Trade-Off Between Fuel and Time Optimization

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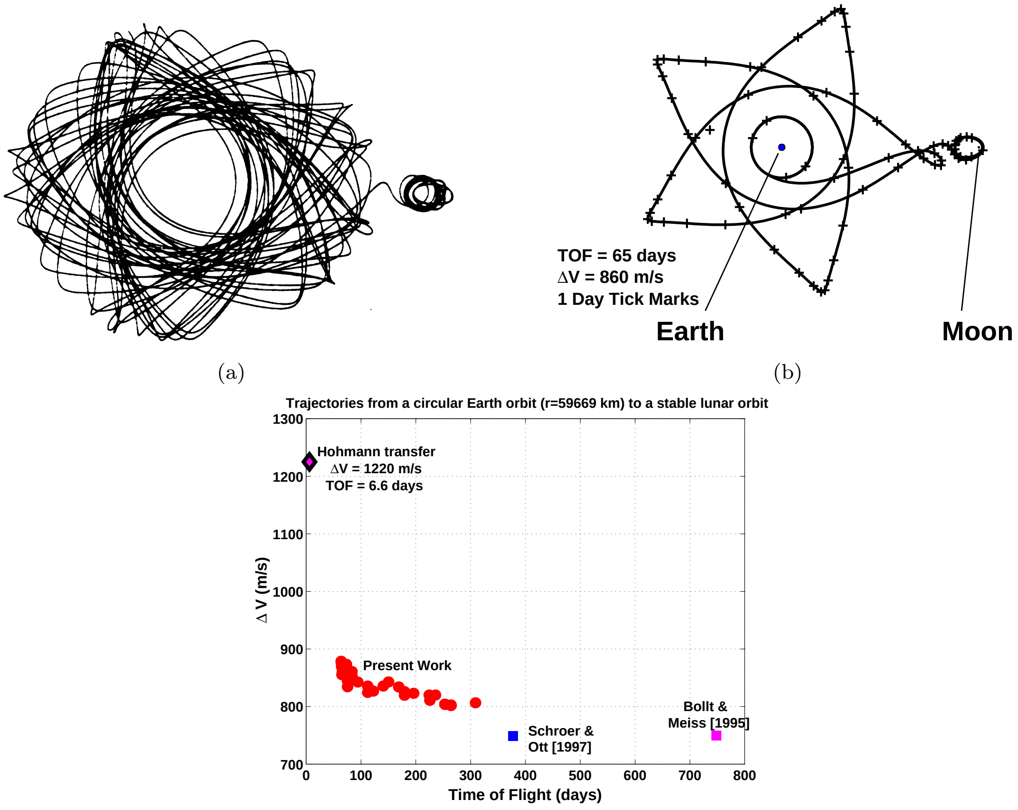
Several studies of new methods for space mission trajectory design have shown that low fuel consumption (measured as low ΔV's) can be achieved at the expense of a long time of flight by taking advantage of N-body effects and repeated gravity assists[1,4,8,11,13]. The trajectories for the *Hiten*[2,3] and *SMART-1*[1,14] missions to the Moon are examples of missions constructed using these nonlinear astrodynamic effects.

The flight time required for some low ΔV missions can be prohibitively long. In the present study, we seek insight into the trade-off between ΔV and flight time for an example problem. We study trajectories from an Earth orbit to the Moon using the planar, circular, restricted three-body model. Our goal is to use as much knowledge of the phase space structure as possible and compare results with two key earlier studies.

Bollt and Meiss[5] considered the transfer from a circular Earth orbit of radius 59669 km to a quasi-periodically precessing ellipse around the moon, with a perilune of 13970 km. Their method takes advantage of the fact that long trajectories in a compact phase space are recurrent. Starting with a long unperturbed chaotic trajectory that eventually reaches the target, they use small well chosen ΔV's to cut recurrent loops from the trajectory. They find a transfer (see Figure 1(a)) that achieves ballistic capture requiring 749.6 m/s, 38% less total velocity boost than a comparable "patched-conics" Hohmann transfer, but requiring a transfer time of 748 days. Schroer and Ott[15] considered this problem with the same initial and final orbits, but found a transfer requiring only 377.5 days, and using roughly the same total ΔV, 748.9 m/s.



(a) (b)

(c)

Figure 1: **Trade-off between fuel and time optimization.** (a) The transfer from a circular earth orbit of radius 59669 km to precessing lunar orbit of perilune 13970 km found by Bollt and Meiss[5] is shown in the rotating frame. The ΔV is 749.6 m/s and the time of flight is 748 days. (b) A transfer between the same initial and final orbits, using a ΔV of 860.1 m/s, but requiring a flight time of 65 days. (c) The ΔV vs. time of flight plot for several "chaotic" trajectories to the moon, compared with the Hohmann transfer designed using a "patched-conics" approach. As can be seen, a trajectory of one-fifth to one-tenth of the flight-time of some previous fuel optimized trajectories can be achieved using only about 100 m/s more ΔV.

Using the method of Ref. 15, together with methods for achieving ballistic capture[9,10], we find a set of transfers for which we plot the ΔV vs. the time of flight in Figure 1(c). Figure 1(b) shows an example trajectory with a flight time of 65 days and a total ΔV of 860.1 m/s. This transfer takes one-tenth of the time as the transfer obtained in Ref. 5 using only slightly more fuel.

This method of determining the ΔV vs. time of flight trade-off has been applied to only one three-body system thus far. As a continuation of this work, we will adapt the method to missions in N-body systems ($N \geq 4$) systems, such as a mission to orbit multiple moons of Jupiter[8,11], e.g., the recently proposed *Jupiter Icy Moons Orbiter*[7]. The development of sophisticated control technology for this mission would not only make it possible to consider a realistic mission for orbiting three of Jupiter's planet-size moons – Callisto, Ganymede and Europa – one after the other, it would also reduce fuel costs compared to the previously proposed *Europa Orbiter* mission[12,17]. Furthermore, the rest of the outer solar system could be opened up to detailed exploration in later missions using this approach[6].

A transfer of this type is sensitive to thruster maneuver implementation errors and perturbations due to unmodeled dynamics, thus autonomous navigation and control capability may be necessary. The first step toward the design of robust missions is the trajectory correction maneuver problem, in which errors are modeled and an optimal control algorithm corrects for those errors[16]. We will incorporate this method in a future study, providing a computational design tool to make low ΔV trajectories more feasible for missions.

References

[1] Belbruno, E.A. [1987], Lunar capture orbits, a method of constructing Earth Moon Trajectories and the Lunar Gas Mission, *19th AIAA/DGLR/JSASS International Electric Propulsion Conference, Colorado Springs, CO*, Paper AIAA 87-1054.

[2] Belbruno, E.A. [1994], The dynamical mechanism of ballistic lunar capture transfers in the four-body problem from the perspective of invariant manifolds and Hill's regions, *Centre de Recera Matemàtica, Institut D'Estudis Catalans*, Preprint No. 270.

[3] Belbruno, E.A. [2002], Low energy transfers, weak capture and chaos, *53rd International Astronautical Congress, World Space Congress, Houston, TX*, Paper IAC-02-A.6.03.

[4] Belló-Mora, M., F. Graziani, P. Teofilatto, C. Circi, M. Porfilio, and M. Hechler [2000], A systematic analysis of weak stability boundary transfers to the Moon, *51st International Astronautical Congress, Rio de Janeiro, Brazil,* Paper IAF-00-A.6.03.

[5] Bollt, E.M. and J.D. Meiss [1995], Targeting chaotic orbits to the Moon through recurrence. *Phys. Lett. A* **204**, 373–378.

[6] Burris, J. and C. Arberg [2002], "Missions to Kuiper Belt Now, Europa Within the Decade Are Key to Space Discoveries," National Academy of Sciences press release, July 11, 2002. (http://www4.nas.edu/news.nsf/isbn/0309084954?OpenDocument)

[7] Cowing, K. [2003], "NASA Set to Unveil 'Jupiter Tour' Mission," January 29, 2003. (http://www.spaceref.com/news/viewnews.html?id=714)

[8] Gómez, G., W.S. Koon, M.W. Lo, J.E. Marsden, J. Masdemont, and S.D. Ross [2001], Invariant manifolds, the spatial three-body problem and space mission design. *Advances in the Astronautical Sciences* **109**(1), 3–22, Paper AAS 01-301.

[9] Koon, W.S., M.W. Lo, J.E. Marsden, and S.D. Ross [2000], Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics. *Chaos* **10**(2), 427–469.

[10] Koon, W.S., M.W. Lo, J.E. Marsden, and S.D. Ross [2001], Low energy transfer to the Moon. *Celest. Mech. Dyn. Astron.* **81**(1-2), 63–73.

[11] Koon, W.S., M.W. Lo, J.E. Marsden, and S.D. Ross [2002], Constructing a low energy transfer between Jovian moons. *Contemp. Math.* **292**, 129–145.

[12] Ludwinski, J., M. Guman, J. Johannesen, R. Mitchell, and R. Staehle [1998], The Europa Orbiter Mission Design, *49th International Astronautical Congress, Melbourne, Australia,* Paper 98-4.2.02.

[13] Ross, S.D., W.S. Koon, M.W. Lo, and J.E. Marsden [2003], Design of a Multi-Moon Orbiter, *13th AAS/AIAA Space Flight Mechanics Meeting, Ponce, Puerto Rico,* Paper AAS 03-143.

[14] Schoenmaekers, J., D. Horas, and J.A. Pulido [2001], SMART-1: With Solar Electric Propulsion to the Moon, *16th International Symposium on Space Flight Dynamics, Pasadena, CA.*

[15] Schroer, C.G. and E. Ott [1997], Targeting in Hamiltonian systems that have mixed regular/chaotic phase spaces. *Chaos* **7**(4), 512–519.

[16] Serban, R., W.S. Koon, M.W. Lo, J.E. Marsden, L.R. Petzold, S.D. Ross, and R.S. Wilson [2002], Halo orbit correction maneuvers using optimal control. *Automatica* **38**(4), 571–583.

[17] Sweetser, T., R. Maddock, J. Johannesen, J. Bell, P. Penzo, A. Wolf, S. Williams, S. Matousek, and S. Weinstein [1997], Trajectory Design for a Europa Orbiter Mission: A Plethora of Astrodynamic Challenges, *AAS/AIAA Space Flight Mechanics Meeting, Huntsville, AL*, Paper AAS 97-174.