Dynamical structure and its uses for insight, discovery, and control

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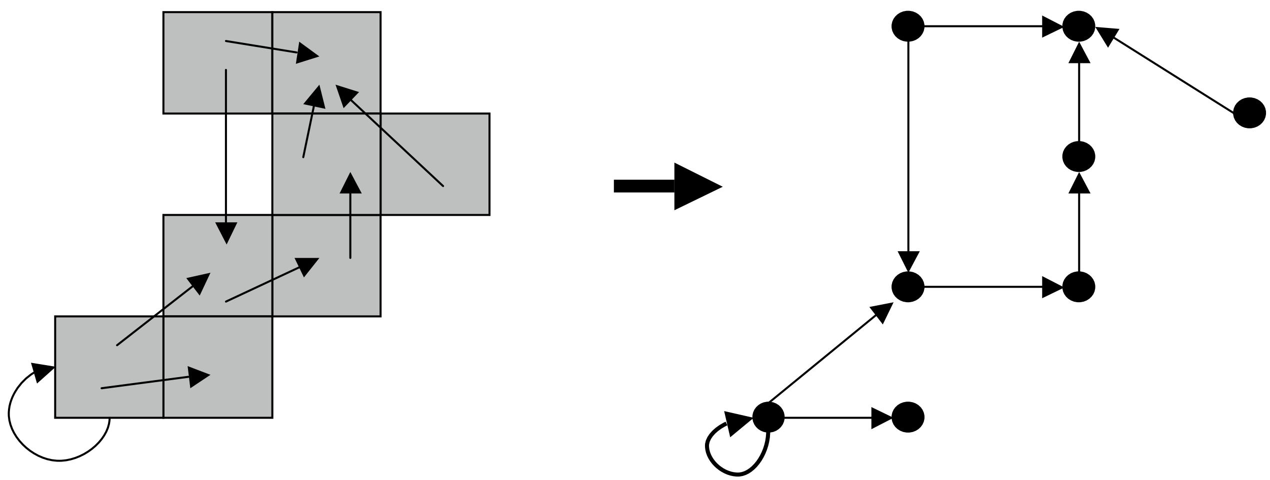
Motivation: application to data

- **Dynamical structure**: how phase space is connected / organized
- Fixed points, periodic orbits, or other invariant sets
  and their stable and unstable manifolds organize phase space
- Many systems defined from data or large-scale simulations
  — experimental measurements, observations
- e.g., from fluid dynamics, biology, social sciences
- Other tools (probabilistic, networks) could be useful in some settings



Phase space transport in 4+ dimensions

- Two examples

  — a biomechanical system

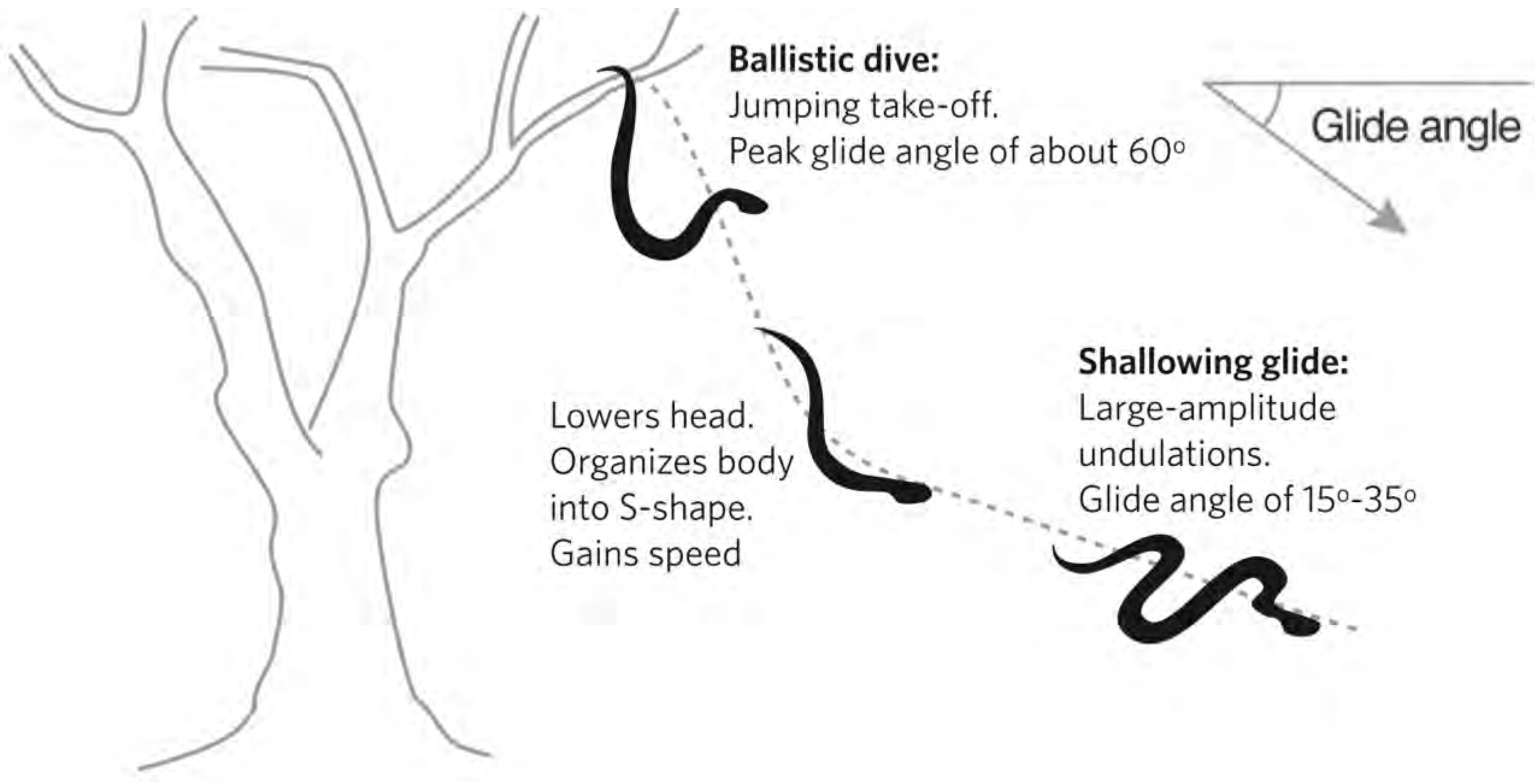
  — escape from a multi-dimensional potential well

- Then some examples from fluids and agriculture

Flying snakes

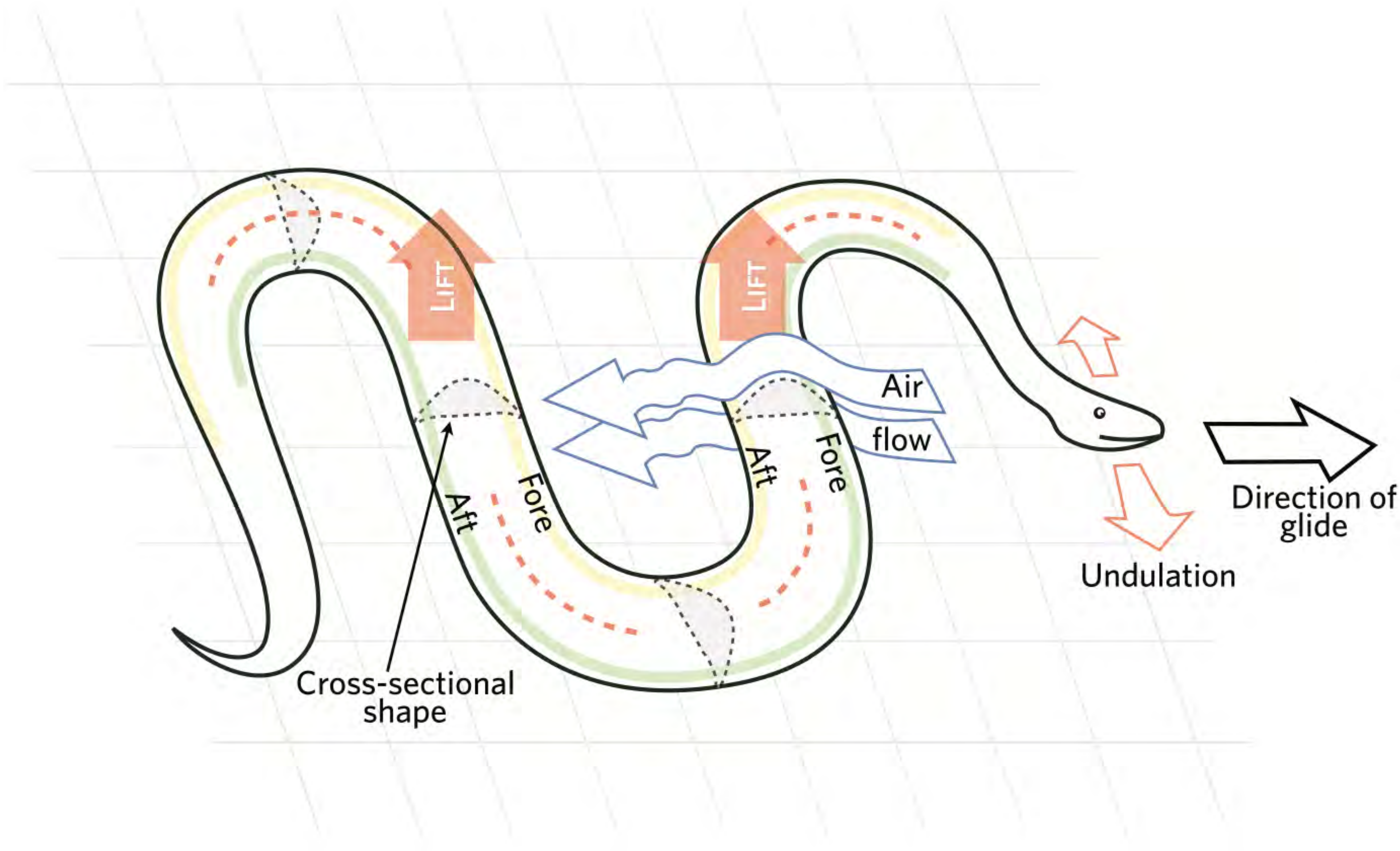
Joint work with Farid Jafari, Jake Socha, Pavlos Vlachos

Flying snakes



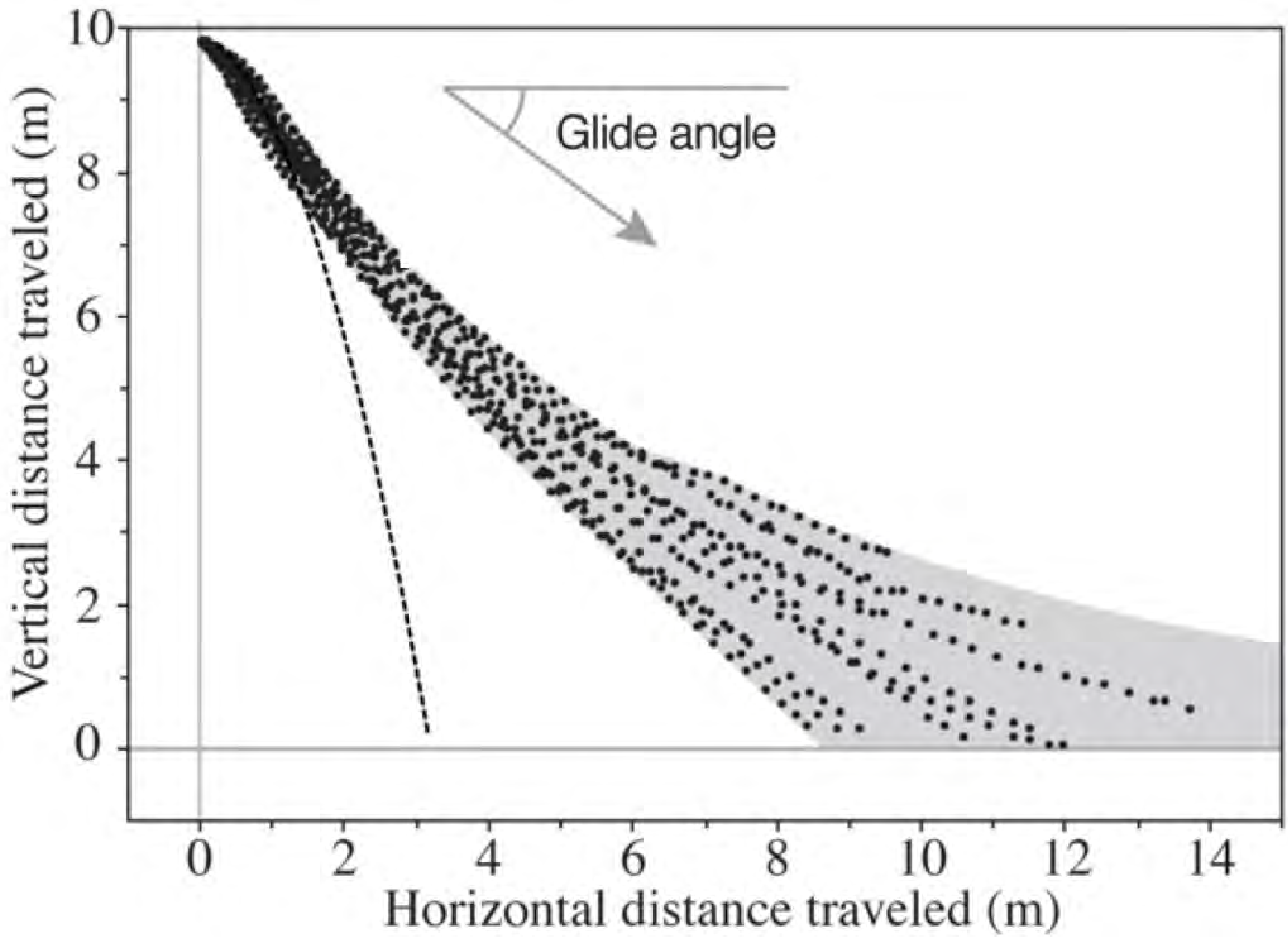
Krishnan, Socha, Vlachos, Barba [2014] Physics of Fluids

Flying snakes: undulation



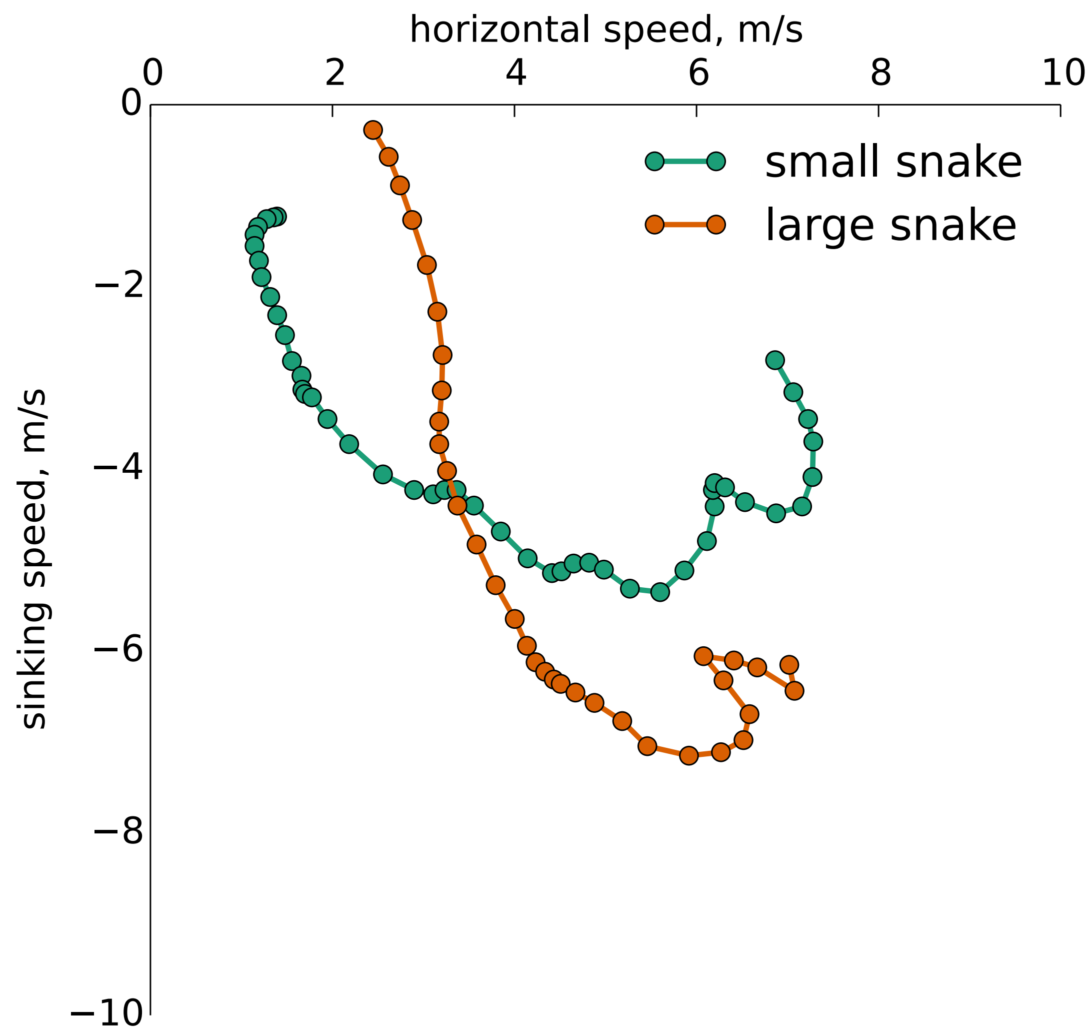
Krishnan, Socha, Vlachos, Barba [2014] Physics of Fluids

Flying snakes: experimental trajectories



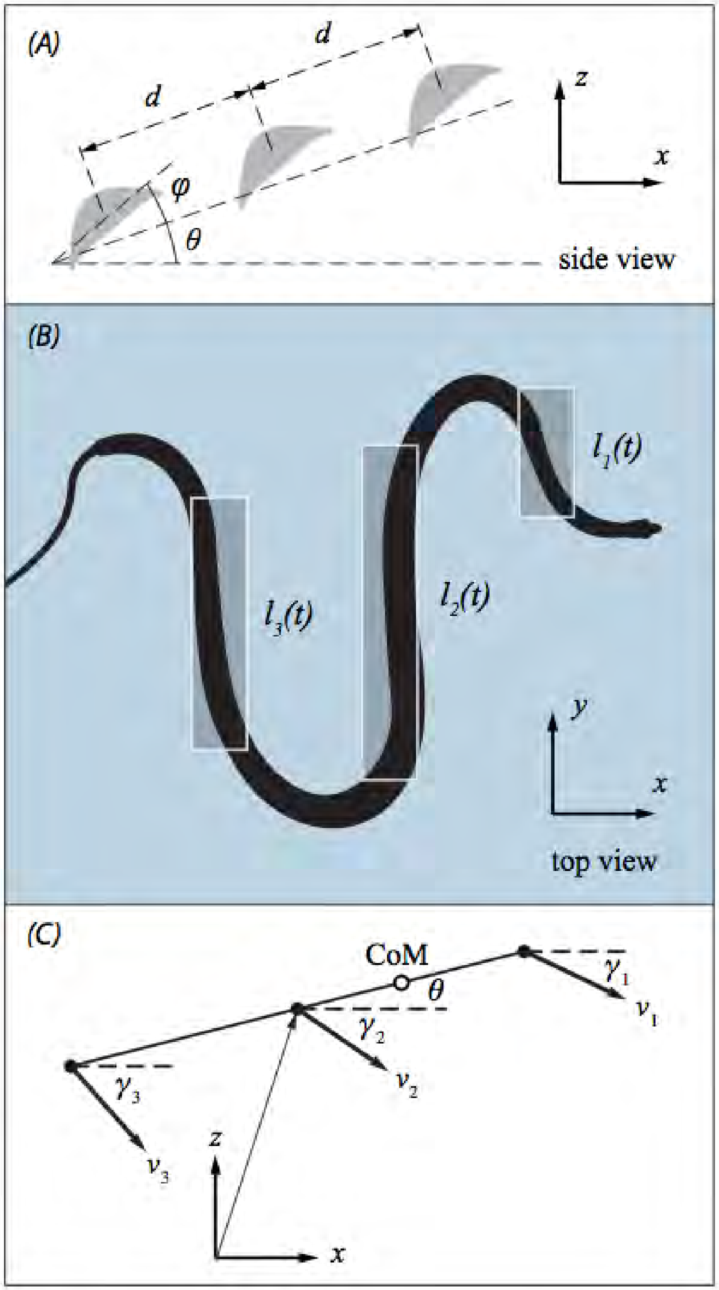
Socha [2011] Integrative and Comparative Biology

Flying snakes: velocity space



Socha [2011] Integrative and Comparative Biology

Flying snakes: minimal model



Consider a minimal model capturing the essential coupled translational-rotational dynamics — an undulating tandem wing configuration.

Given by 4-dimensional time-periodic system

$$
\begin{aligned}
\dot{v}_x &= u_1(\theta, \Omega, v_x, v_z, t) \\
\dot{v}_z &= u_2(\theta, \Omega, v_x, v_z, t) \\
\dot{\theta} &= u_3(\Omega) = \Omega \\
\dot{\Omega} &= u_4(\theta, \Omega, v_x, v_z, t)
\end{aligned}
$$

with translational kinematics $\dot{x} = v_x$, $\dot{z} = v_z$.

System is passively stable in pitch θ with equilibrium manifold $\{\Omega = 0\}$.

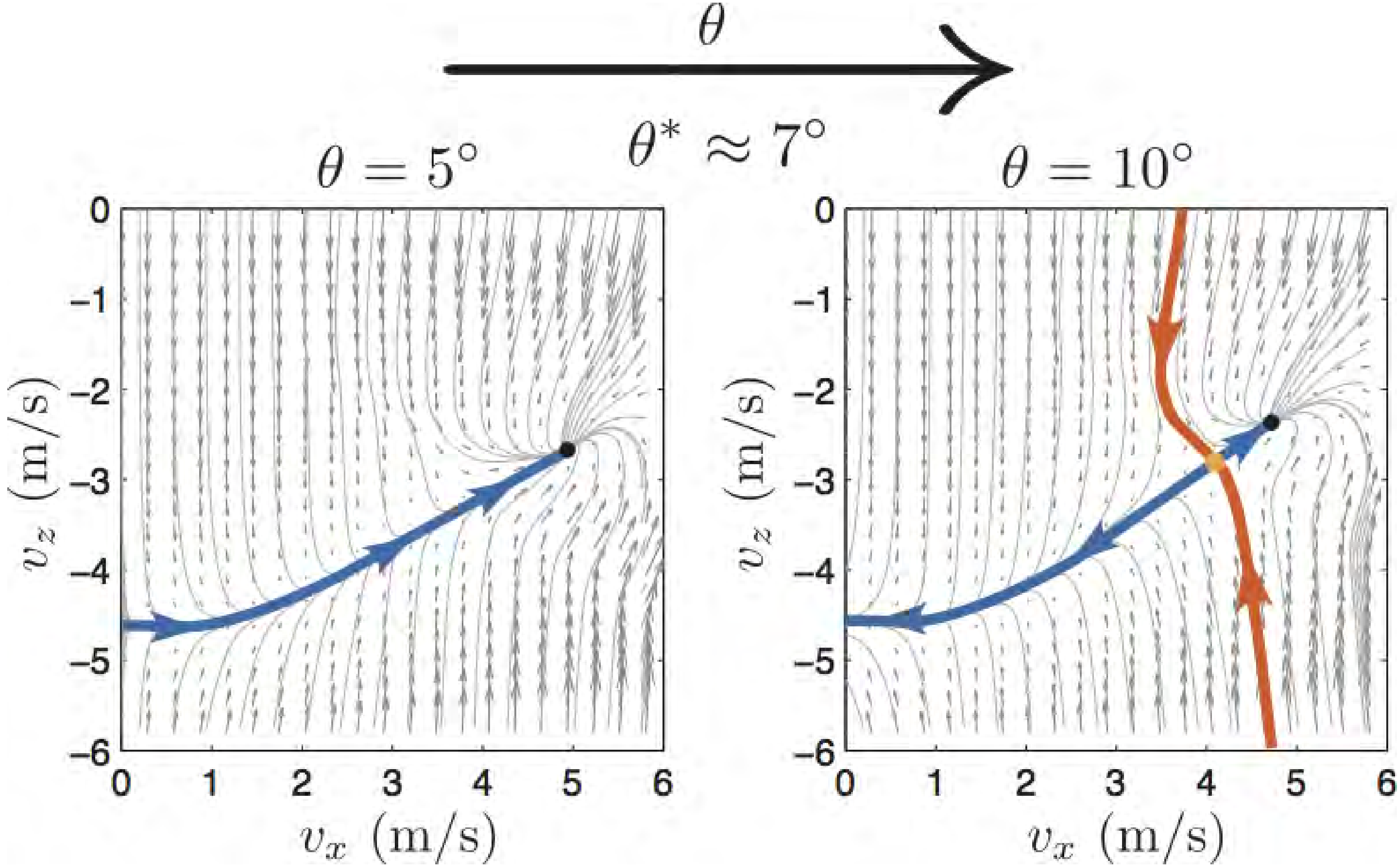
Translational dynamics are more complicated, but there does seem to be a 'shallowing manifold'.

Jafari, Ross, Vlachos, Socha [2014] Bioinspir. & Biomim.

Flying snakes: achieving equilibrium glide

Flying snakes: falling like a stone

Flying snakes: separatrix behavior

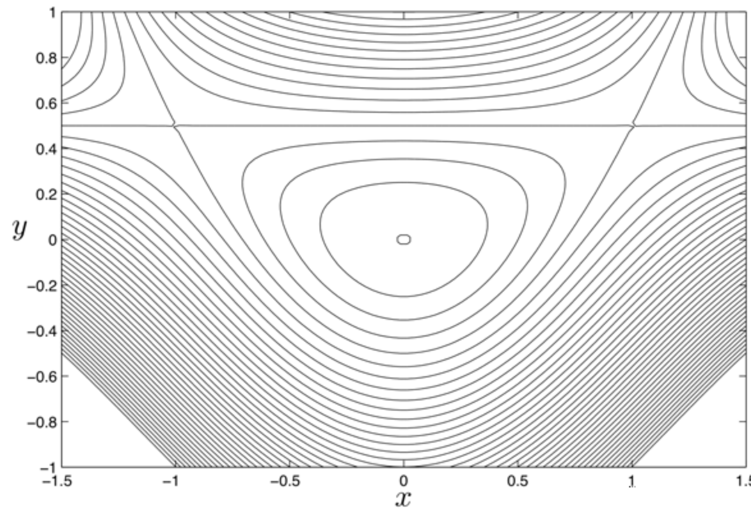


saddle-node bifurcation at θ^* along shallowing manifold

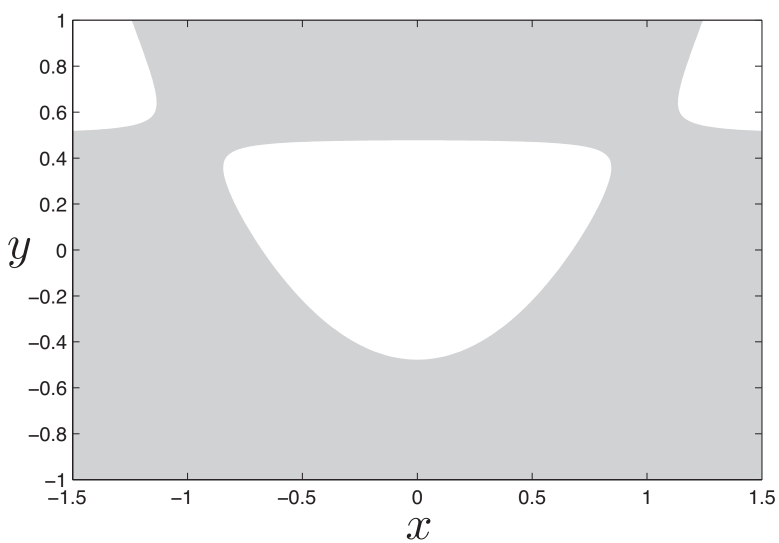
Ship motion and capsize

Tubes leading to capsize

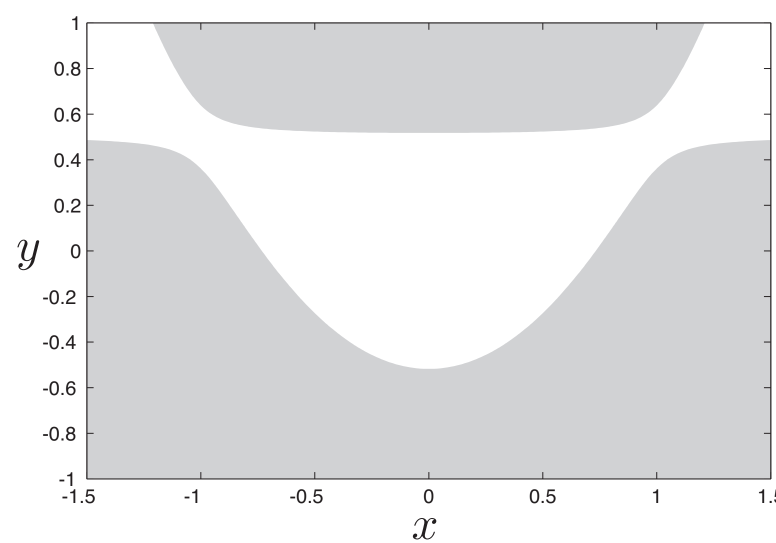
- Model built around Hamiltonian,
  $$H = p_x^2/2 + R^2 p_y^2/4 + V(x,y),$$
  where $x = \mathrm{roll}$ and $y = \mathrm{pitch}$ are coupled



$V(x,y)$

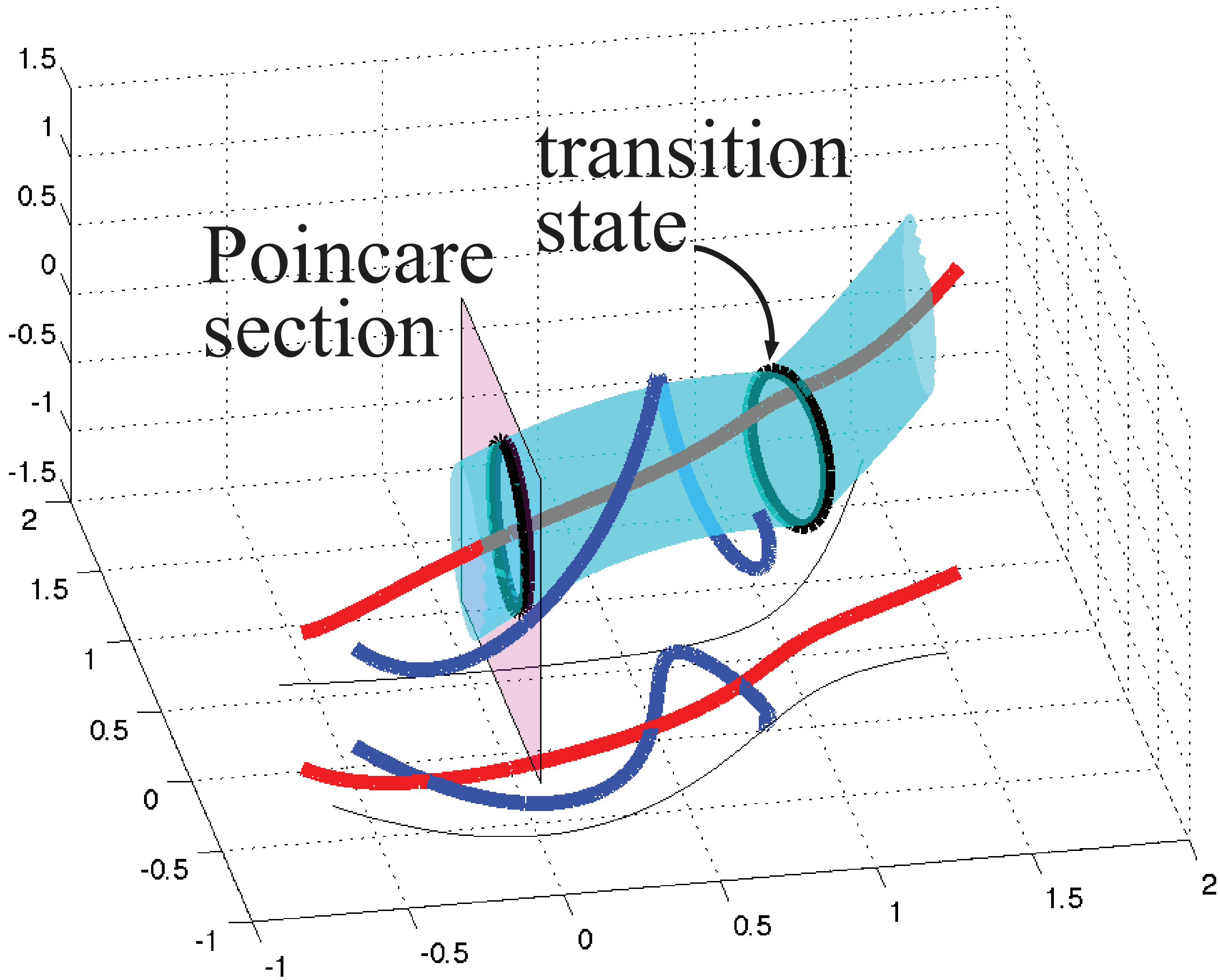


$E < E_c$



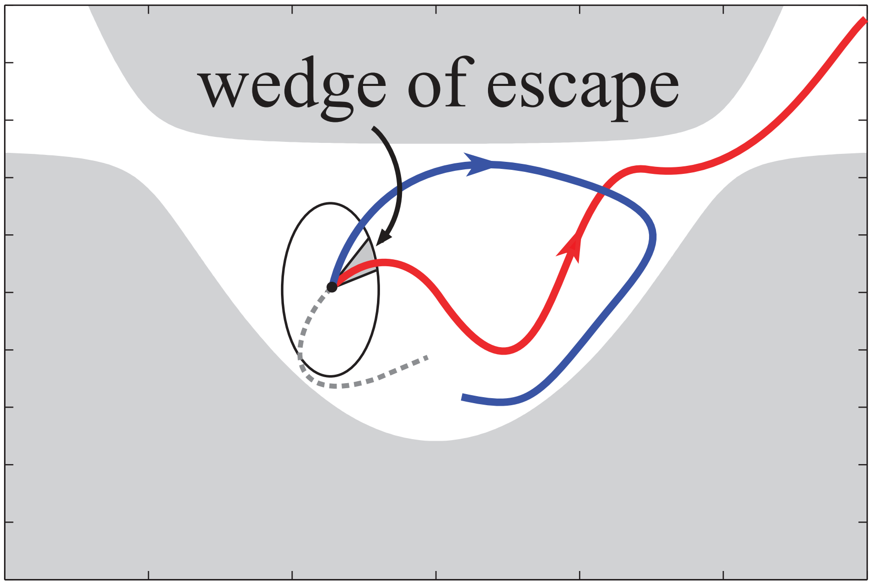
$E > E_c$

Tubes leading to capsize



Tubes leading to capsize

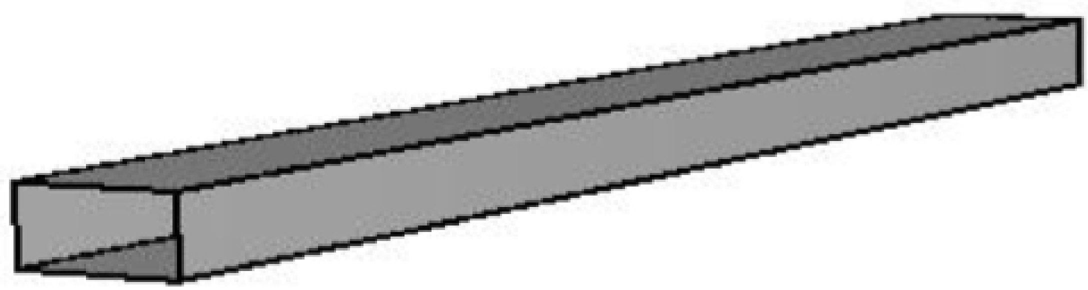
- Wedge of trajectories leading to imminent capsize

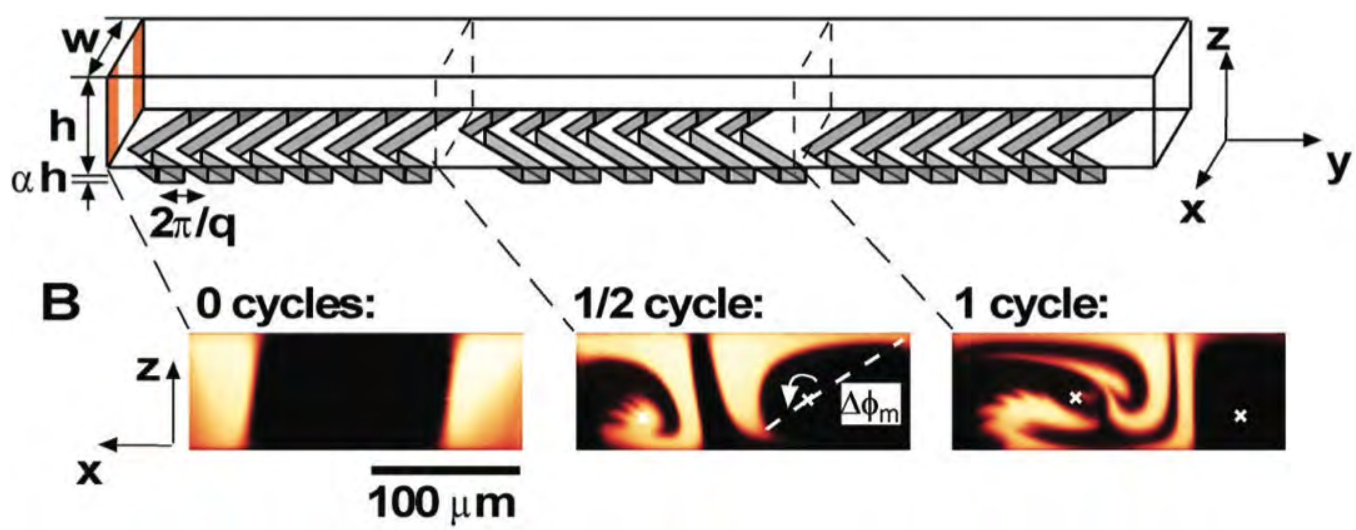


- Tubes are a useful paradigm for predicting capsize even in the presence of random forcing, e.g., random ocean waves
- Could inform **control schemes to avoid capsize** in rough seas

2D fluid example – almost-cyclic behavior

- A microchannel mixer: microfluidic channel with spatially periodic flow structure, e.g., due to grooves or wall motion[1]
- How does behavior change with parameters?

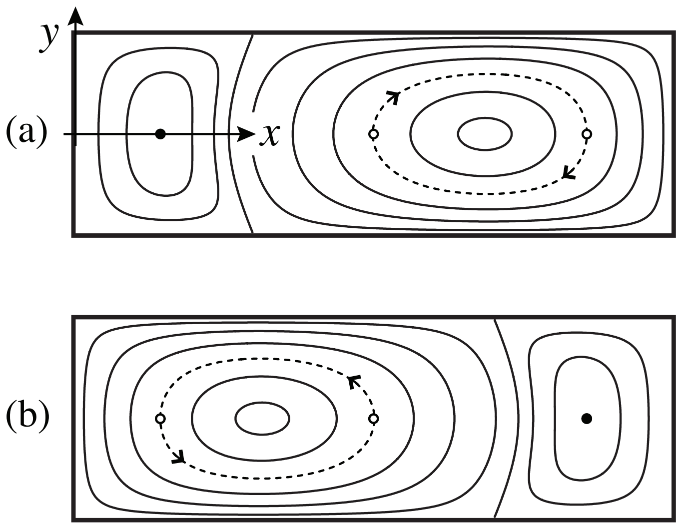




[1] Stroock et al. [2002], Stremler et al. [2011]

2D fluid example – almost-cyclic behavior

- A microchannel mixer: modeled as periodic Stokes flow



streamlines for $\tau_f = 1$

tracer blob ($\tau_f > 1$)

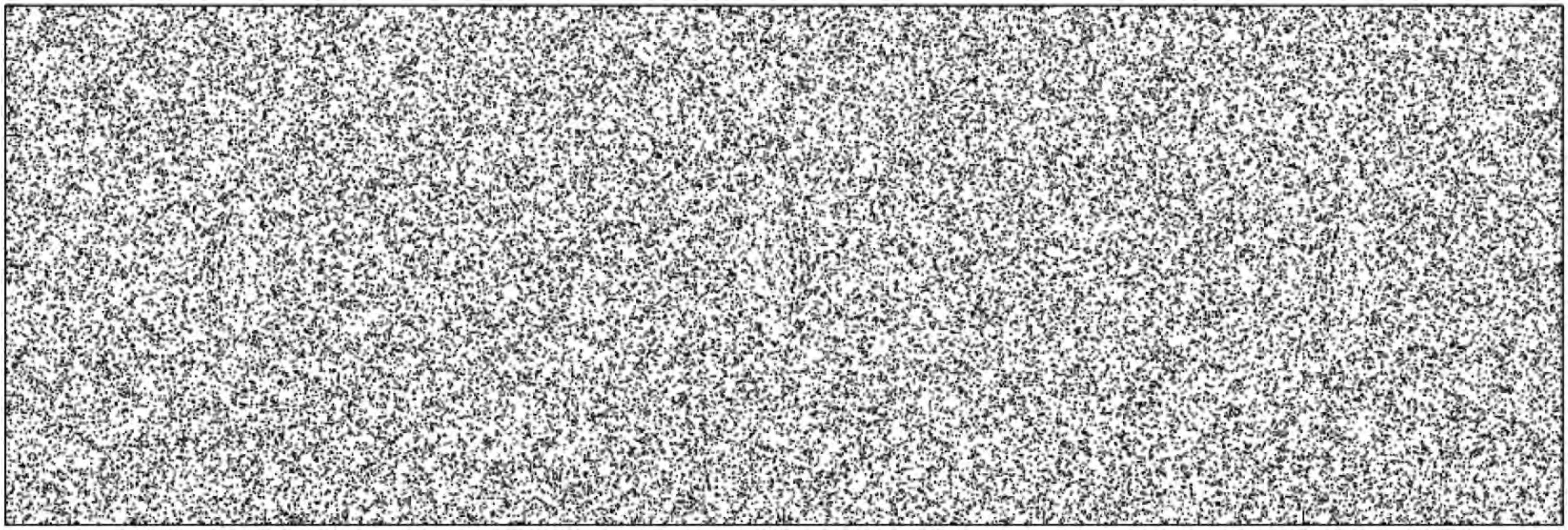
- piecewise constant vector field (repeating periodically)

  top streamline pattern during first half-cycle (duration $\tau_f/2$)

  bottom streamline pattern during second half-cycle (duration $\tau_f/2$), then repeat

- System has parameter τ_f, period of one cycle of flow, which we treat as a bifurcation parameter — there's a critical point $\tau_f^* = 1$

2D fluid example – almost-cyclic behavior



Poincaré section for $\tau_f < 1 \Rightarrow$ no obvious structure!

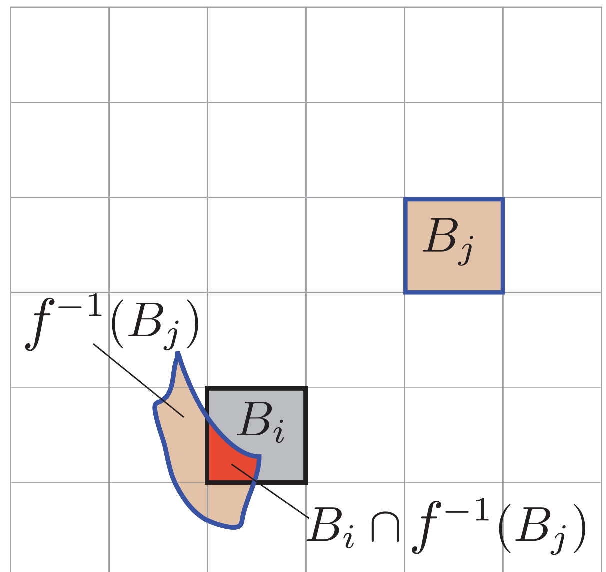
- Poincaré map: Over large range of parameter, no obvious cyclic behavior
- So, is the phase space featureless?

Almost-invariant sets / almost-cyclic sets

- No, we can identify **almost-invariant sets** (AISs) and **almost-cyclic sets** (ACSs)[1]
- Create box partition of phase space $\mathcal{B} = \{B_1, \dots B_q\}$, with q large
- Consider a q-by-q **transition (Ulam) matrix**, P, where

$$P_{ij} = \frac{m(B_i \cap f^{-1}(B_j))}{m(B_i)},$$

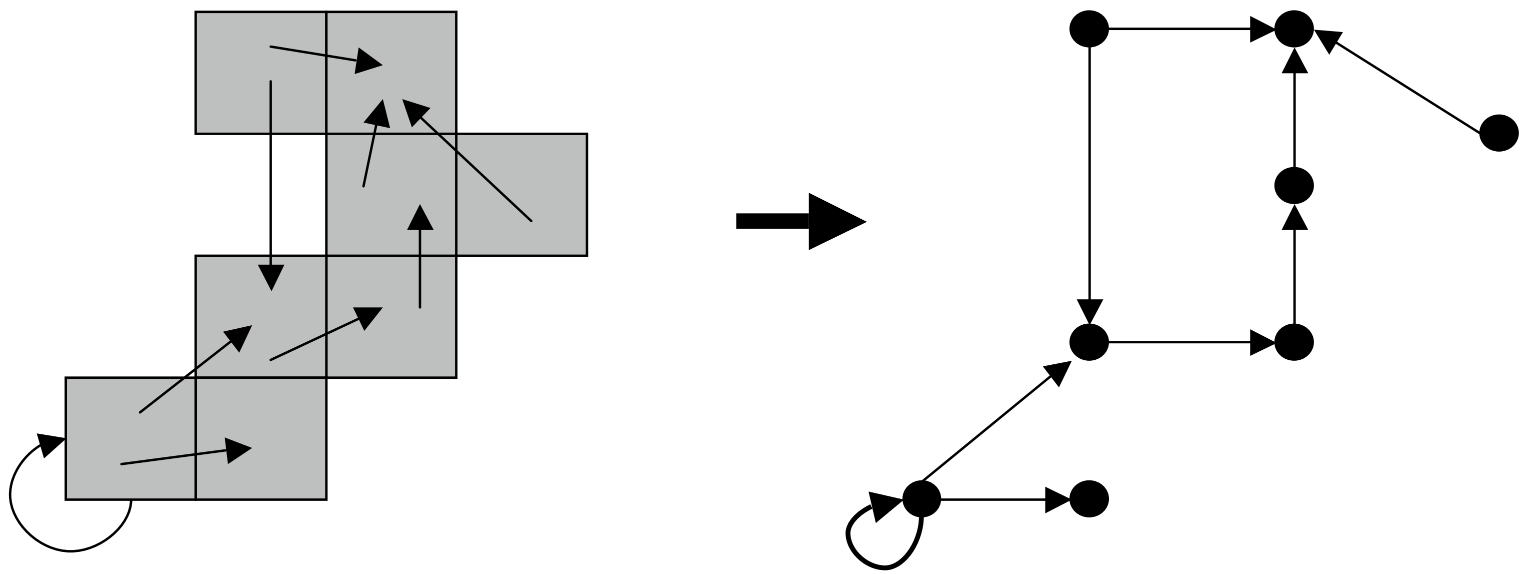
the *transition probability* from B_i to B_j using, e.g., $f = \phi_t^{t+T}$, often computed numerically



- P approximates $\mathcal{P}$, Perron-Frobenius transfer operator — which evolves densities, ν, over one iterate of f, as $\mathcal{P}\nu$
- Typically, we use a reversibilized operator R, obtained from P

[1] Dellnitz & Junge [1999], Froyland & Dellnitz [2003]

Identifying AISs by graph- or spectrum-partitioning

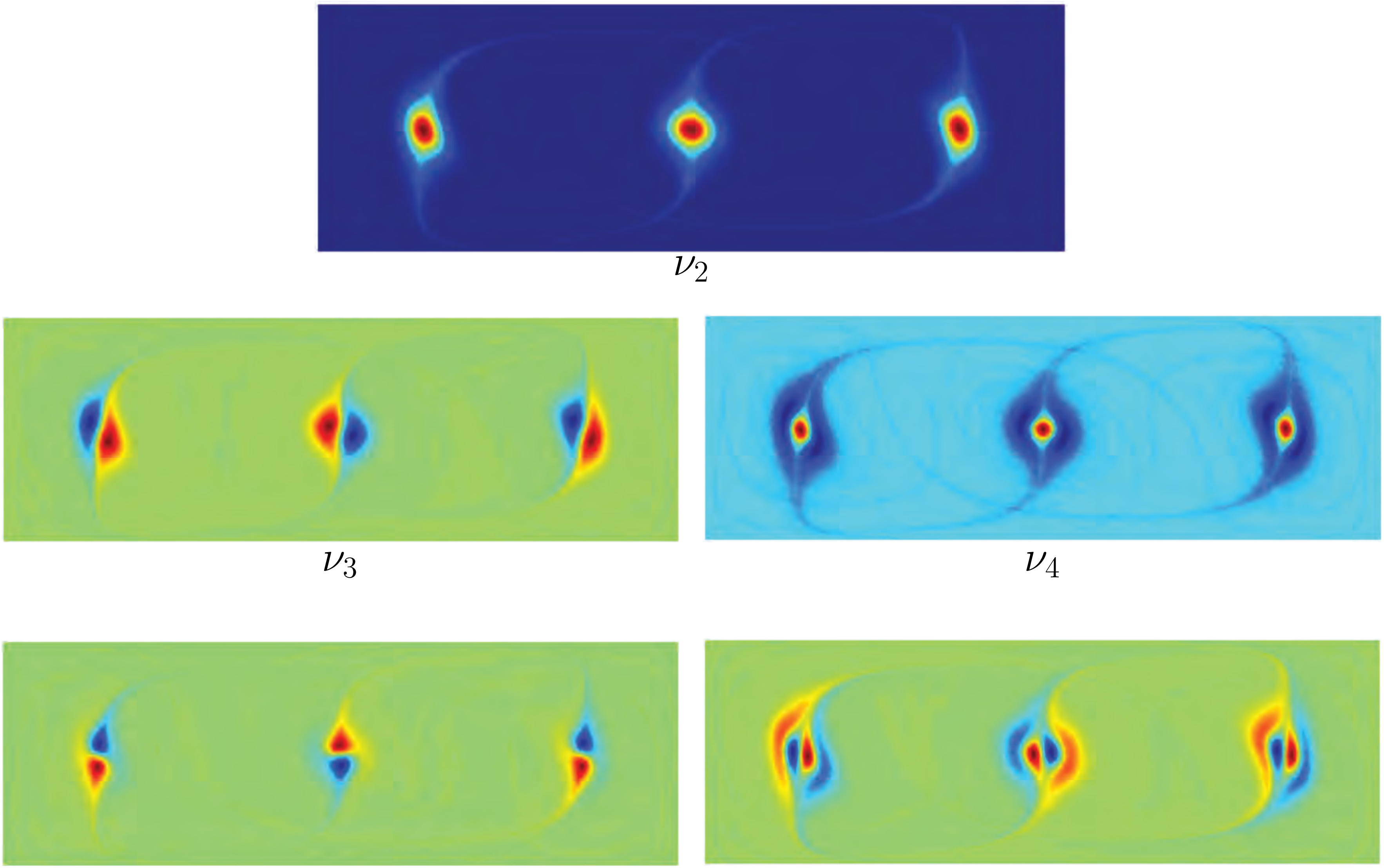


- P admits graph representation where nodes correspond to boxes B_i and transitions between them are edges of a directed graph
- Graph partitioning methods can be applied[1]
- can obtain AISs/ACSs and transport between them
- spectrum-partitioning as well (eigenvectors of large eigenvalues)[2]

[1] Dellnitz, Junge, Koon, Lekien, Lo, Marsden, Padberg, Preis, Ross, Thiere [2005] Int. J. Bif. Chaos
[2] Dellnitz, Froyland, Sertl [2000] Nonlinearity

Identifying AISs by graph- or spectrum-partitioning

Top eigenvectors of transfer operator reveal structure

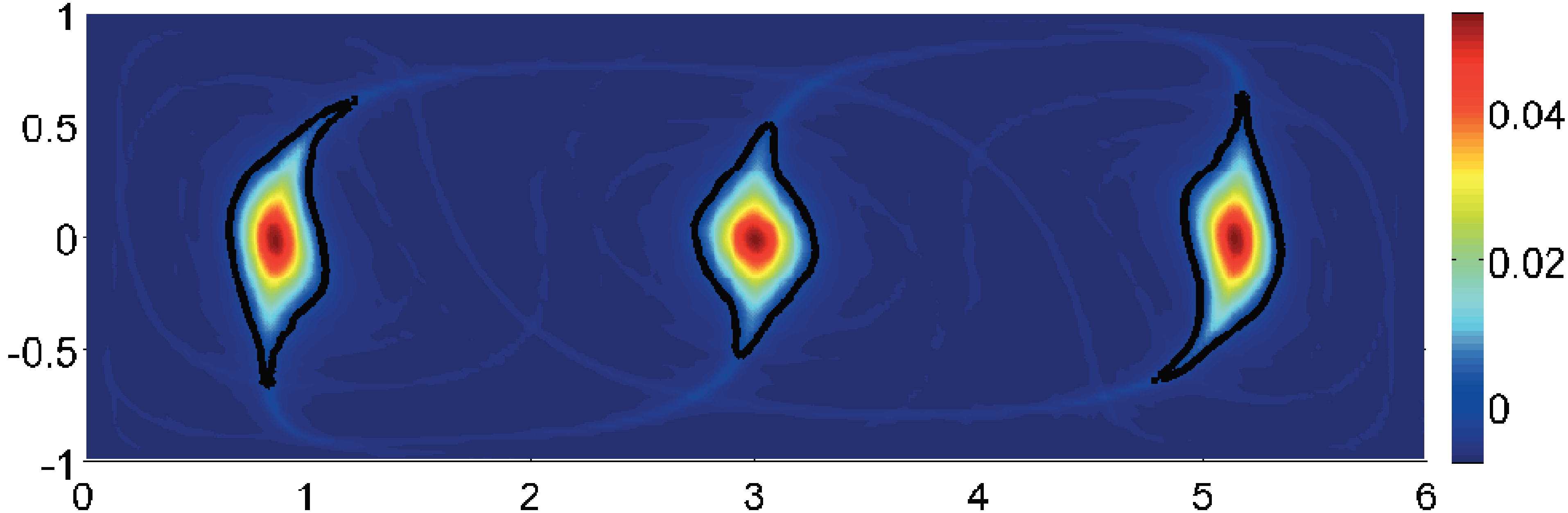


ν_2

ν_3 ν_4

ν_5 ν_6

Almost-cyclic sets stir fluid like rods



The zero contour (black) is the boundary between the two almost-invariant sets.

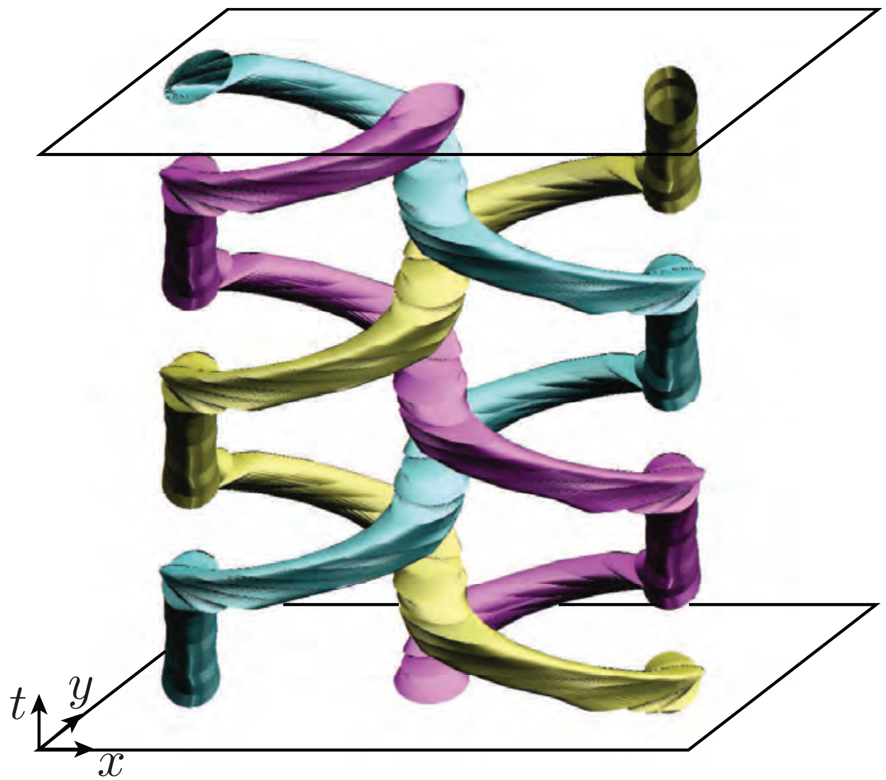
- Three-component AIS made of 3 ACSs each of period 3

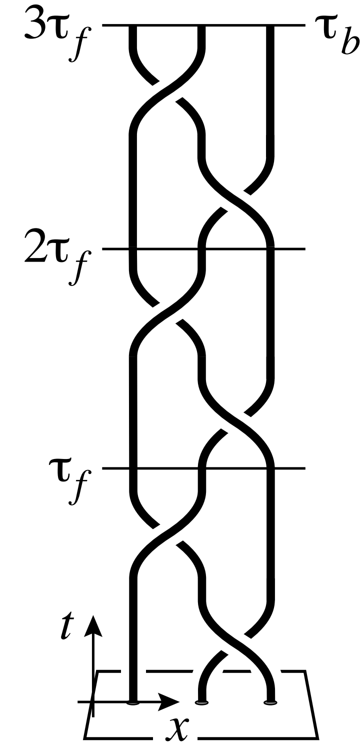
Almost-cyclic sets stir fluid like rods

Almost-cyclic sets, in effect, stir the surrounding fluid like ‘ghost rods’

In fact, there’s a theorem (Thurston-Nielsen classification theorem) that provides a topological lower bound on the mixing based on braiding in space-time

Almost-cyclic sets stir fluid like rods

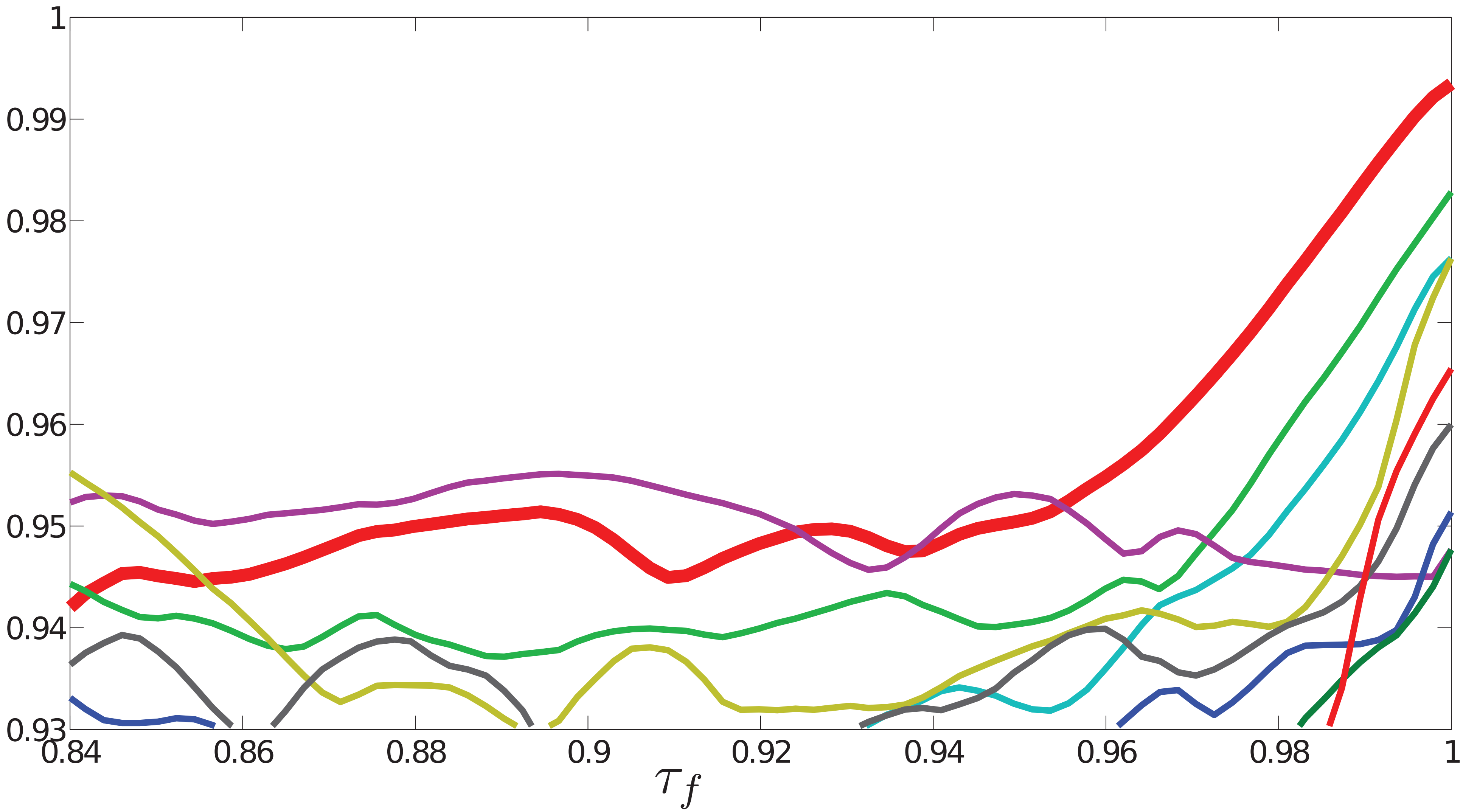




Thurston-Nielsen theorem applies only to periodic points
— But seems to work, even for approximately cyclic blobs of fluid[1]

[1] Stremler, Ross, Grover, Kumar [2011] Phys. Rev. Lett.

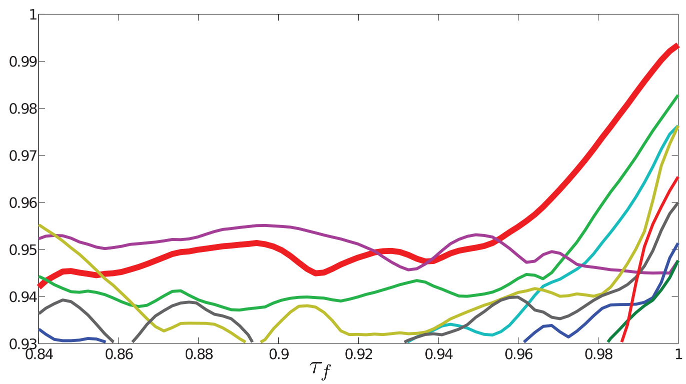
Eigenvalues/eigenvectors vs. parameter



Top eigenvalues of transfer operator as parameter τ_f changes

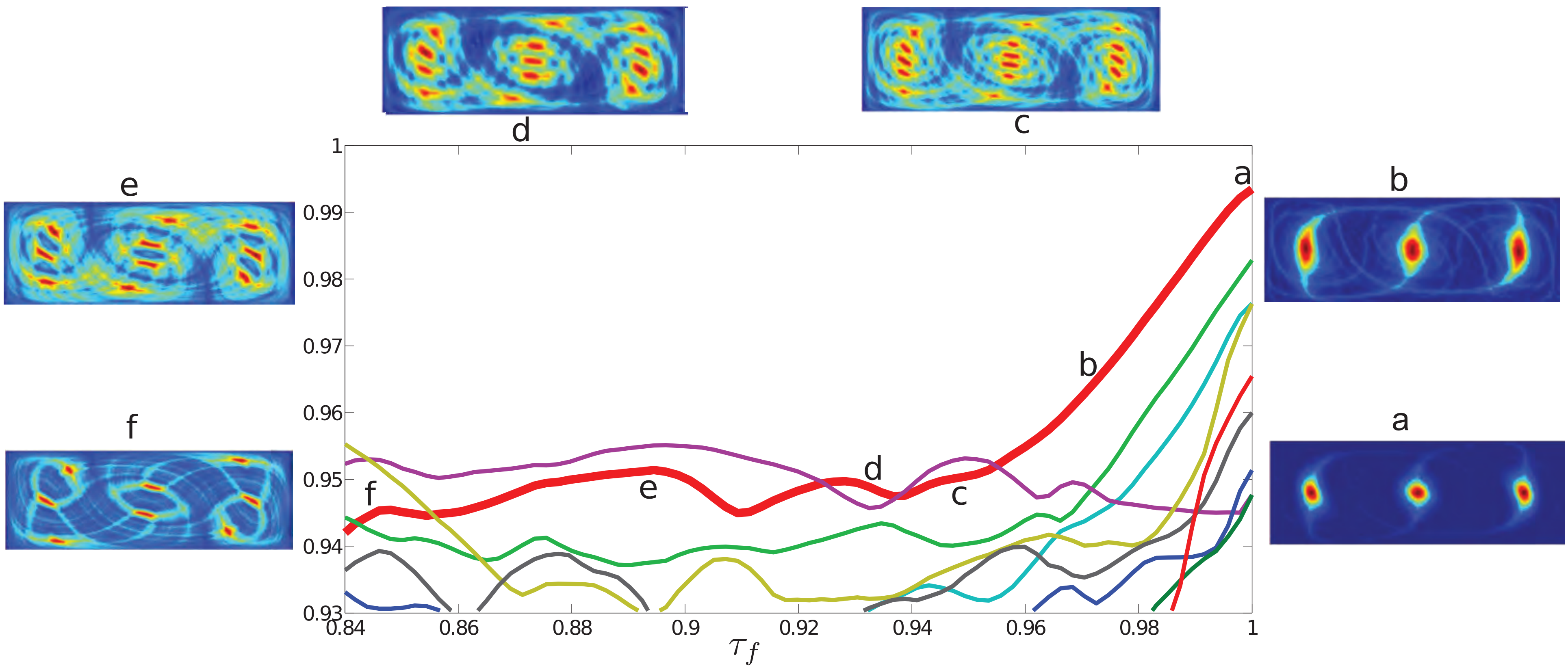
Lines colored according to continuity of eigen**vector**

Eigenvalues/eigenvectors vs. parameter



Genuine eigenvalue crossings? Eigenvalues generically avoid crossings if there is no symmetry present (Dellnitz, Melbourne, 1994)

Eigenvalues/eigenvectors vs. parameter



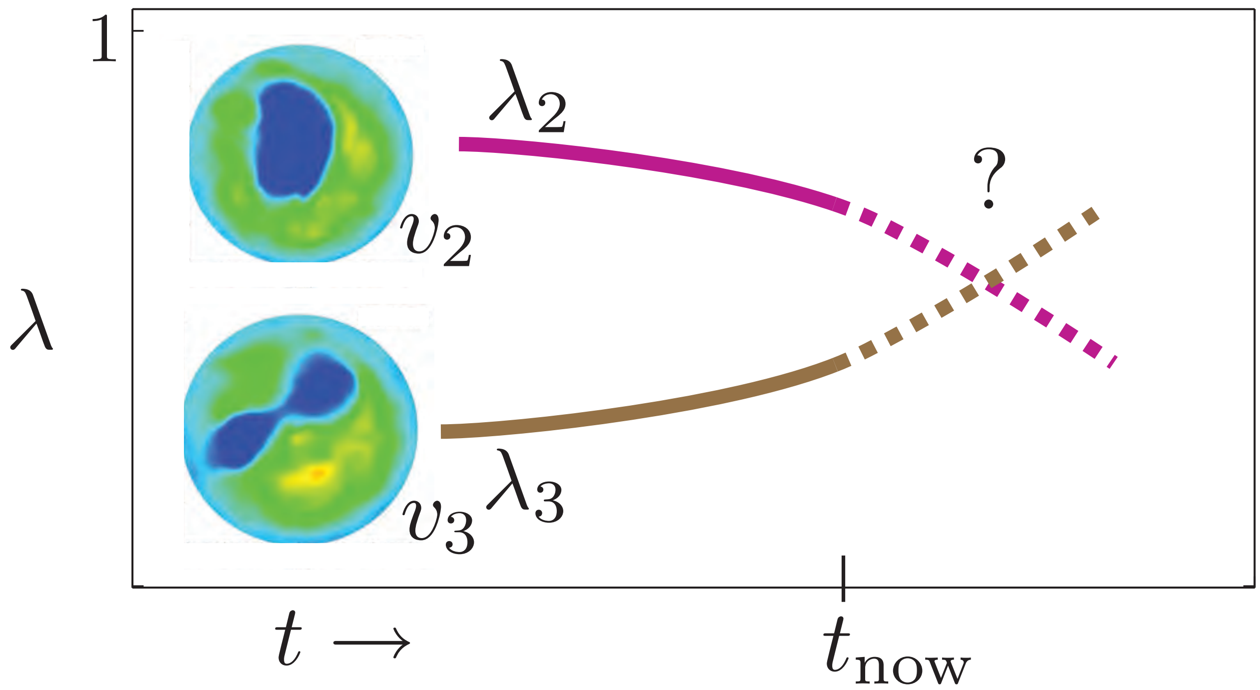
change in eigenvector along thick red branch (a to f), as τ_f decreases.

Grover, Ross, Stremler, Kumar [2012] Chaos

Predict critical transitions in geophysical transport?

Ozone data (Lekien and Ross [2010] Chaos)

Predict critical transitions in geophysical transport?



- Different eigenmodes can correspond to dramatically different behavior.
- Some eigenmodes increase in importance while others decrease
- Can we predict dramatic changes in system behavior?
- e.g., predicting major changes in geophysical transport patterns??

Chaotic fluid transport: aperiodic setting

- Identify regions of high sensitivity of initial conditions
- The finite-time Lyapunov exponent (FTLE),

$$\sigma_t^T(x) = \frac{1}{|T|} \log \left\| D\phi_t^{t+T}(x) \right\|$$

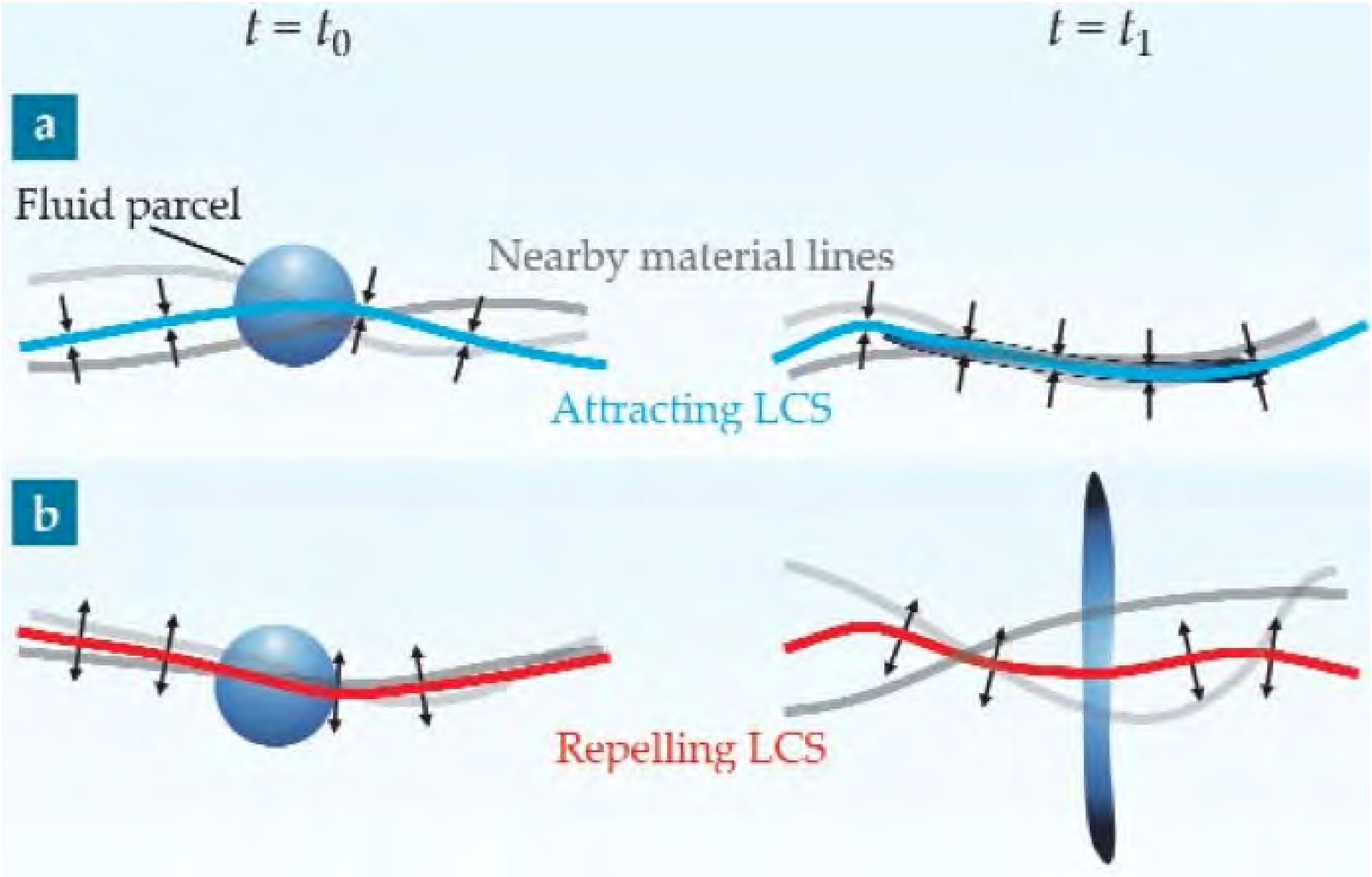
  measures the maximum stretching rate over the interval T of trajectories starting near the point x at time t

- Ridges of σ_t^T reveal hyperbolic codim-1 surfaces; finite-time stable/unstable manifolds; '**Lagrangian coherent structures**' or **LCSs**[2]

[2] cf. Bowman, 1999; Haller & Yuan, 2000; Haller, 2001; Shadden, Lekien, Marsden, 2005

Repelling and attracting structures

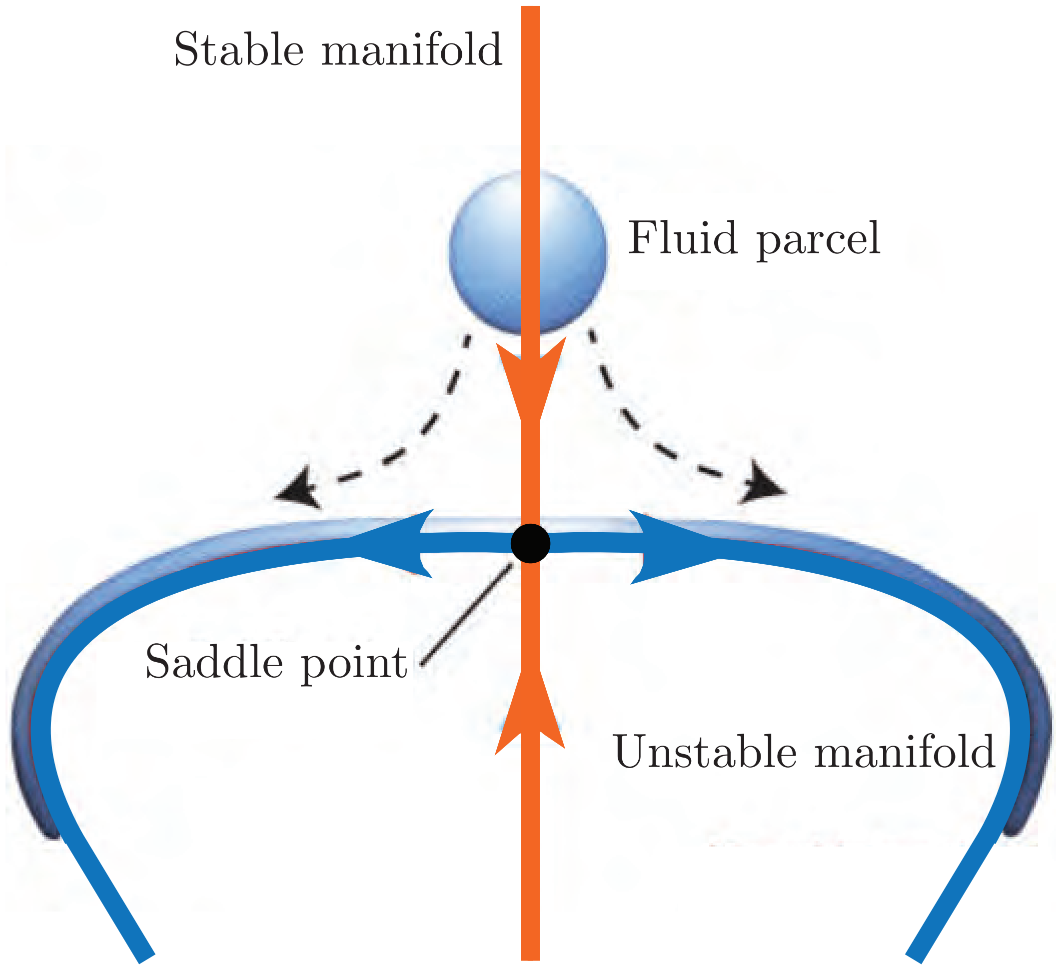
- attracting structures for $T < 0$
  repelling structures for $T > 0$



Peacock and Haller [2013]

Repelling and attracting structures

- Stable manifolds are repelling structures
  Unstable manifolds are attracting structures



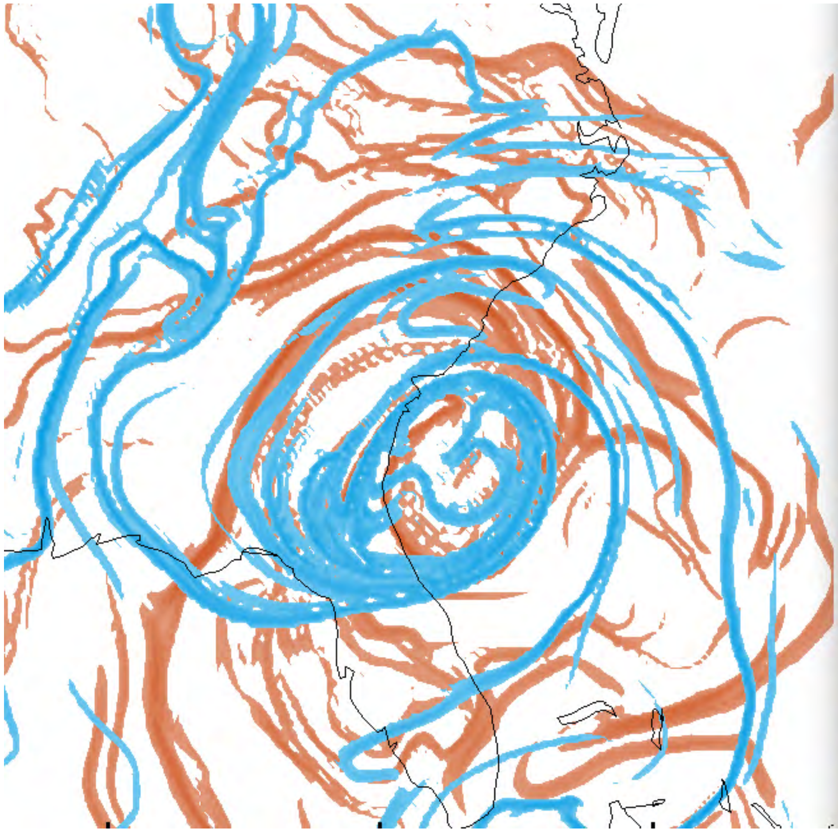
Peacock and Haller [2013]

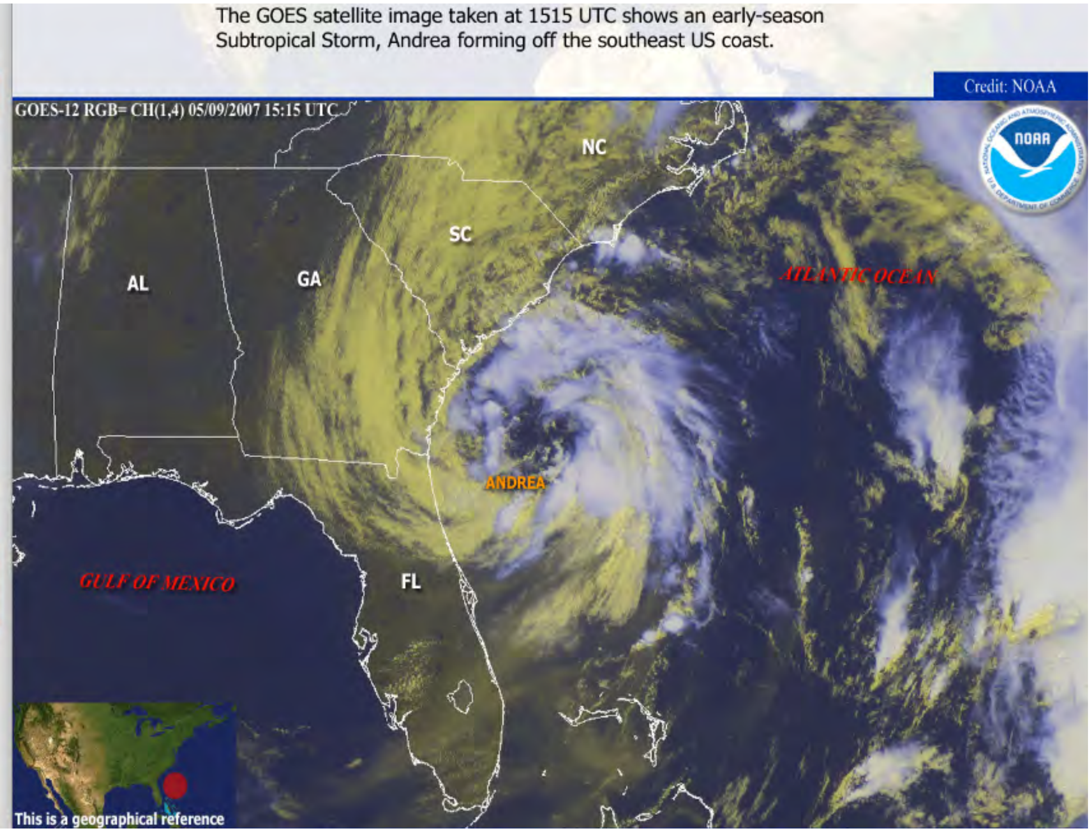
Atmospheric flows: continental U.S.

LCSs: orange = repelling, blue = attracting

2D curtain-like structures bounding air masses

Atmospheric flows and lobe dynamics





orange = repelling LCSs, blue = attracting LCSs

satellite

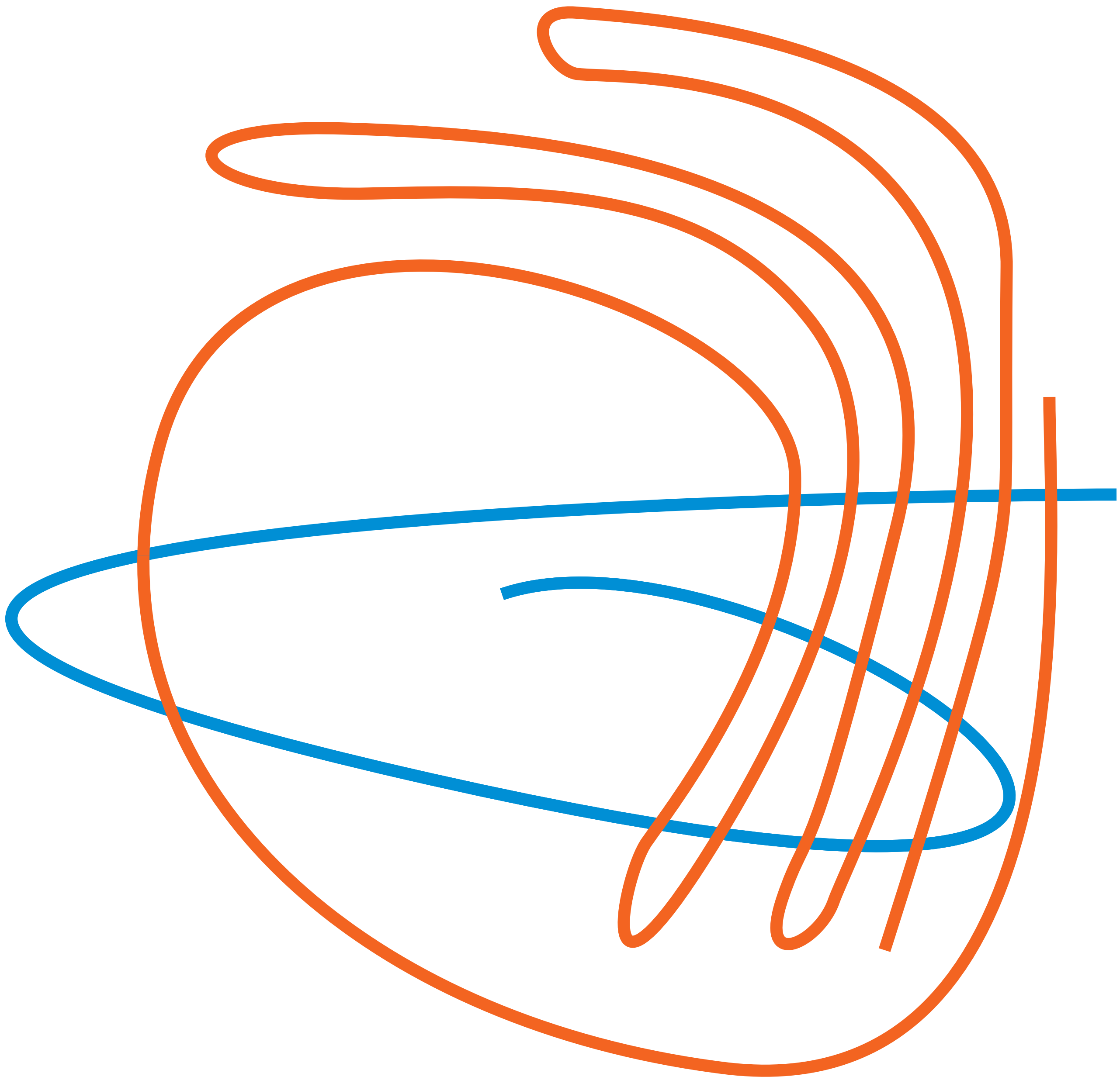
Andrea, first storm of 2007 hurricane season

cf. Sapsis & Haller [2009], Du Toit & Marsden [2010], Lekien & Ross [2010], Ross & Tallapragada [2012]

Atmospheric flows and lobe dynamics

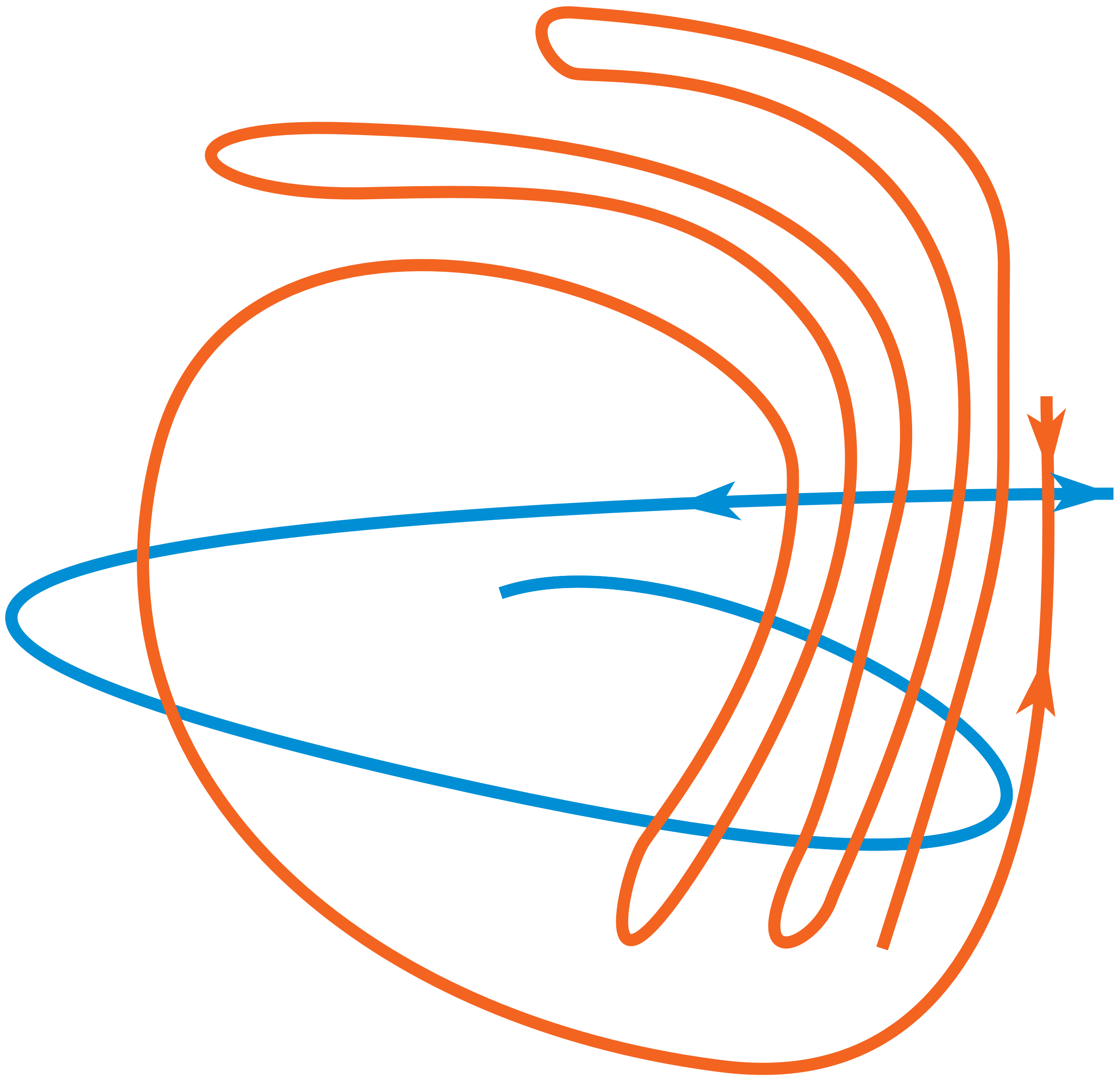
Andrea at one snapshot; LCS shown (orange = repelling, blue = attracting)

Atmospheric flows and lobe dynamics



orange = repelling (stable manifold),   blue = attracting (unstable manifold)

Atmospheric flows and lobe dynamics



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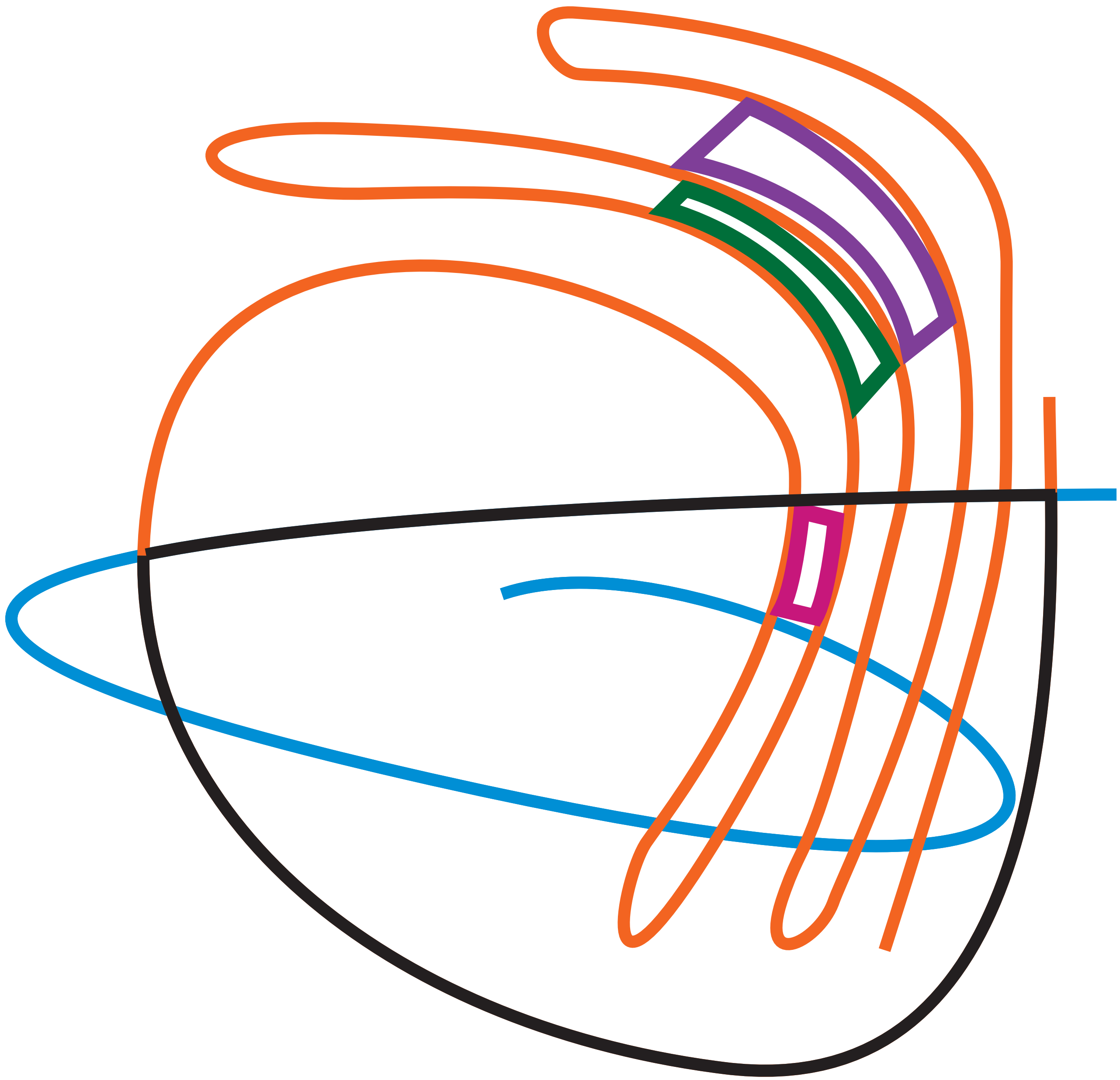
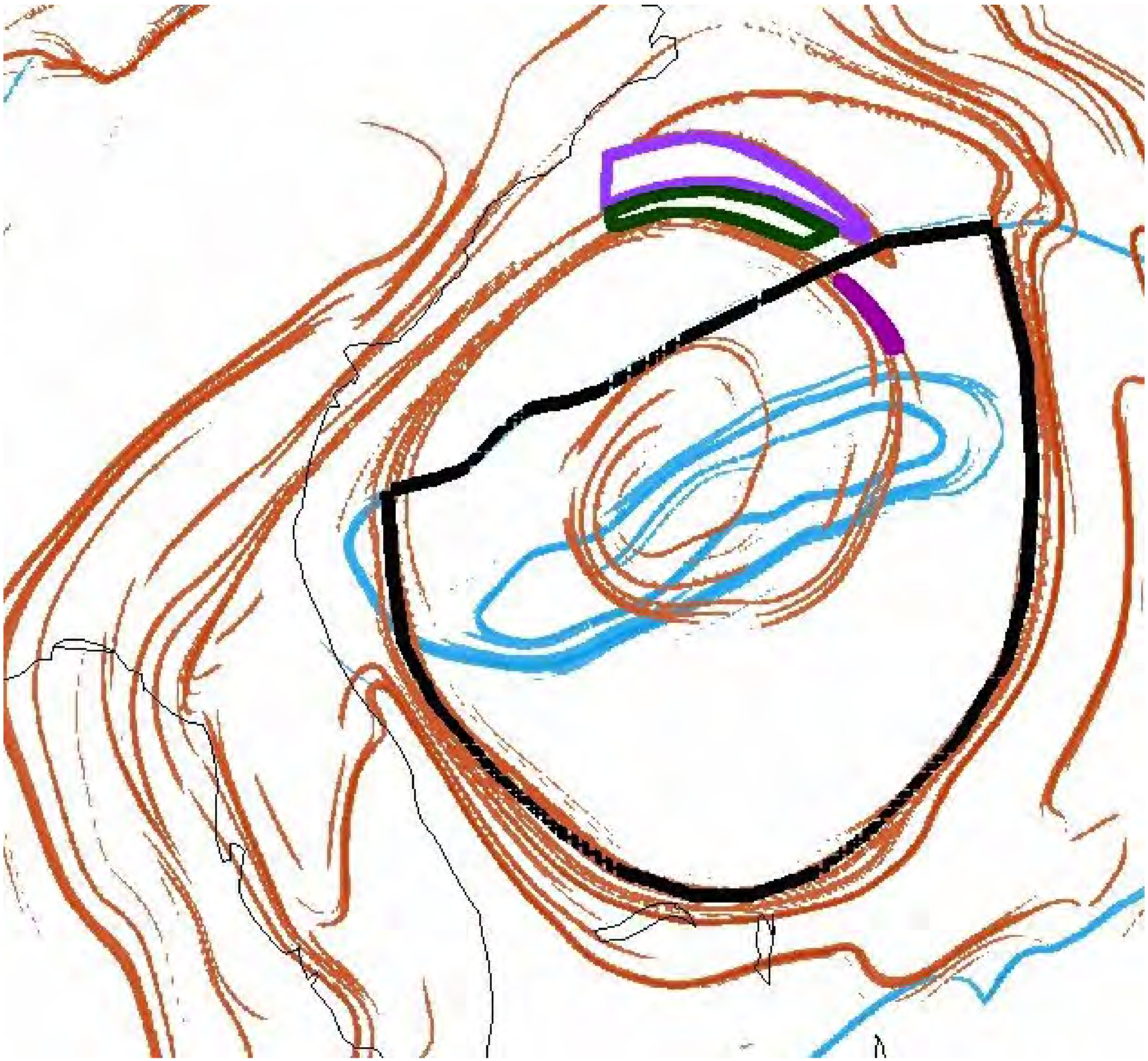
Atmospheric flows and lobe dynamics



Portions of lobes colored; magenta = outgoing, green = incoming, purple = stays out

Atmospheric flows and lobe dynamics

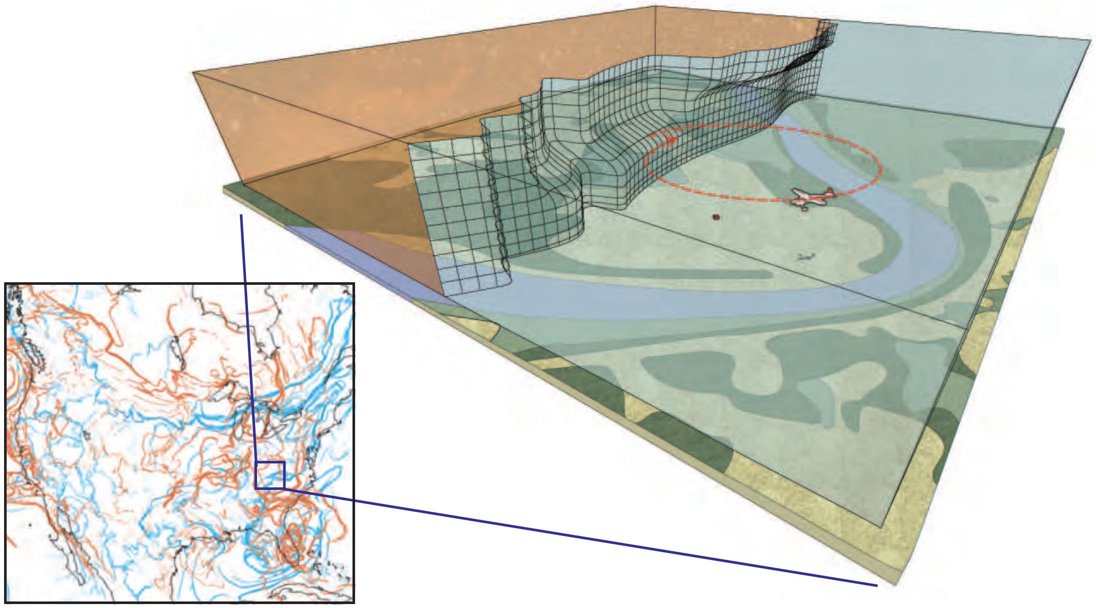


Portions of lobes colored; magenta = outgoing, green = incoming, purple = stays out

Atmospheric flows and lobe dynamics

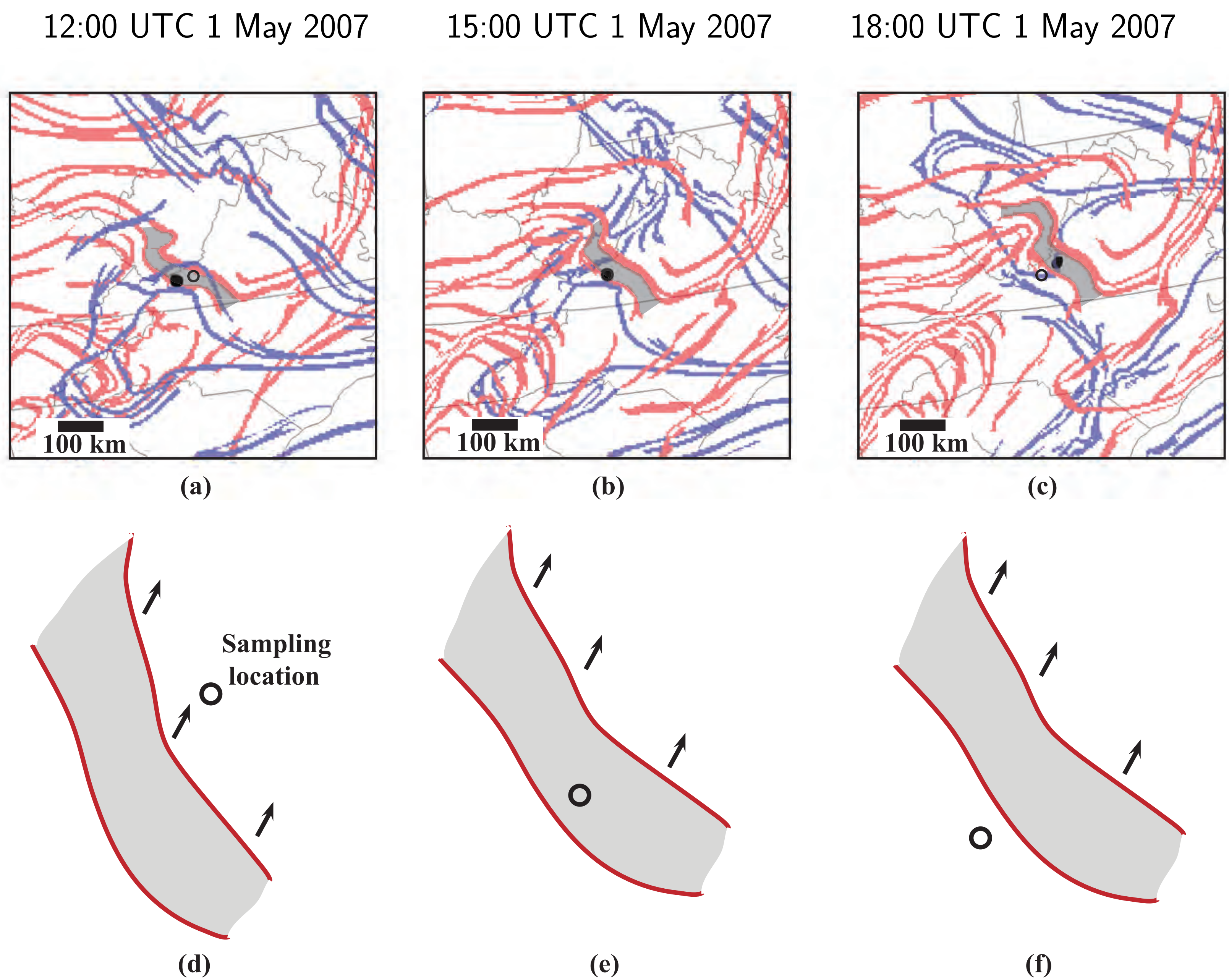
Sets behave as lobe dynamics dictates

Airborne diseases moved about by coherent structures



Joint work with David Schmale, Plant Pathology / Agriculture at Virginia Tech

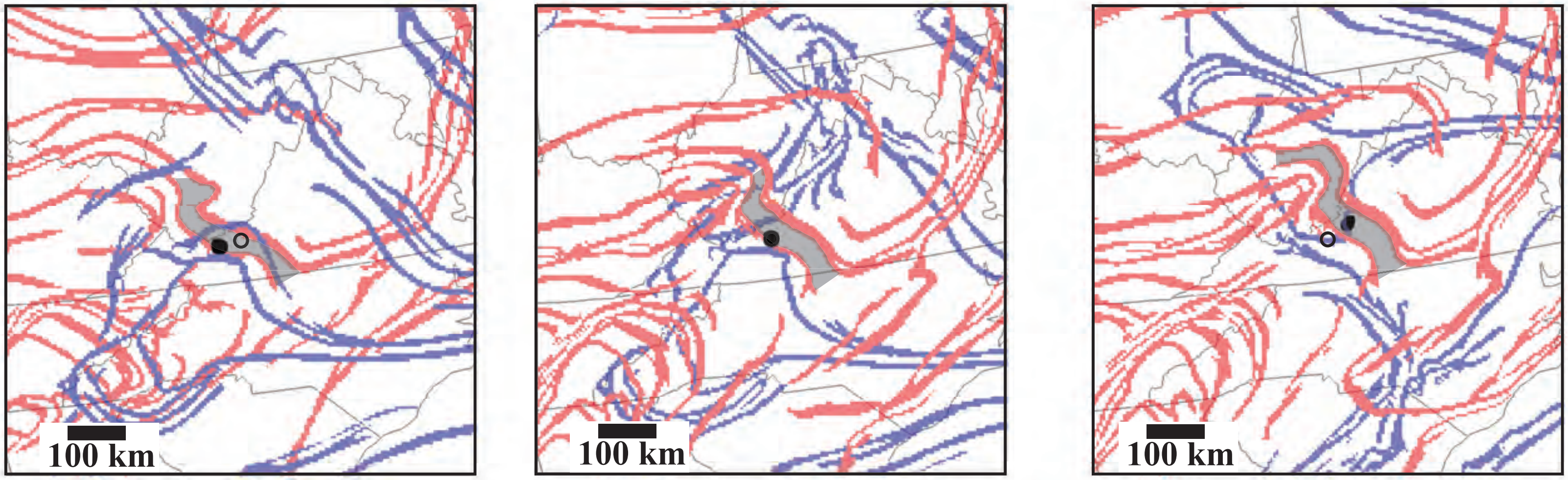
Coherent filament with high pathogen values



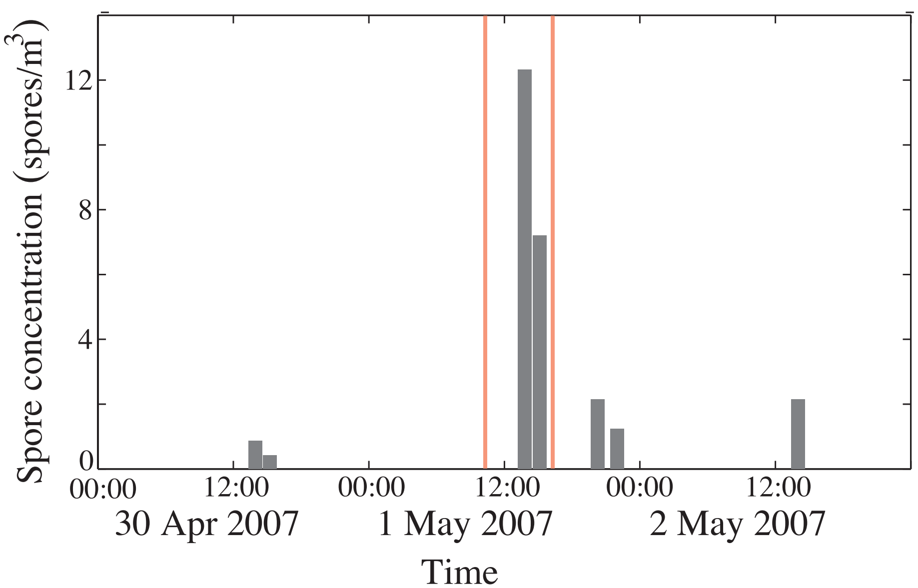
Tallapragada et al [2011] Chaos; Schmale et al [2012] Aerobiologia; BozorgMagham et al [2013] Physica D

Coherent filament with high pathogen values

12:00 UTC 1 May 2007 15:00 UTC 1 May 2007 18:00 UTC 1 May 2007



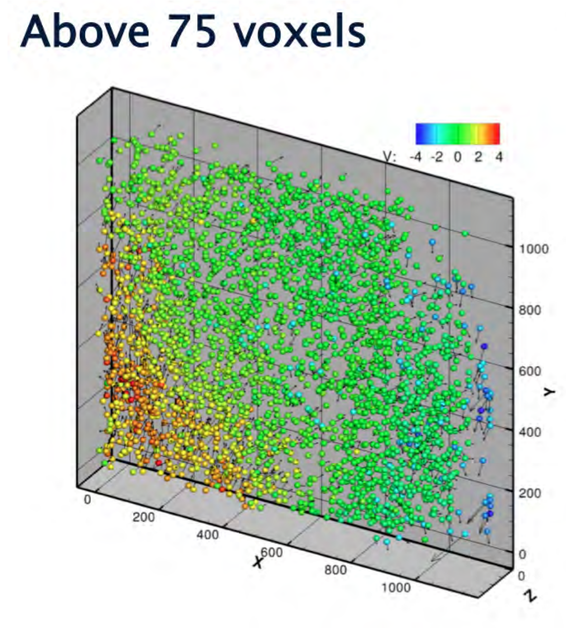
(a) (b) (c)

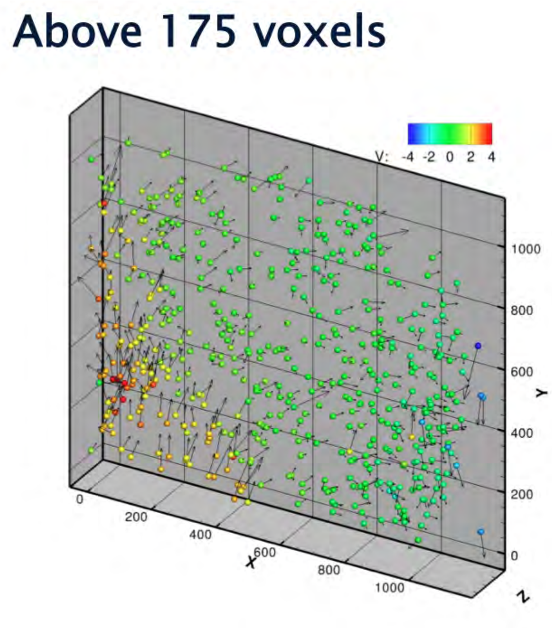


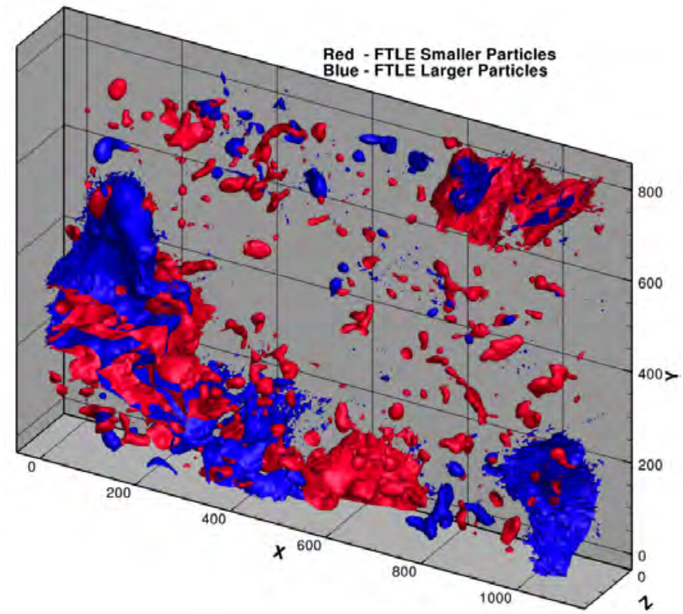
Tallapragada et al [2011] Chaos; Schmale et al [2012] Aerobiologia; BozorgMagham et al [2013] Physica D

Laboratory fluid experiments

3D Lagrangian structure for non-tracer particles:
— Inertial particle patterns (do not follow fluid velocity)





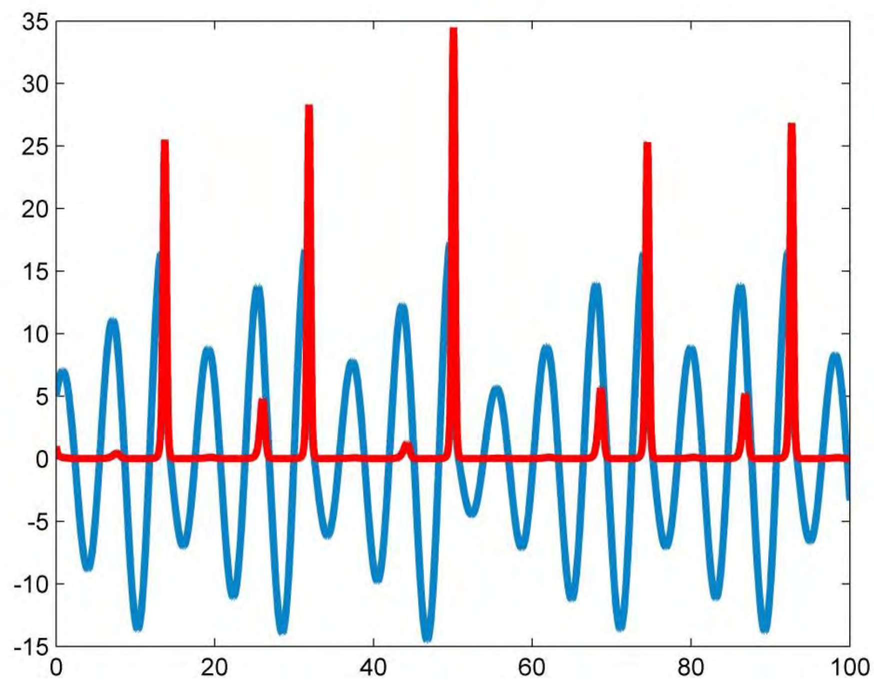


e.g., allows further exploration of physics of multi-phase flows[3]

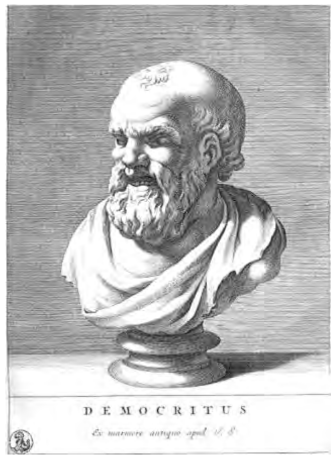
[3] Raben, Ross, Vlachos [2014,2015] Experiments in Fluids

Detecting causality

- Ultimate goal: detecting causality between two time series,



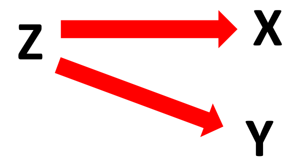
I would rather discover one causal law than be King of Persia.
Democritus (460-370 B.C.)



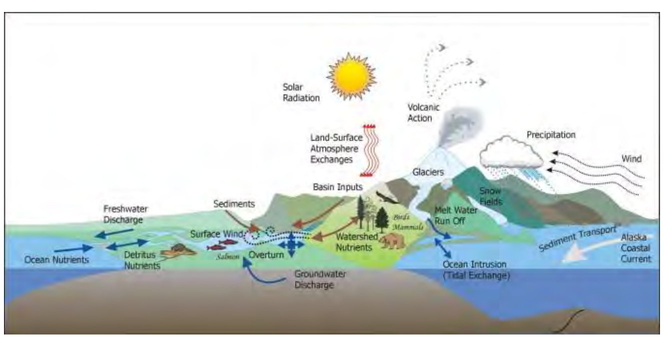
Detecting causality

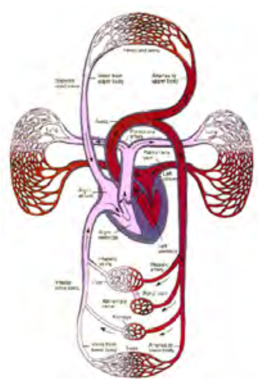
- We have just two time series,
  - Which signal is the driver,
  - Causality direction,

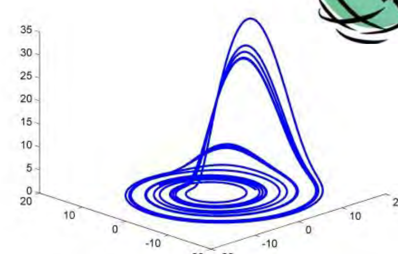
  - Direct causality vs. common external forcing,



  - …
- Signals from:
  - Measurements: temperature, pressure, salinity, velocity, …
  - Maps,
  - ODE’s, PDE’s, …



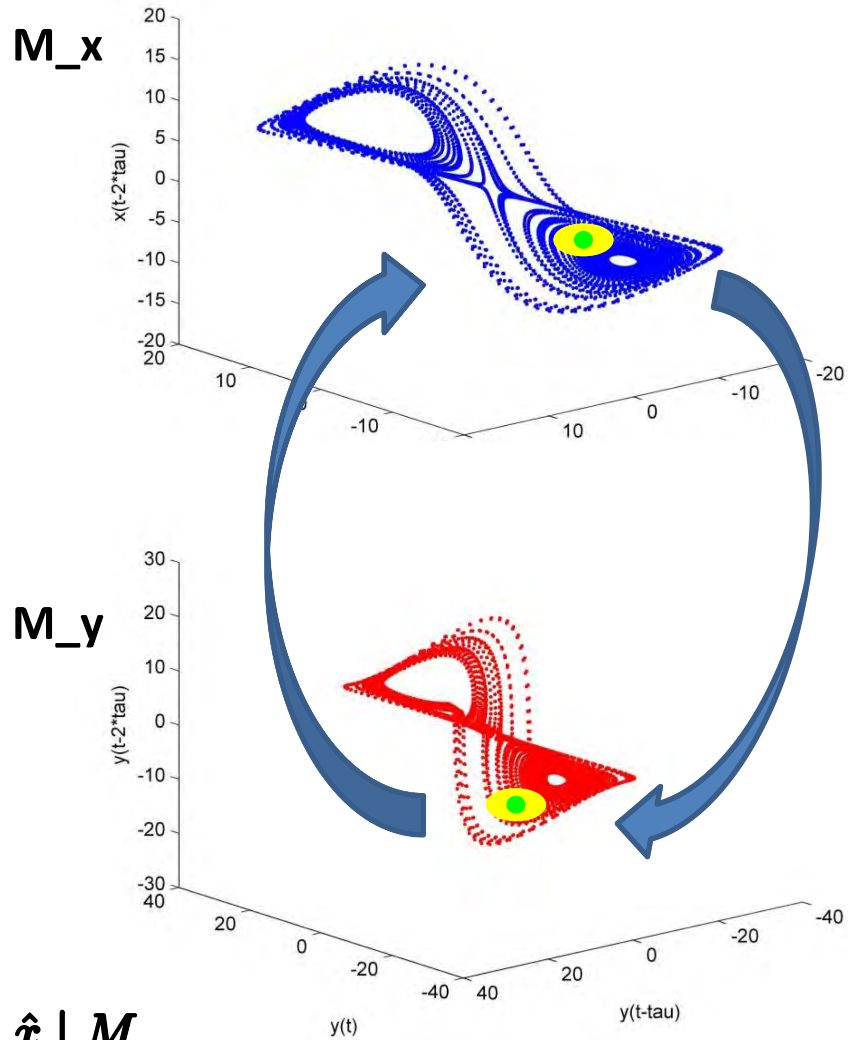




Detecting causality – cross-mapping approach

- If two signals are from a same n-D manifold, then there would be some correspondence between shadow manifolds (reconstructed phase spaces),

Estimating states across manifolds using nearest neighbors:



Sugihara et al. 2012

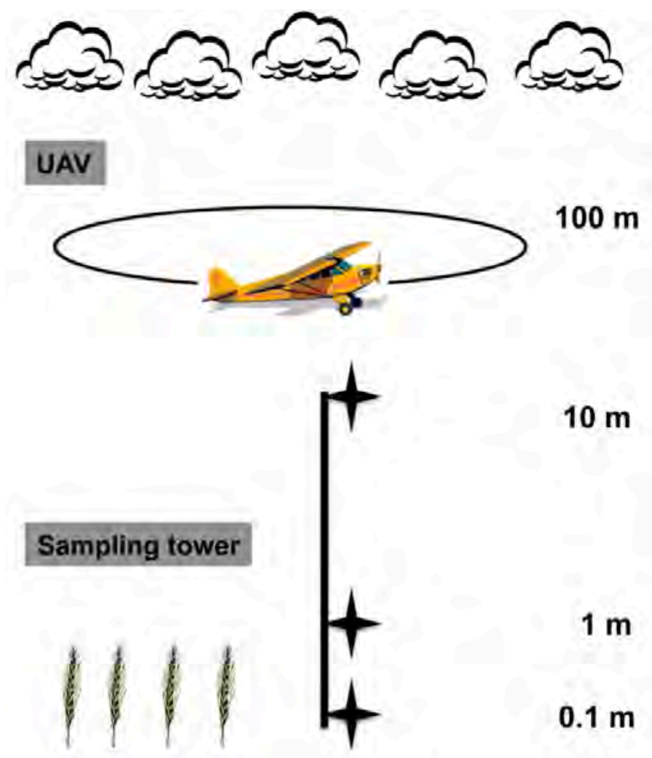
- If x(t) causally influences y(t) then **signature** of x(t) inherently exists in y(t),

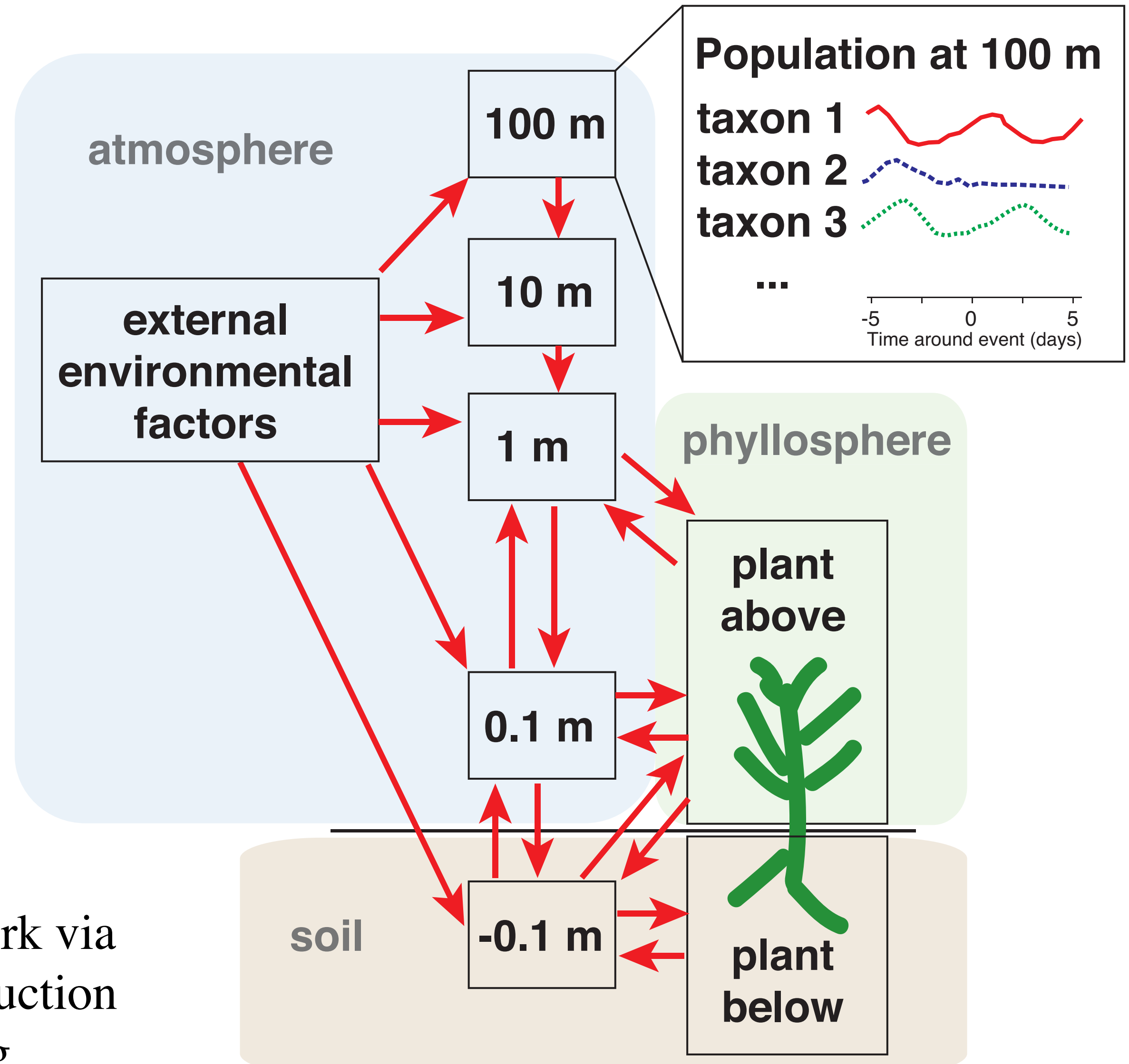
$$\dot{\mathbf{y}}(t) = \bar{f}(\mathbf{x}, \mathbf{y}, \ldots)$$

$$y(t+1) = \bar{g}(x(t), y(t))$$

- **If so, historical record of y(t) values can reliably estimate the state of x** → $\hat{x} \mid M_y$

Detecting causality – agricultural example





Determining the causal network via nonlinear state space reconstruction and convergent cross mapping

Phase space geometry — looking forward

- Many inter-related concepts
  - apply to data-based finite-time settings — just more interesting
  - almost-invariant sets, almost-cyclic sets, braids, LCS, transfer operators, phase space transport networks, dependence on parameters, separatrices, basins of stability
- Opportunities:
  - use in control
  - value-added way of viewing and comparing data
  - detecting causality
- Applications:
  - agriculture, ecology
  - predicting critical transitions in geophysical flow patterns
  - comparative biomechanics, ...