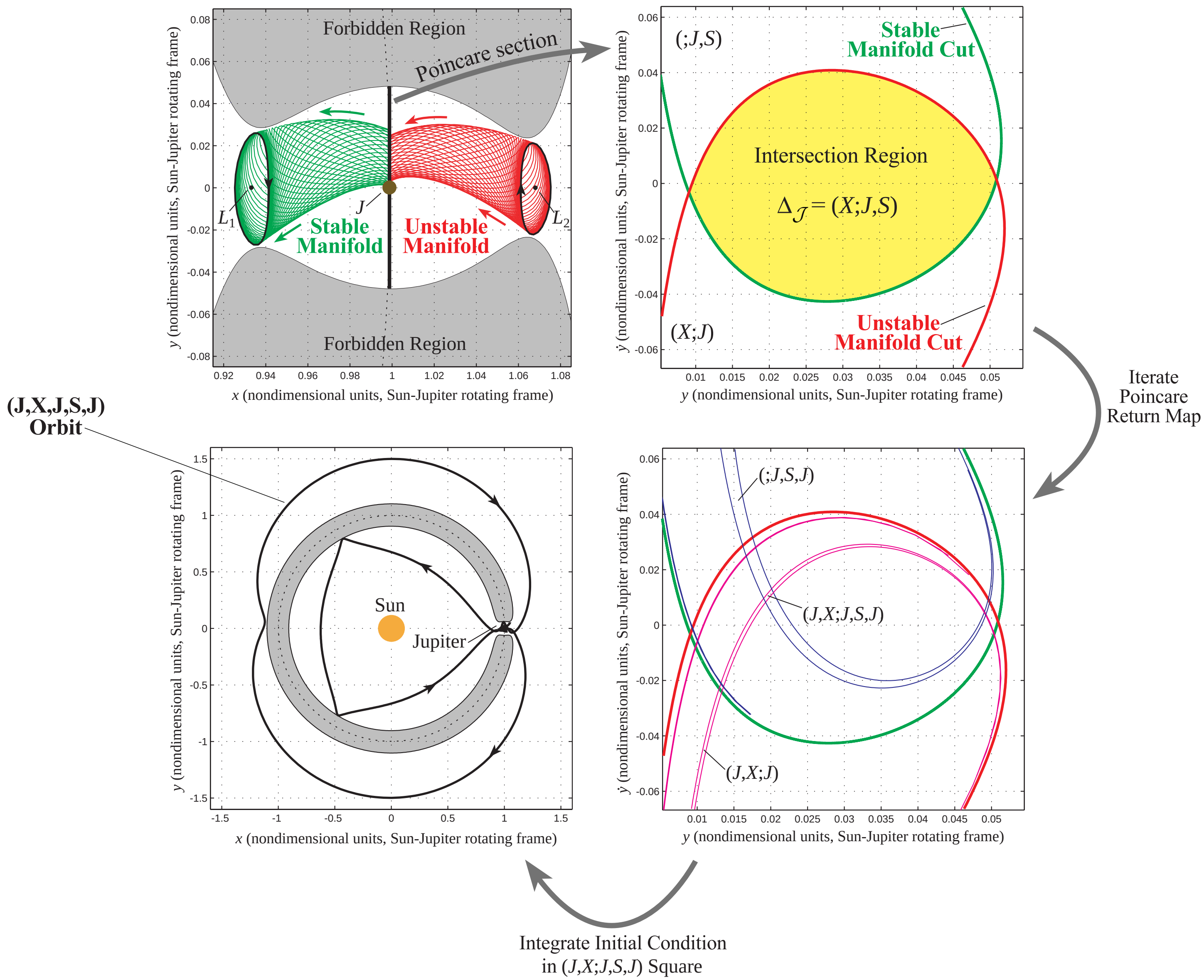
Construction of Orbits with Prescribed Itineraries

- Note that the **image of the $\mathcal{R}_2$ cap** and the **pre-image of the $\mathcal{R}_1$ cap** intersect. We label this **intersection region** $\Delta_{\mathcal{J}}$.
  - This is the set of orbits which came from the exterior region, are now in the Jupiter region, and are headed for the interior (Sun) region, so we label this region $(\mathbf{X}; \mathbf{J}, \mathbf{S})$.
  - Integrating an initial condition in this set forwards and backwards would give us an orbit with the desired itinerary $(\mathbf{X}, \mathbf{J}, \mathbf{S})$.



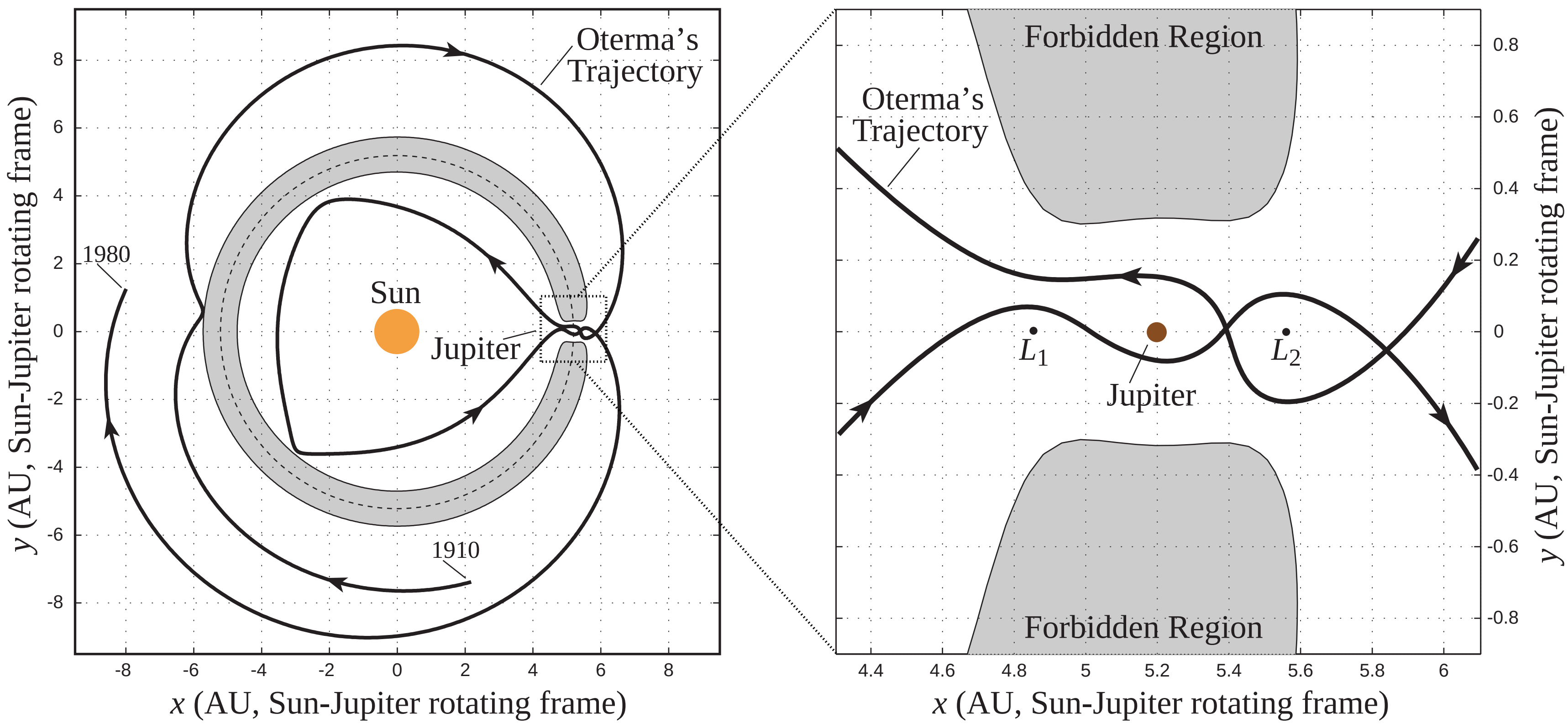
Construction of Orbits with Prescribed Itineraries

- We can partition the intersection region even further using this technique. Smaller regions of orbits with longer itineraries emerge.



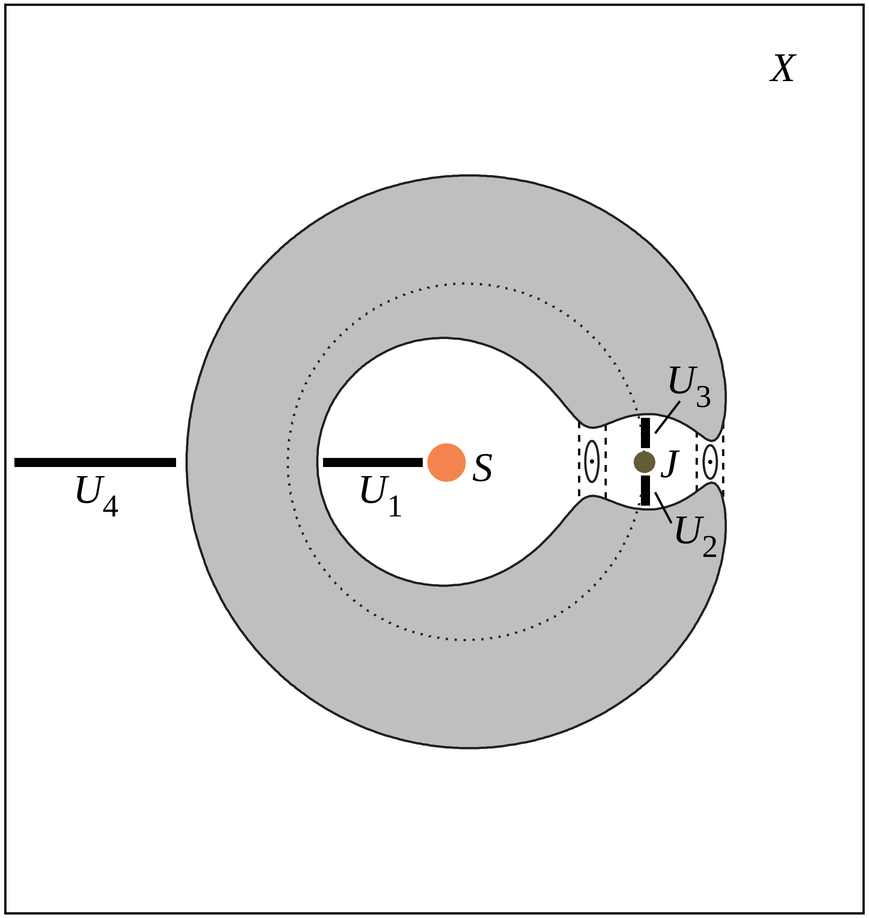
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

- We wish to illustrate these techniques by constructing an orbit with an itinerary $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$.
  - This is the itinerary for the comet ***Oterma*** in the Sun-Jupiter system during the time interval of interest.



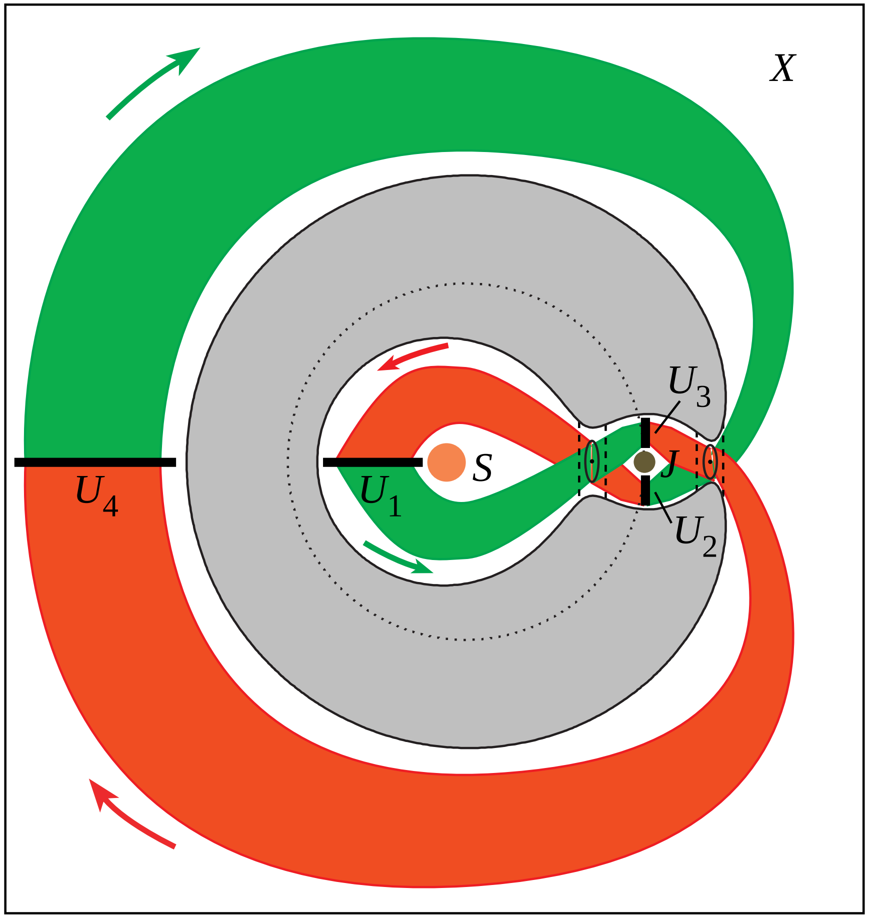
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

- Before beginning, we review the notation regarding the location of the Poincaré sections used for the construction.
  - U_1 in interior (Sun) region, U_4 in exterior region
  - U_2 and U_3 in Jupiter region



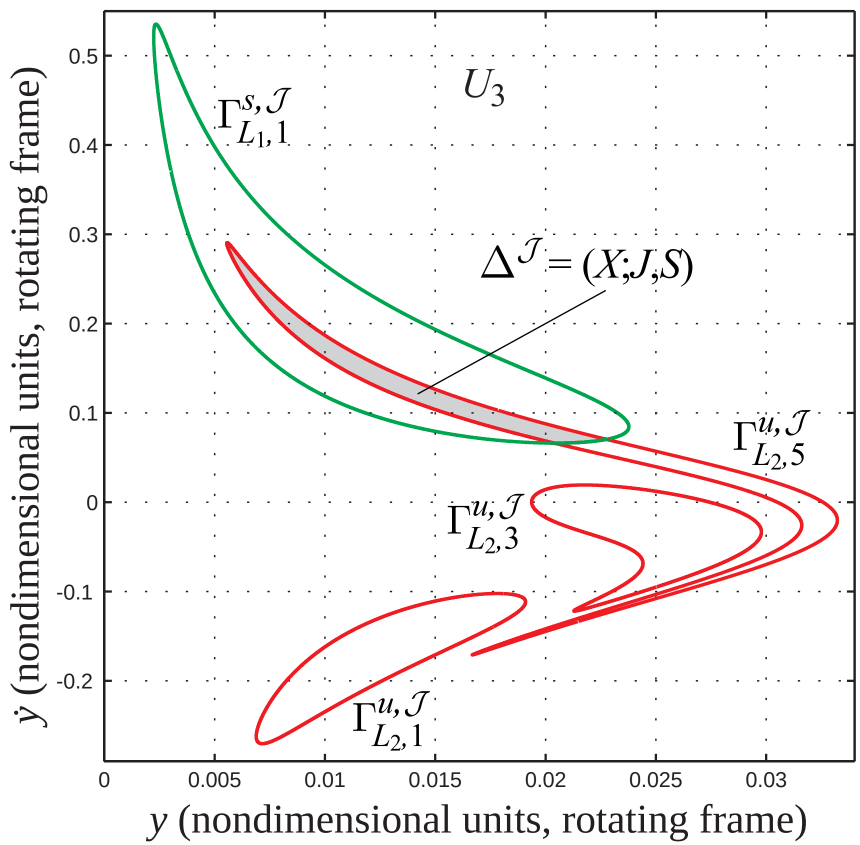
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

- Note also the locations of the **stable** and **unstable** manifolds of the L_1 and L_2 periodic orbits.
  - Note the mirror symmetry about the Sun-Jupiter line.



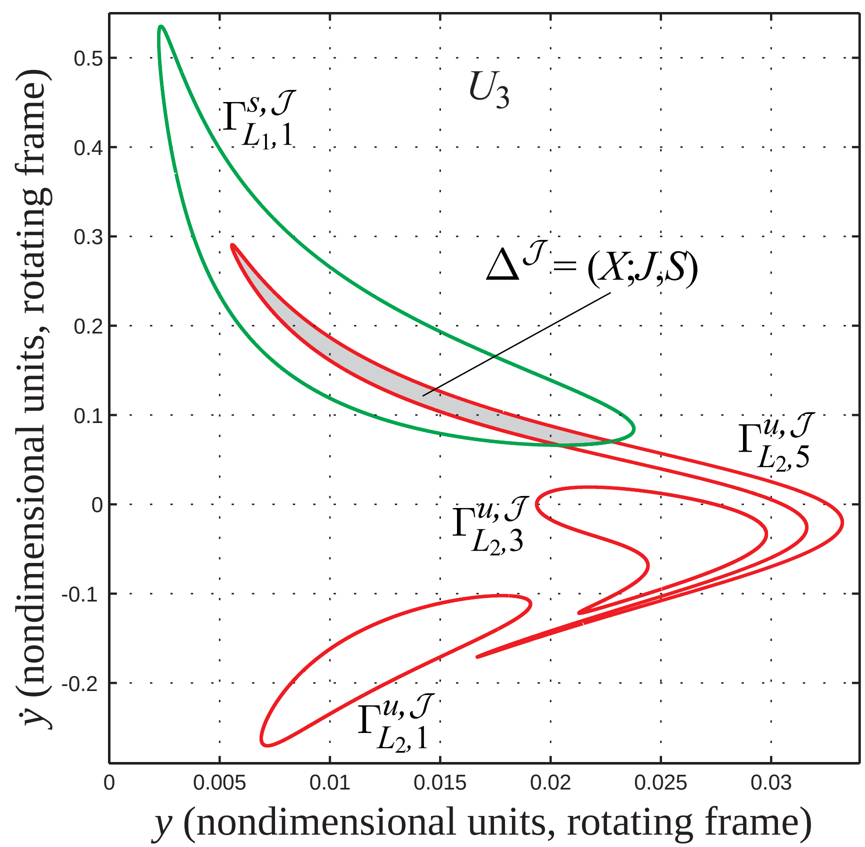
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

- We compute the first few transversal cuts of the L_1 **stable** and L_2 **unstable** periodic orbit manifolds on the U_3 Poincaré section in the Jupiter region.
  - Notice the intersection region $\Delta^{\mathcal{J}}$, in which all orbits have the central block itinerary $(\mathbf{X}; \mathbf{J}, \mathbf{S})$.



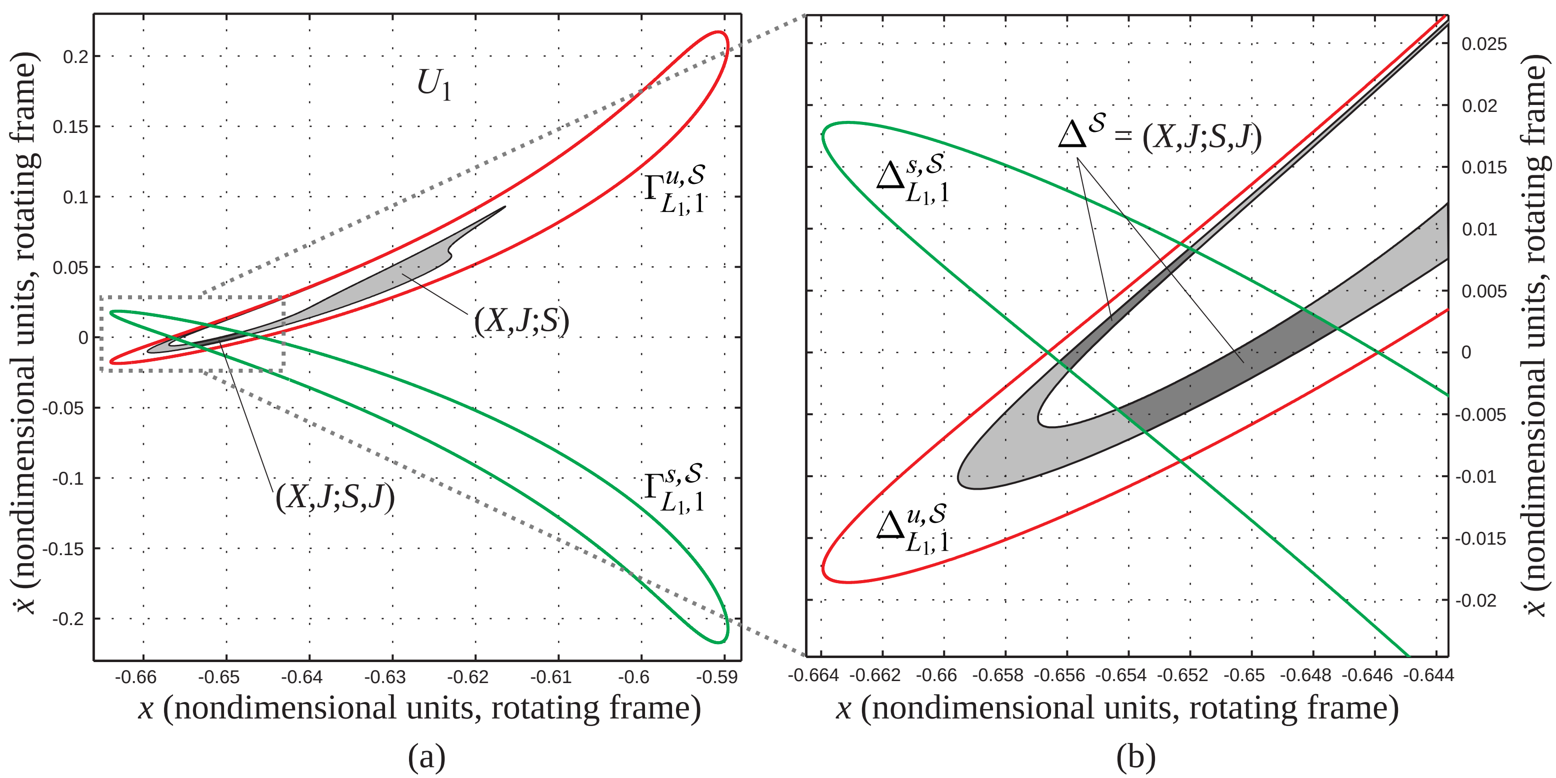
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

- All orbits entering the Sun region must pass through the **first cut of the L_1 p.o. stable manifold**.
  - Notice that the L_2 **p.o. unstable manifold** swings around Jupiter several times before intersecting the **first cut of the L_1 p.o. stable manifold**.
  - The intersection, containing $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbits, is shown shaded.



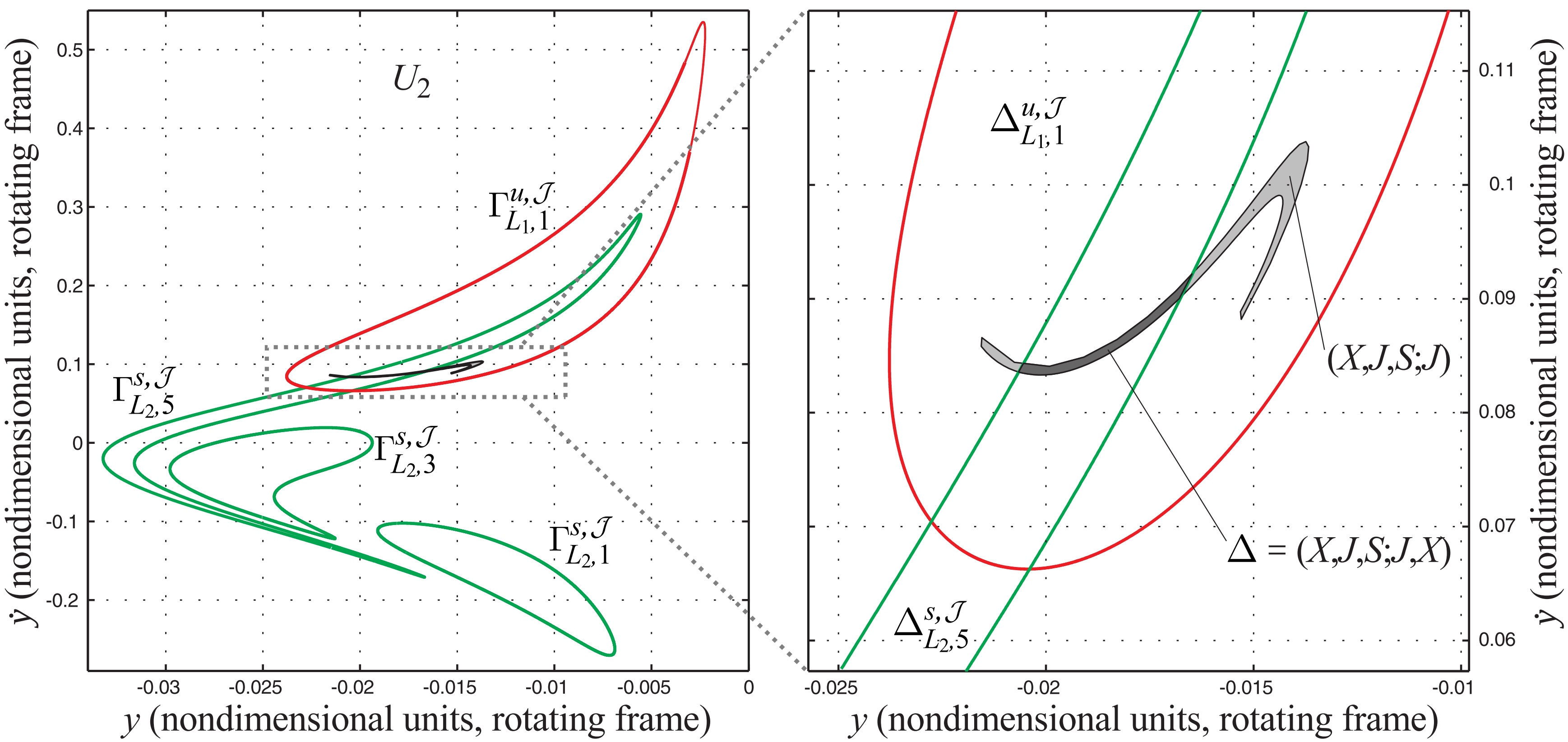
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

- Follow the **shaded** $\Delta^{\mathcal{J}} = (\mathbf{X}; \mathbf{J}, \mathbf{S})$ region forward under the flow until it intersects the U_1 Poincaré section in the Sun region.
  - The image $(\mathbf{X}, \mathbf{J}; \mathbf{S})$ is contained entirely inside the region bounded by the L_1 **p.o. unstable manifold**, as expected.
  - **Pieces** that also intersect the L_1 **p.o. stable manifold** will return to the Jupiter region and are labelled $\Delta^{\mathcal{S}} = (\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J})$.



Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

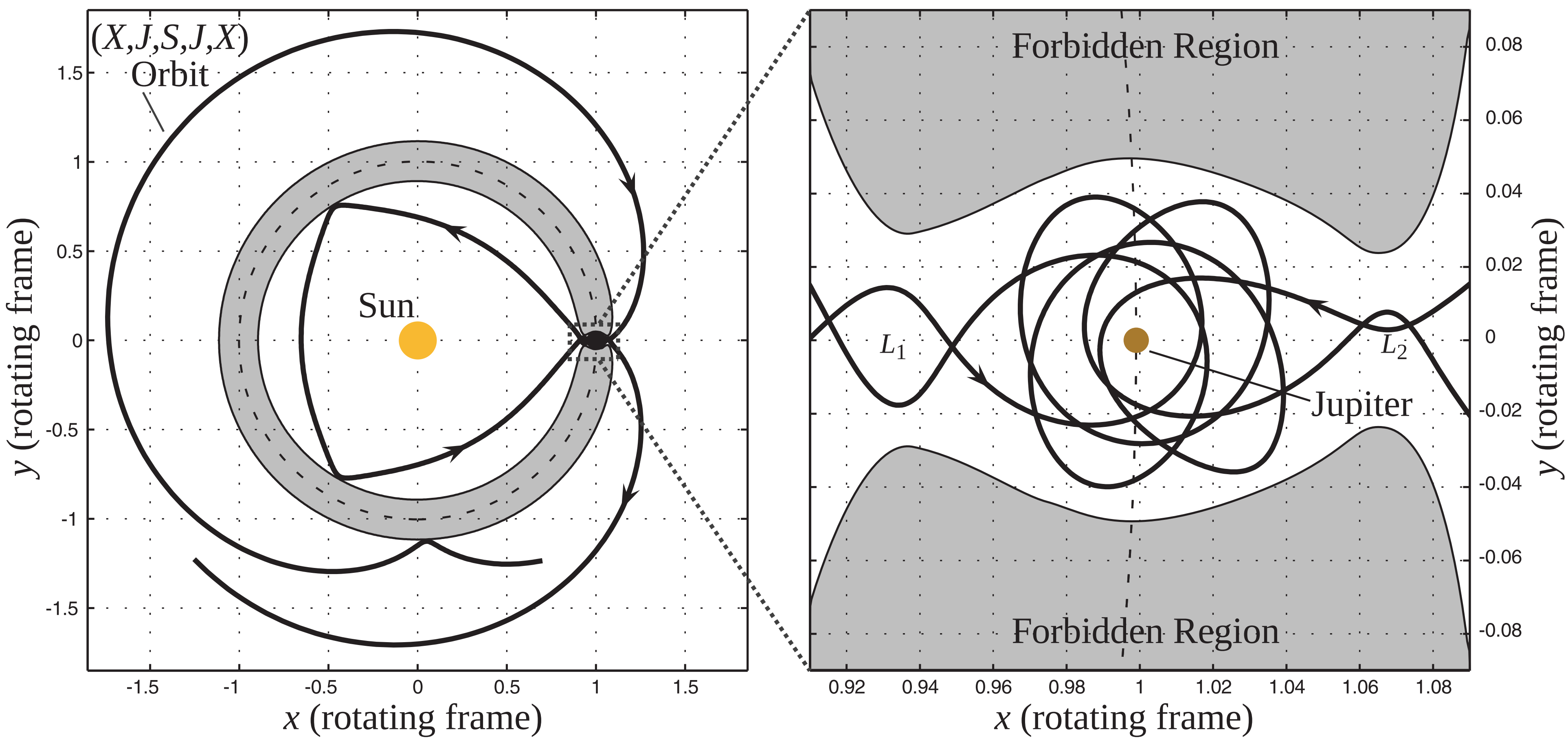
- Follow the **larger strip of** $\Delta^{\mathcal{S}} = (\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J})$ forward until it returns to the Jupiter region, intersecting the U_2 Poincaré section.
  - The **piece** contained inside the region bounded by the L_2 **p.o. stable manifold** will return to the exterior region.
  - Region $\Delta = (\mathbf{X}, \mathbf{J}, \mathbf{S}; \mathbf{J}, \mathbf{X})$ contains orbits with desired itinerary.



(a) (b)

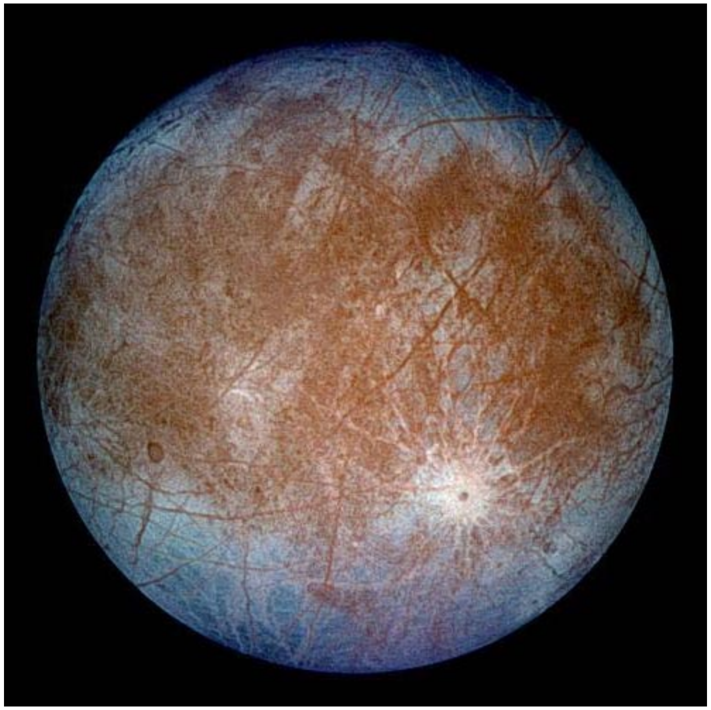
Example Itinerary: $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$

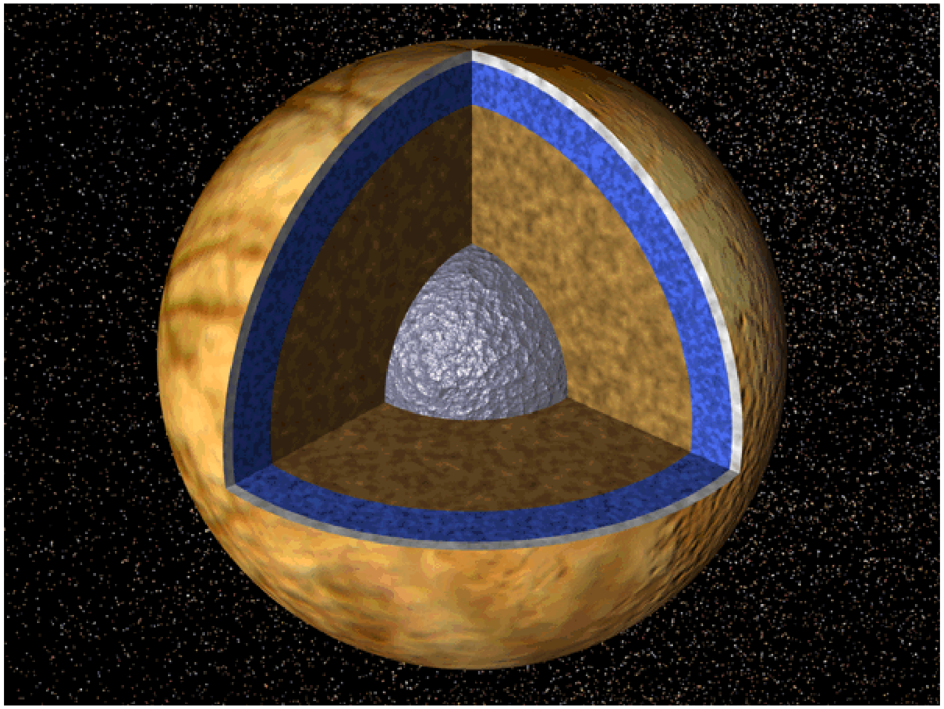
- Pick any initial condition in the region $\Delta = (\mathbf{X}, \mathbf{J}, \mathbf{S}; \mathbf{J}, \mathbf{X})$ and integrate forward and backward.
  - We obtain orbit with the desired prescribed itinerary $(\mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X})$.



Application: Petit Grand Tour of Jovian Moons

- Recently, there has been interest in sending a scientific spacecraft to orbit and study **Europa**.
  - The Jovian moon may have subterranean oceans and life.



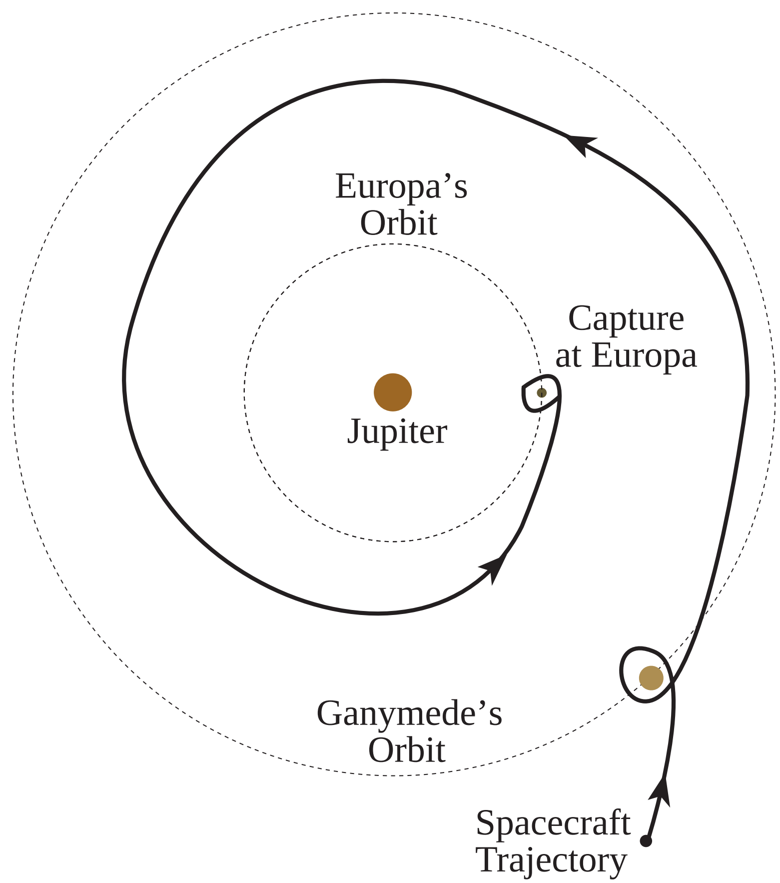


Application: Petit Grand Tour of Jovian Moons

- The trajectory design involved in effecting an orbital **capture by Europa** presents formidable challenges to the traditional two-body "patched conic" approach.
  - Regimes of motion involved depend heavily on three-body dynamics.
- **New three-body perspective** is necessary to design a feasible trajectory from Earth to Europa using a very limited fuel budget.
  - Fortunately, Jupiter's vicinity and the Galilean moons provide enough interesting orbital dynamics that the construction of a relatively low fuel mission is possible.

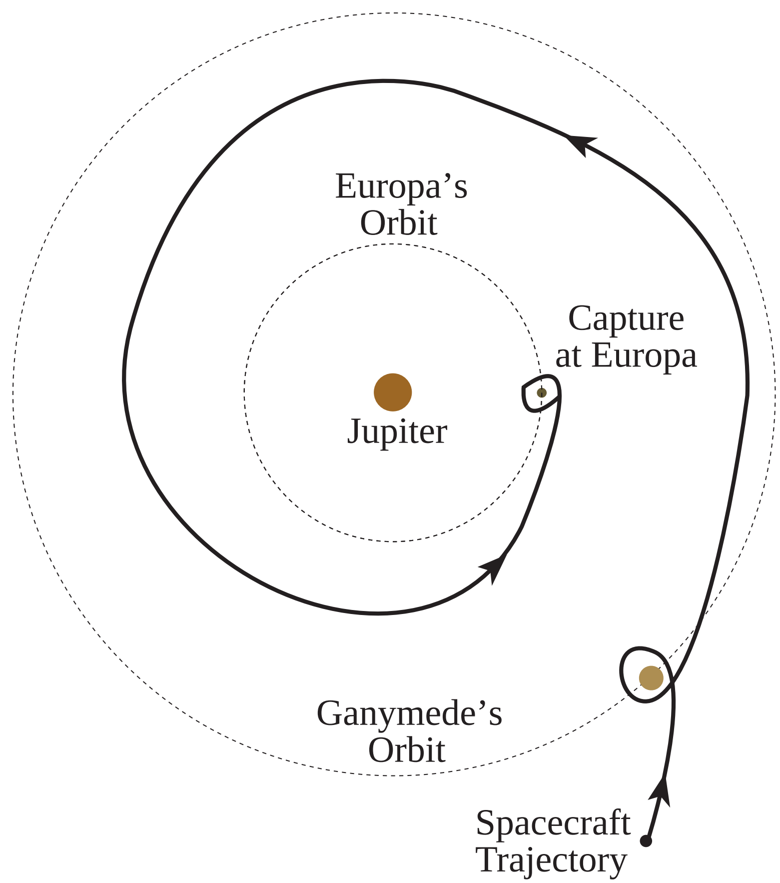
Petit Grand Tour: Ganymede and Europa

- For this initial exploration, we study the final leg of the trip, once the spacecraft is in a large orbit around Jupiter.
- The **goal** is to perform a **tour** of two of the Jovian moons, Ganymede and Europa.
  - Ending with a **capture orbit around Europa**.



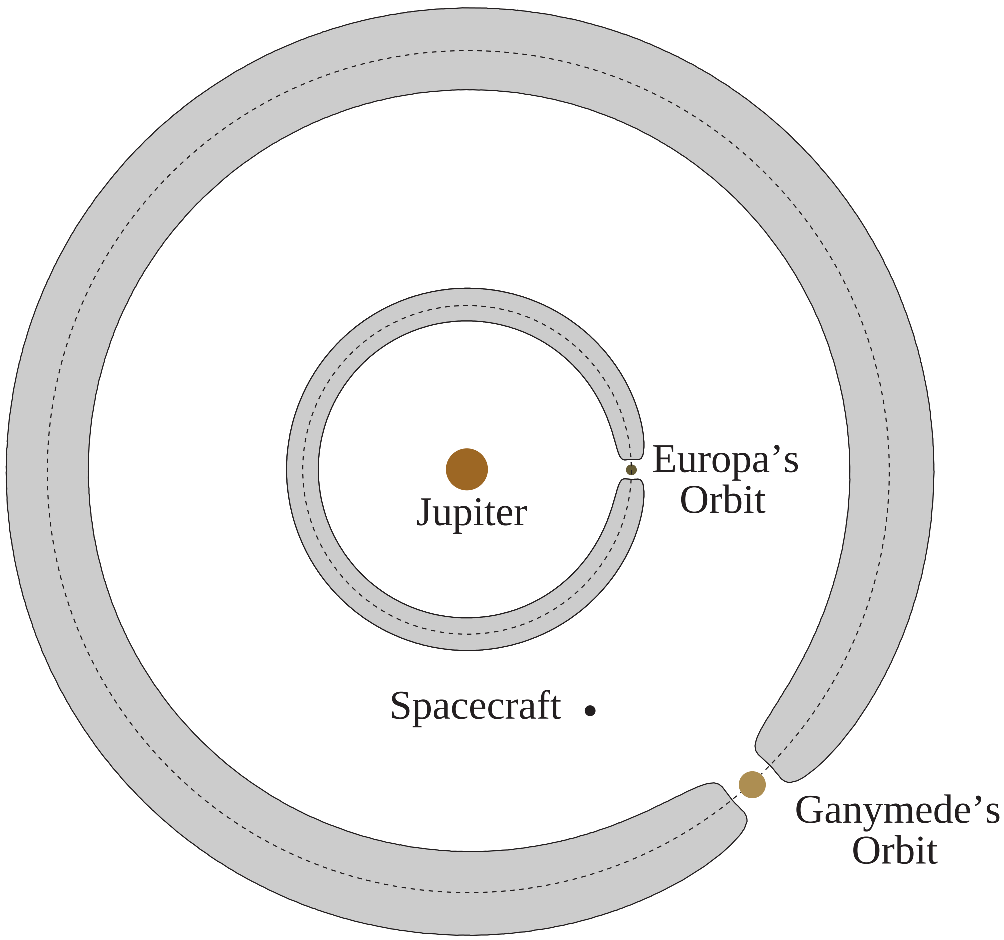
Petit Grand Tour: Ganymede and Europa

- **Itinerary**:
  - Starting from outside Ganymede's orbit, get captured for one orbit around Ganymede.
  - Leaving Ganymede, transfer to Europa.
  - Get captured by Europa for several orbits.



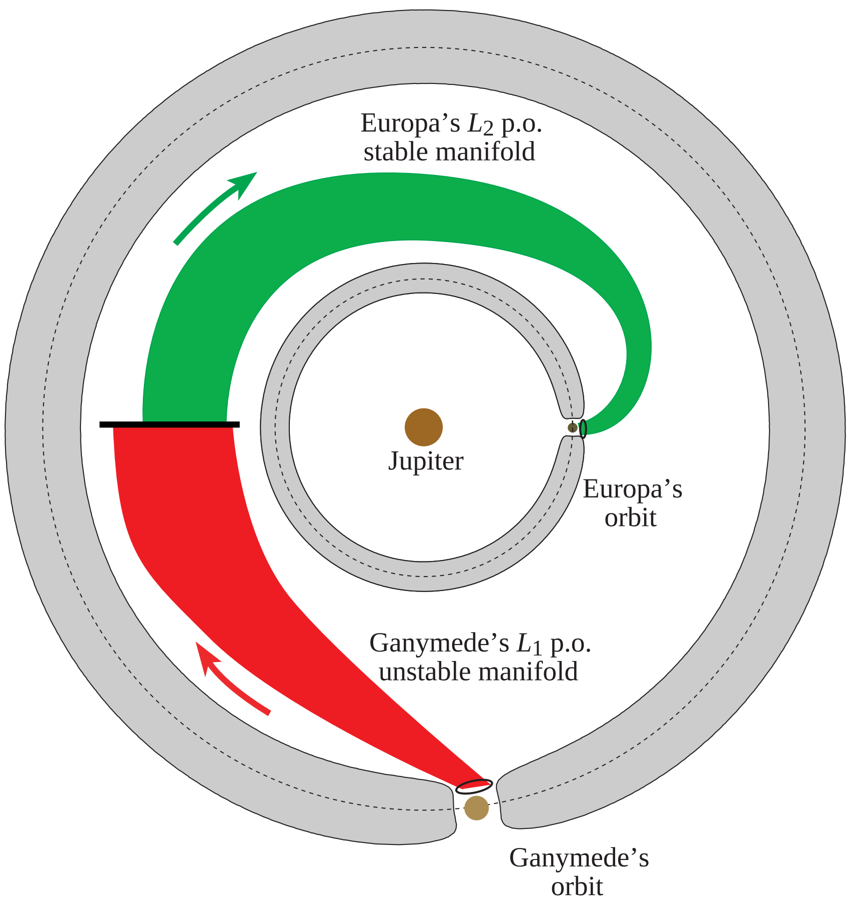
Starting Model: Coupled Three-Body Problem

- Solution space of full 4-body problem is little known.
- Look at coupled 3-body problem:
  - Two nested 3-body systems, Jupiter-Ganymede-spacecraft and Jupiter-Europa-spacecraft, approximate the dynamics well.



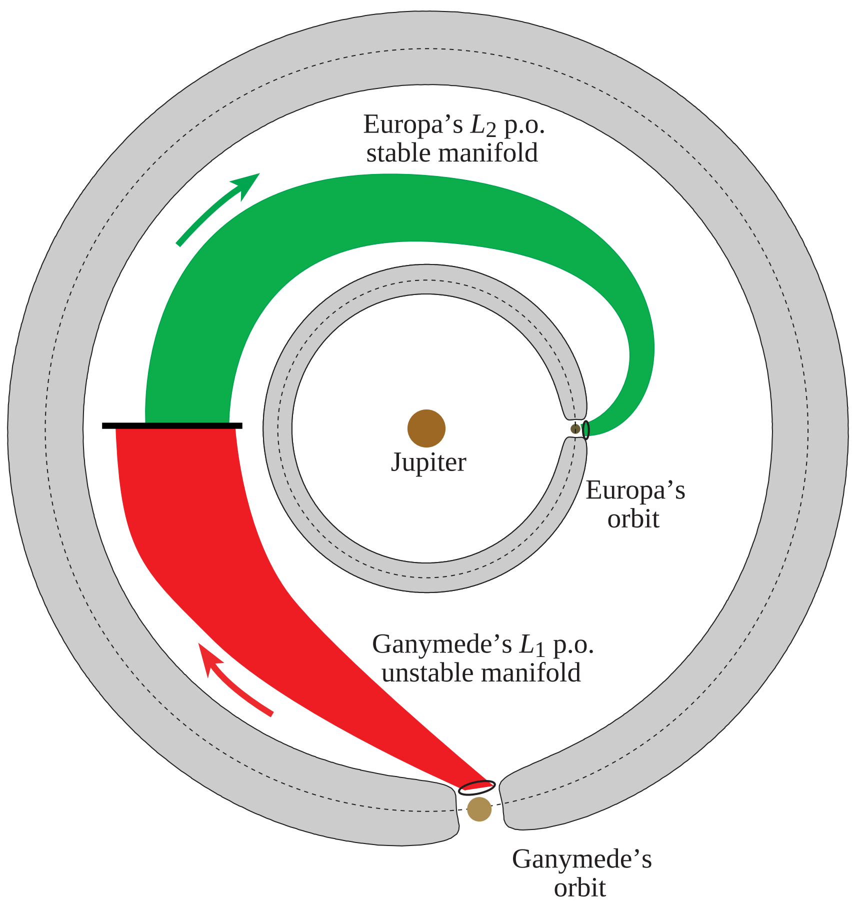
Petit Grand Tour: Ganymede and Europa

- The **transfer trajectory** from Ganymede to Europa is the most difficult part of the tour to construct. That is where we begin.
  - We wish to go from the "capture region" of Ganymede to the "capture region" of Europa.
  - Use the intersection of **unstable** and **stable** **manifold tubes**.



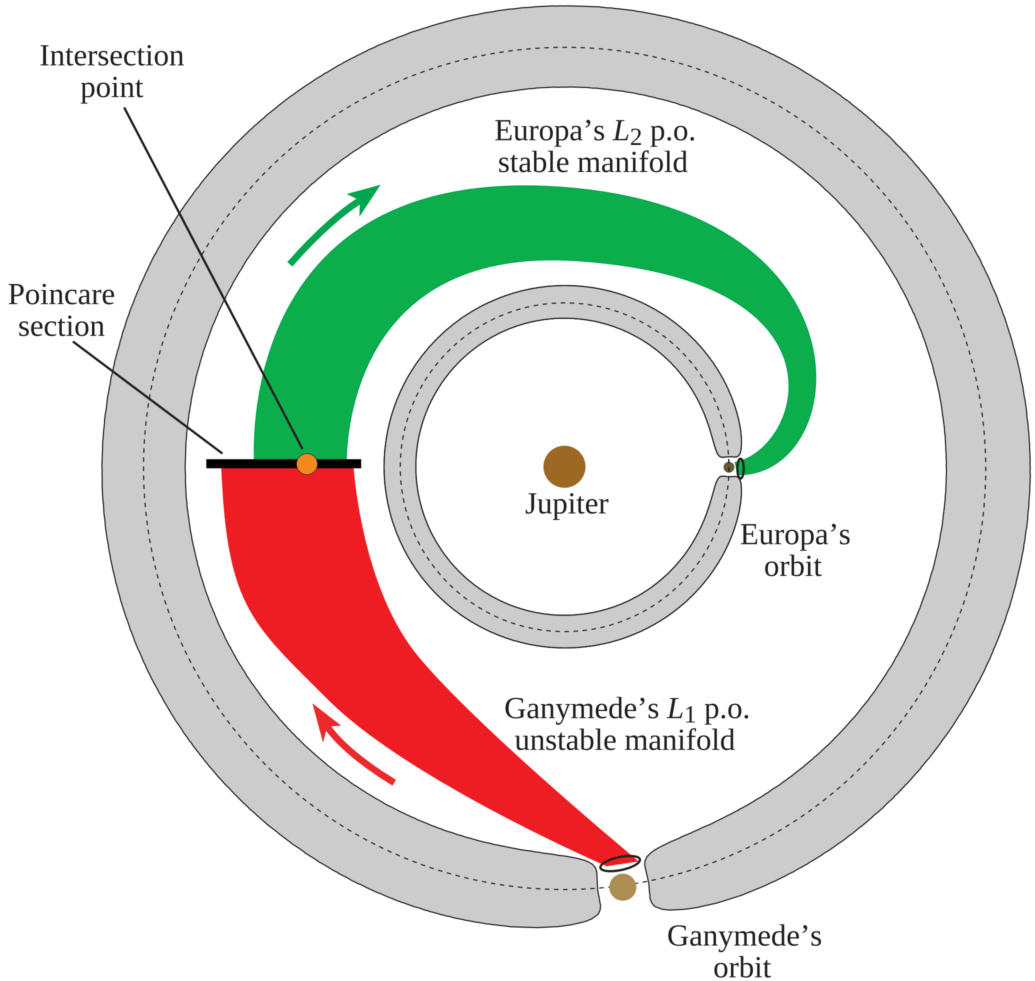
Intersection of Tubes

- Find intersection of **Ganymede's L_1 periodic orbit unstable manifold** and **Europa's L_2 periodic orbit stable manifold**.
  - The two manifolds are at different energies, so a rocket burn (ΔV) will be necessary.



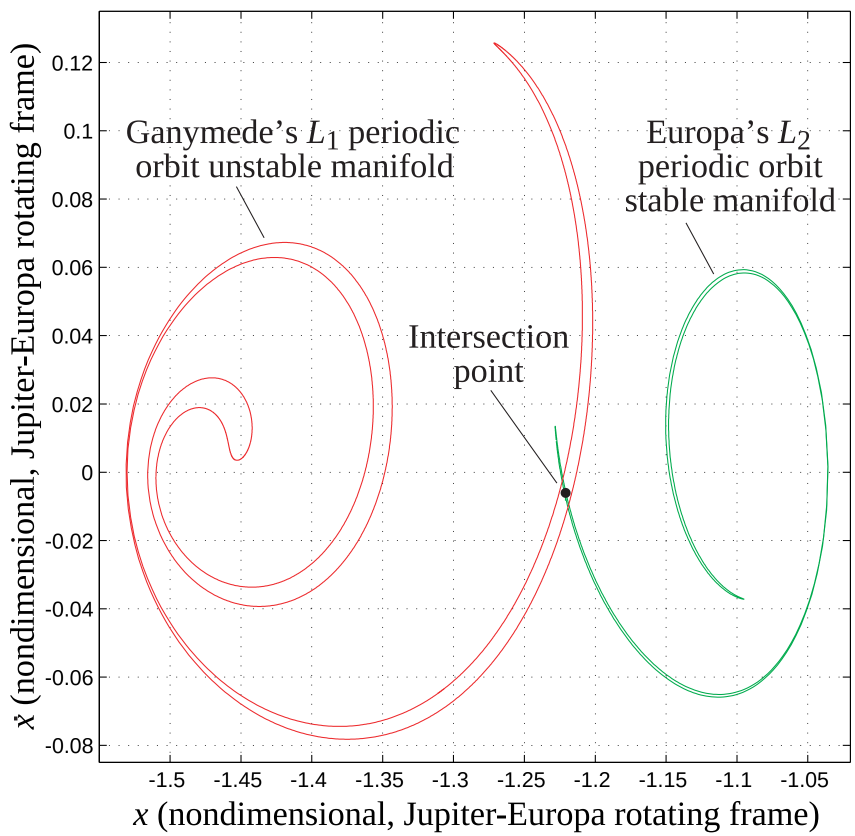
Intersection of Tubes: Poincaré Section

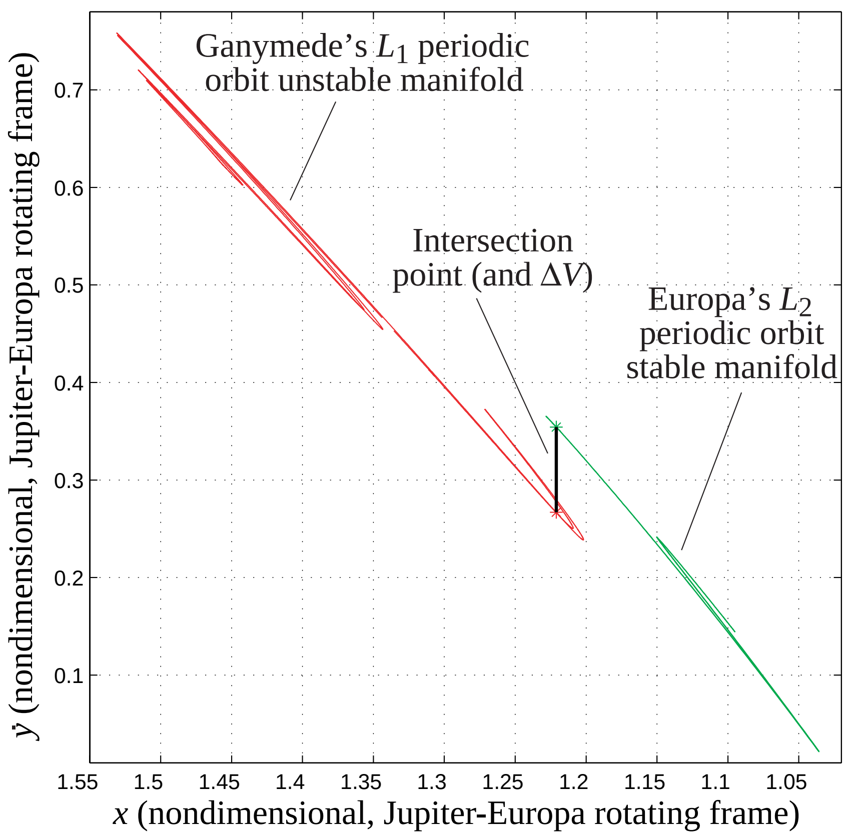
- On a suitably chosen Poincaré section
  - Look for an intersection between the manifolds.
  - Compute magnitude of rocket burn (ΔV).



Intersection of Tubes: Poincaré Section

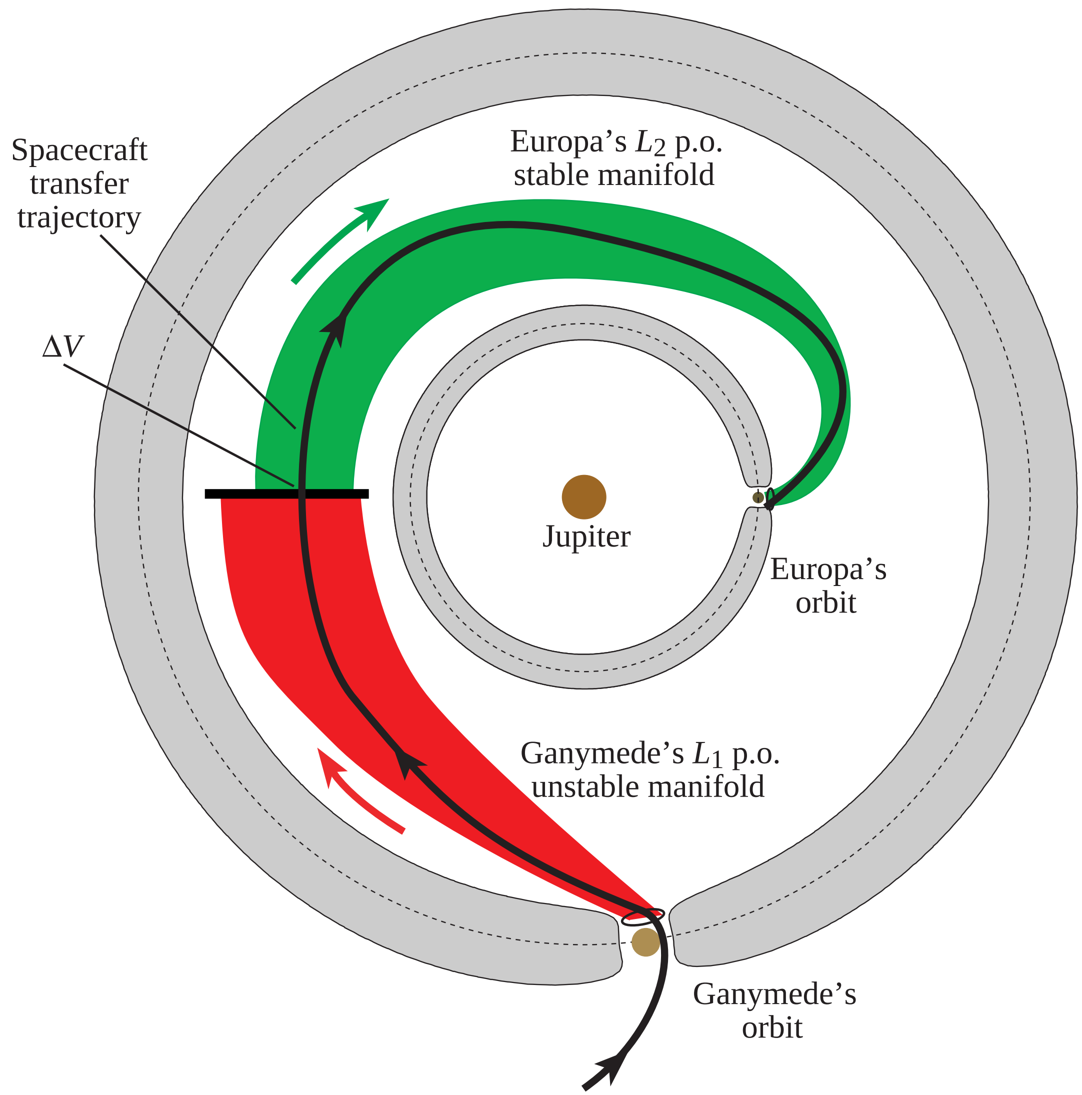
- Poincaré section: Vary configuration of the moons until,
  - $(x, \dot{x})$-plane: Intersection found!
  - $(x, \dot{y})$-plane: Velocity discontinuity since energies are different.
  - A rocket burn ΔV of this magnitude will make transfer trajectory **"jump" from one tube to the other**.





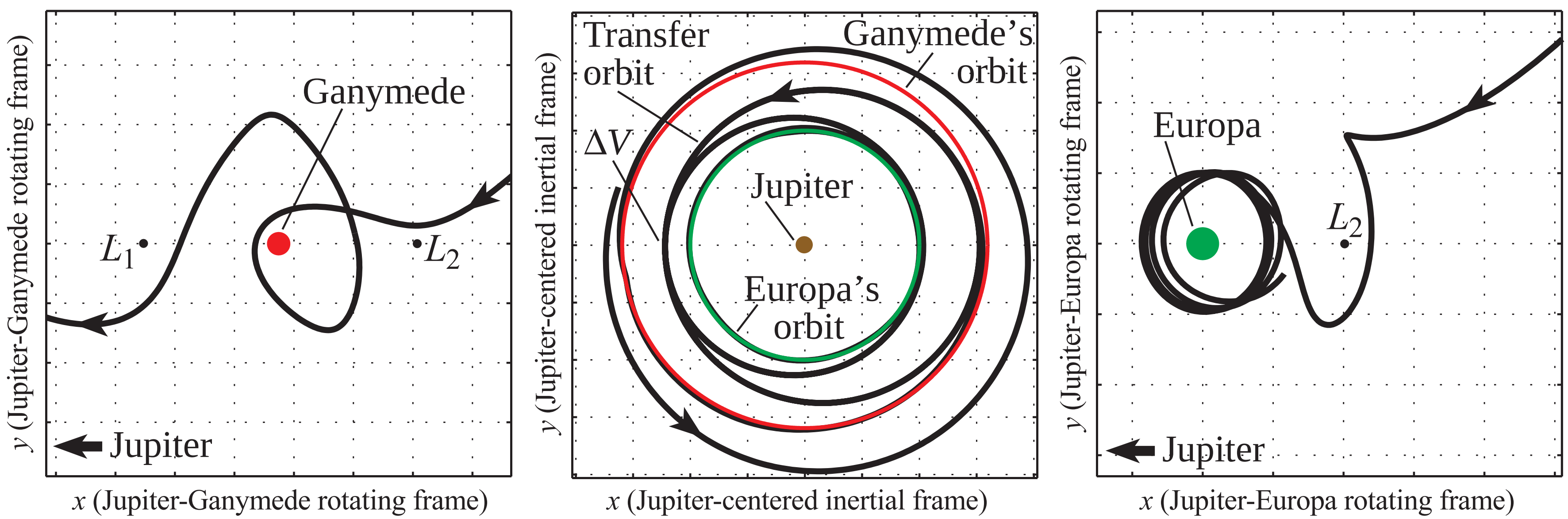
Intersection of Tubes: Transfer Trajectory

- Using an initial condition in the intersection region, forward and backward integrate to generate a **transfer trajectory**.
  - Integrate full 4-body problem: Jupiter-Ganymede-Europa-S/C.



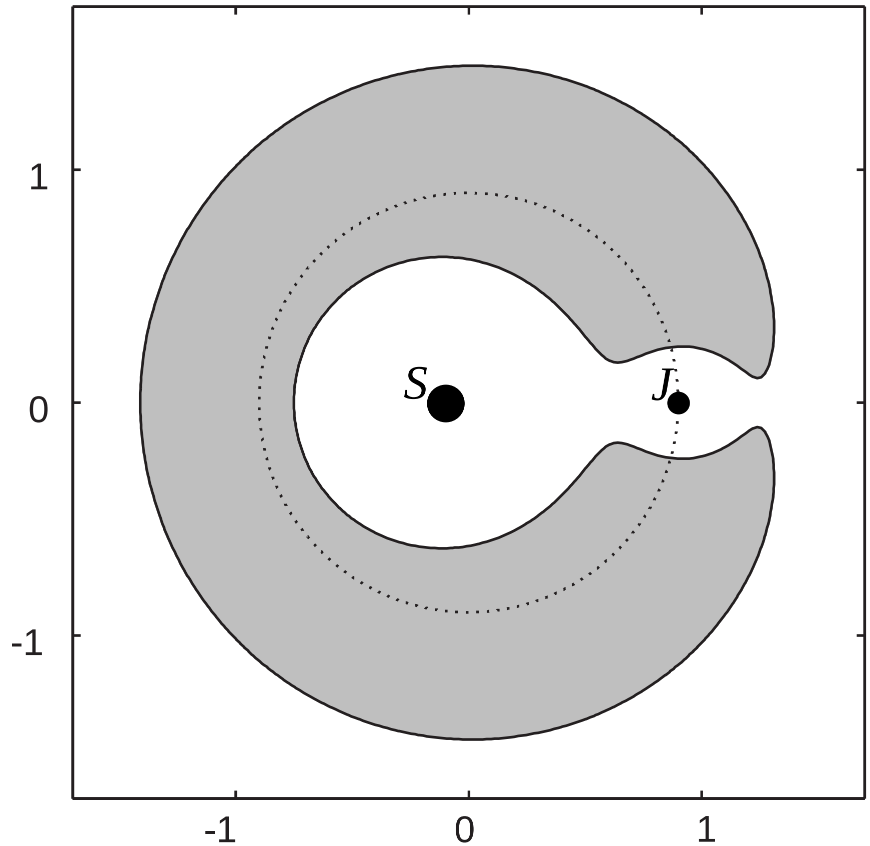
Intersection of Tubes: Transfer Trajectory

- Rocket burn required for transfer trajectory: $\Delta \mathbf{V} = \mathbf{1200\ m/s}$
- Compare with standard Hohmann transfer: $\Delta \mathbf{V} = \mathbf{2800\ m/s}$
  - Coupled 3-body approach uses only **40% of the fuel** of the conventional coupled 2-body approach. **Significant savings!**
- Trajectory shown below in both inertial and rotating frames.
  - Transfer trajectory takes only **25 days**.



Summary: Using Tubes to Construct Trajectories

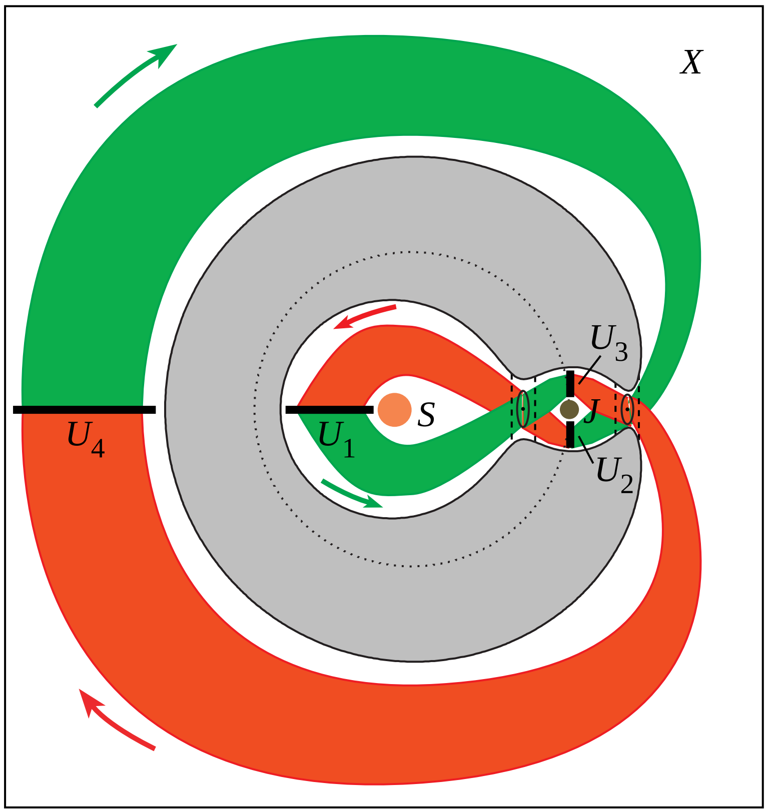
- The study of the planar circular restricted 3-body problem (PCR3BP) provides a **systematic method** for the numerical construction of a transfer trajectory for a space mission.
- We work in the **Case 3 energy regime** of the PCR3BP.

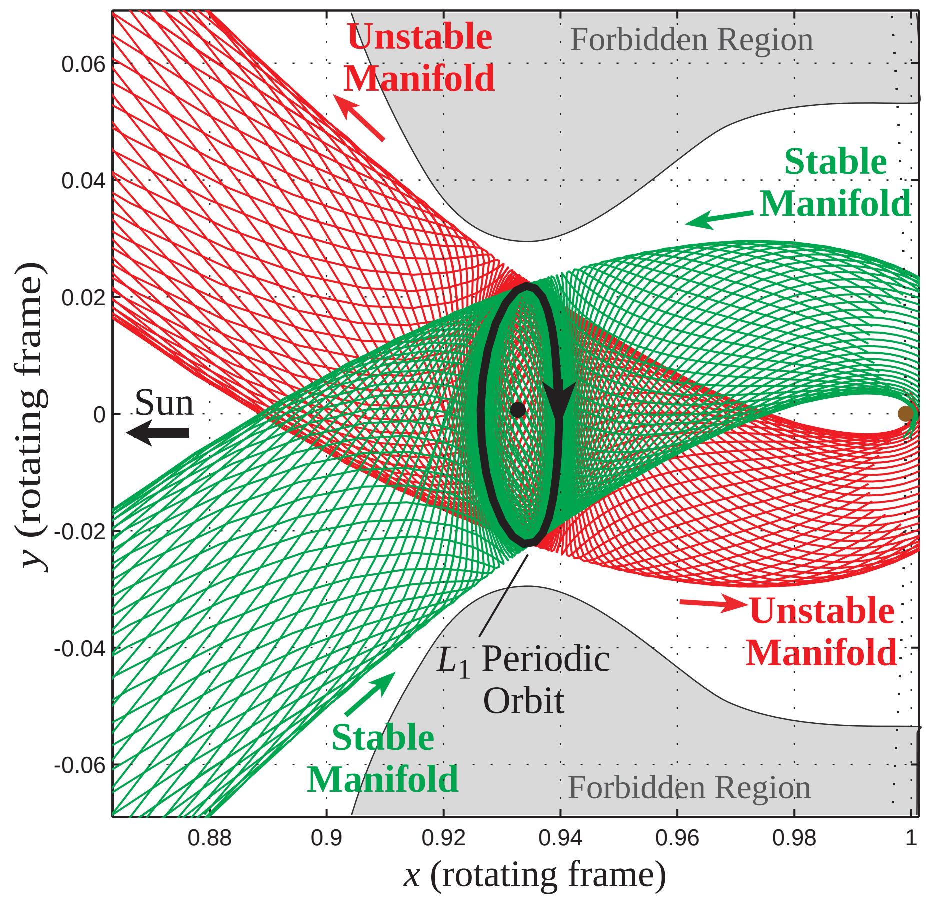


Case 3 : $E_2 < E < E_3$ or $C_2 > C > C_3$

Summary: Using Tubes to Construct Trajectories

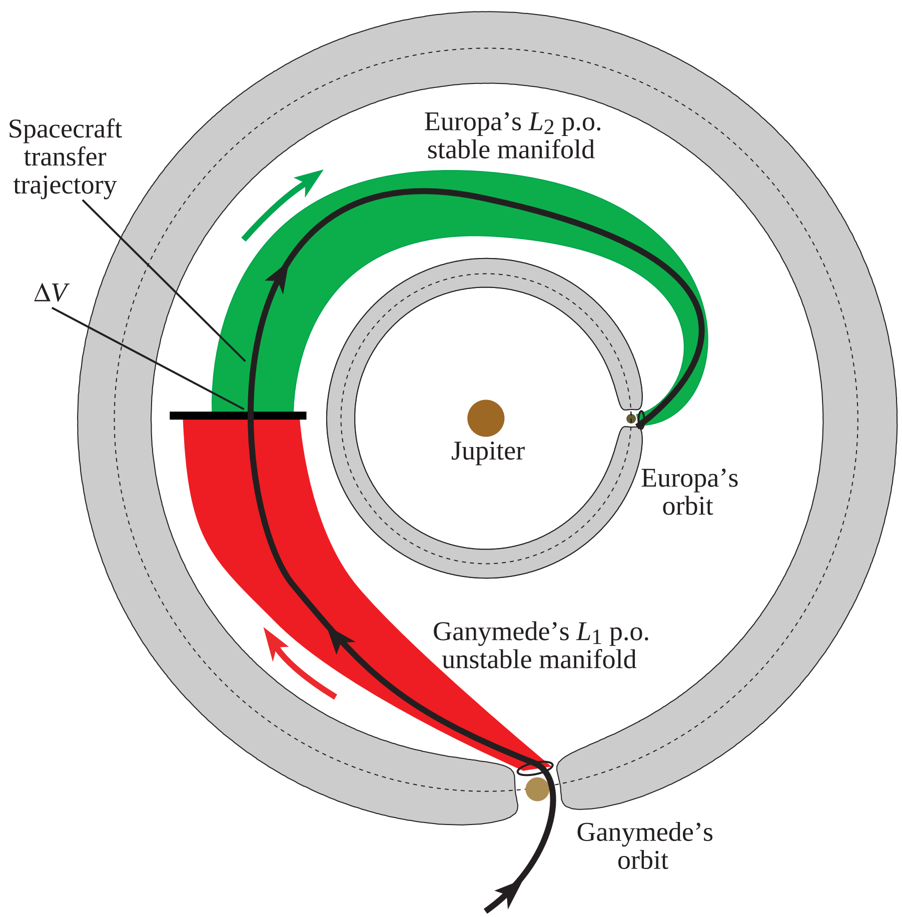
- L_1 and L_2 periodic orbit **stable** and **unstable** manifold tubes partition the energy surface into **transit** and **nontransit** orbits.
  - Transit orbits, contained inside the tubes, transit between the interior, capture, and exterior regions.





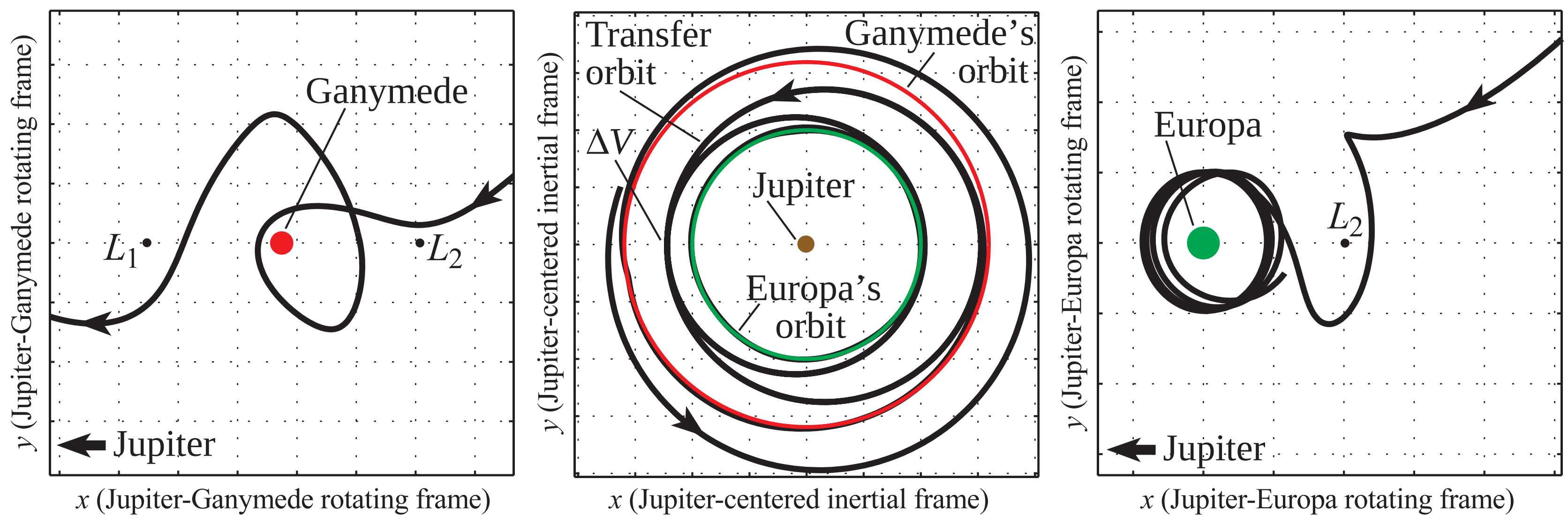
Summary: Using Tubes to Construct Trajectories

- Construct "leap-frogging" transfer trajectories between moons:
  - Jump from inside of **unstable manifold tube** of one 3-body system to the inside of **stable manifold tube** of an adjacent 3-body system.
  - Use rocket burn (ΔV) at tube intersection point.



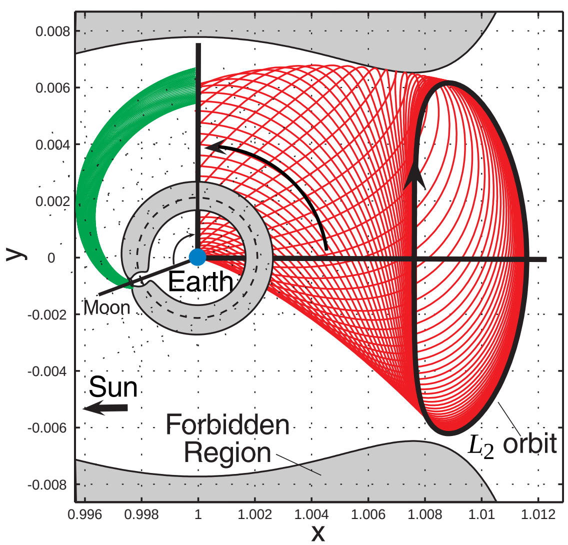
Summary: Using Tubes to Construct Trajectories

- e.g., **Petit Grand Tour** of Jovian moons Ganymede and Europa.
- Uses less than half the fuel of standard 2-body approach.

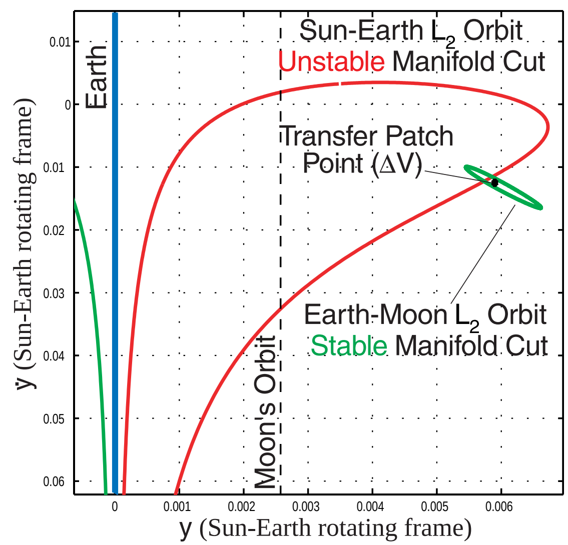


Summary: Using Tubes to Construct Trajectories

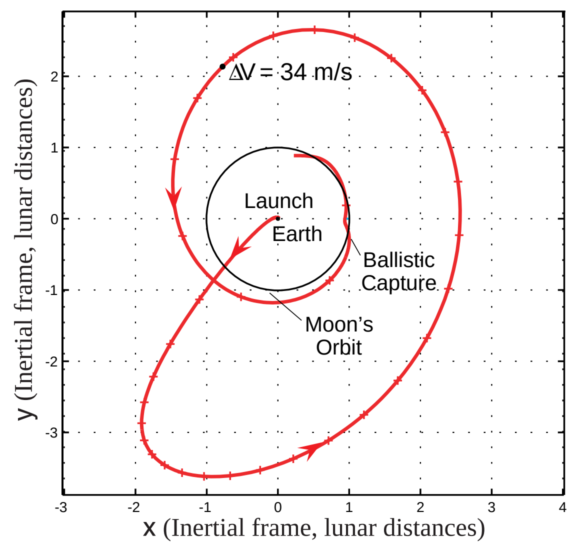
- Construction of many other low-fuel transfers is possible.
  - Earth to Moon transfer: **“Shoot the Moon”** in an upcoming lecture.



(a)



(b)



(c)

Looking Ahead: Natural Transfers in the Solar System

- Natural (zero-fuel) transfers may exist in the Solar System.
  - Comet transitions between resonances of Jupiter (e.g., *Oterma*).
  - Transfer of comets between Jupiter and Saturn systems.
  - Transfer of material between Asteroid Belt and Kuiper Belt.