THE GENESIS TRAJECTORY AND HETEROCLINIC CONNECTIONS

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Genesis will be NASA's first robotic sample return mission. The purpose of this mission is to collect solar wind samples for two years in an L_1 halo orbit and return them to the Utah Test and Training Range (UTTR) for mid-air retrieval by helicopters. To do this, the *Genesis* spacecraft makes an excursion into the region around L_2. This transfer between L_1 and L_2 requires no deterministic maneuvers and is provided by the existence of heteroclinic cycles defined below. The *Genesis* trajectory was designed with the knowledge of the conjectured existence of these heteroclinic cycles. We now have provided the first systematic, semi-analytic construction of such cycles. The heteroclinic cycle provides several interesting applications for future missions. First, it provides a rapid low-energy dynamical channel between L_1 and L_2 such as used by the *Genesis Discovery Mission*. Second, it provides a dynamical mechanism for the temporary capture of objects around a planet without propulsion. Third, interactions with the Moon. Here we speak of the interactions of the Sun-Earth Lagrange point dynamics with the Earth-Moon Lagrange point dynamics. We motivate the discussion using Jupiter comet orbits as examples. By studying the natural dynamics of the Solar System, we enhance current and future space mission design.

INTRODUCTION

The key feature of the *Genesis* trajectory (see Figure 1) is the Return Trajectory which makes a 3 million km excursion between L_1 and L_2 in order to reach UTTR during day-light hours. The extraordinary thing about this 5 month excursion is that it requires no deterministic maneuvers! In the language of dynamical systems theory, this transfer is said to shadow a heteroclinic connection between the L_1 and L_2 regions.

There is, in fact, a vast theory about heteroclinic dynamics which among other things are the generators of deterministic chaos in a dynamical system. Our recent work (Koon *et al.*[1]) provides a precise theory of how heteroclinic dynamics arise in the context of the planar circular restricted three-body problem (PCR3BP). In this paper, we apply this theory to explain how the *Genesis* return trajectory works. This provides the beginnings of a systematic approach to the design and generation of this type of trajectories. In the not too distant future, automation of this process will be possible based on this approach. The eventual goal is for the on board autonomous navigation of this type of low-energy Earth sample return missions. But in fact, as we will show, this dynamics affects a much greater class of new mission concepts.

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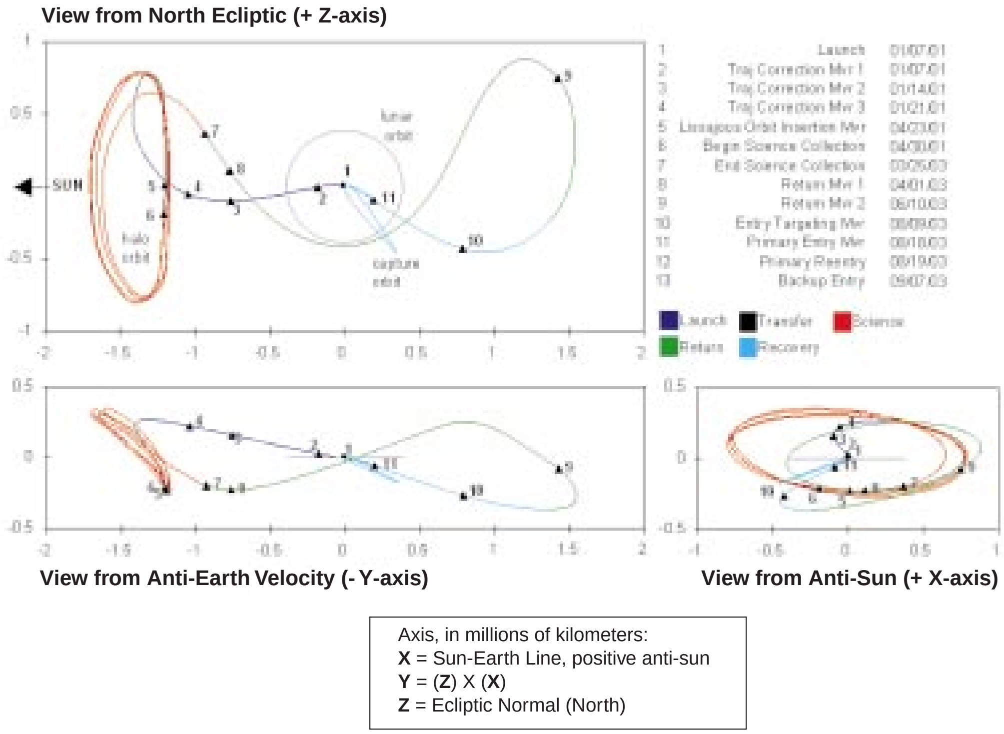


Figure 1: The *Genesis* trajectory in Sun-Earth rotating frame.

To motivate the discussion and to provide an independent example from nature, we examine the orbit of the comets, *Oterma* and *Gehrels 3*. We want to understand how heteroclinic dynamics work in nature in order to develop and verify our theory. An important theme in our work is to learn from nature because it seems that nature has already found the best solution. Whatever we can glean from natural phenomena will contribute immeasurably to the development of new trajectory and mission concepts. In particular, the understanding of the structure of the heteroclinic and homoclinic orbits has given us new insights into the transport mechanisms within the Solar System which can be utilized for space trajectory design.

We will explain the key theorem from Koon *et al.*[1] and apply it to the "temporary capture mechanism" in the astrodynamics context. Of course, this theorem has more to say about the complex orbital dynamics in this regime. It seems that the region of phase space between L_1 and L_2 is full of dynamical channels like a complex system of wormholes or tunnels. These channels exist throughout the Solar System in a vast network connecting all the planets and their satellites (see Lo and Ross[2–3]). Together, they provide a network of low-energy trajectories which may be used for new mission concepts. Although these hidden passageways may be new to us, they have been well trodden by comets and asteroids perhaps since the birth of the Solar System. We apply our understanding of these dynamical channels to a new class of missions which we call "The Petit Grand Tour". The Petit Grand Tour combines the temporary capture mechanism with the concept of the interplanetary network of dynamical channels (or dynamical wormholes) to provide a low-energy mission to tour the moons of Jupiter (or Saturn) in any desired sequence.

HETEROCLINIC CONNECTIONS AND CYCLES

The goal of the *Genesis Mission* is to return to UTTR all of the solar wind samples collected over four revolutions (two years) in an L_1 halo orbit (see Lo *et al.*[4] and references therein). The mid-air retrieval of the ballistic sample return capsule by helicopters requires that the entry must occur during daylight. But the natural dynamics of an L_1 halo orbit require a night-side return. In order to achieve the day-side entry within a reasonable ΔV budget, an excursion into the L_2 region is necessary which added two months to the return phase.

A heteroclinic connection, $\mathcal{H}$, also called a heteroclinic trajectory, is an asymptotic trajectory which connects two periodic orbits which we denote by $\mathcal{A}$ and $\mathcal{B}$ for this discussion. In the event $\mathcal{A}$ and $\mathcal{B}$ are the same periodic orbit, $\mathcal{H}$ is called a homoclinic orbit. $\mathcal{H}$ is a theoretical construct of great importance both in theory and in practical applications. We examine some of the key features of these orbits. It takes $\mathcal{H}$ infinite time to wind off from $\mathcal{A}$ to transfer to $\mathcal{B}$. Once near the vicinity of $\mathcal{B}$, it takes $\mathcal{H}$ an infinite time to wind onto $\mathcal{B}$. They were studied intensely by Poincaré and were central to his discovery of chaos in the 3 body problem. Practically, of course, we are never able to compute the real $\mathcal{H}$ just as we are never able to compute the real periodic orbits of any nonlinear system. However, what we are able to compute are neighboring trajectories, H's, which "shadow" $\mathcal{H}$ to any desired accuracy (within machine accuracy) for the particular problem at hand.

Now a slight diversion on our notation which we will keep to a minimum. We denote a heteroclinic orbit between $\mathcal{A}$ and $\mathcal{B}$ by $\mathcal{H}_{\mathcal{AB}}$ and a heteroclinic orbit between $\mathcal{B}$ and $\mathcal{A}$ by $\mathcal{H}_{\mathcal{BA}}$. In particular, a homoclinic orbit of $\mathcal{A}$ is denoted by $\mathcal{H}_{\mathcal{AA}}$. We distinguish the theoretical orbit, $\mathcal{H}$, and its numerical shadow, H, by the script and block fonts respectively. Returning to our main discussion, when we have a heteroclinic orbit between $\mathcal{A}$ and $\mathcal{B}$ and a heteroclinic orbit between $\mathcal{B}$ and $\mathcal{A}$, the two orbits $\{\mathcal{H}_{\mathcal{AB}}, \mathcal{H}_{\mathcal{BA}}\}$ form a heteroclinic cycle. In particular, homoclinic orbits are already cycles. The importance of cycles both theoretically and practically will be discussed shortly. We shall see that they are very important indeed.

Of course, the existence of heteroclinic behavior was generally known to the halo mission community previously. Typically when integrating an L_1 halo orbit for too long, it escapes the vicinity of the halo orbit and sometimes returns towards Earth before continuing to wind around L_2. Thus the phenomenon is easily observed in numerical experiments. The *WIND Mission* was the first to use this heteroclinic behavior between L_1 and L_2 (Sharer *el al.*[5]). Howell and Barden[6] made a more formal study of heteroclinic connections and found a free connection between a halo orbit and a lissajous orbit using numerical search.

Koon, Lo, Marsden and Ross[1] studied the problem of PCR3BP and used the more systematic method of Poincaré sections from dynamical systems theory to produce heteroclinic orbits between two Lyapunov orbits (periodic orbits around L_1 and L_2 in the plane). The method of Poincaré section reduces the problem by one dimension thereby rendering the problem more tractable. Futhermore, there is a substantial body of known results on Poincaré sections from dynamical systems theory which provide additional knowledge and insight into the specifics of the dynamics. This knowledge and insight provide the foundation for new mission concepts and for the optimization of current mission designs discussed in this paper.

We conclude this section by emphasizing the importance of these seemingly esoteric theoretical constructs, the $\mathcal{H}$ and H orbits. Their importance in astrodynamics is two-fold: (1) to simplify computation, and (2) to generate new mission concepts. Their importance to computation is perhaps best illustrated by the process used to compute the *Genesis* halo orbit which we denote by G. We start the process with a theoretical model lissajous orbit, $\mathcal{G}$, specified by amplitudes and phase angles. We produce an analytic approximation, G_1 using a 3rd order analytic expansion. Next we produce a differentially corrected lissajous orbit, G_2, from G_1. Finally, starting with G_2, we apply the various mission constraints and differentially correct for the *Genesis* halo orbit, G. To summarize, the theoretical model orbit, $\mathcal{G}$, is the starting point from which practical orbits may be constructed via the continuation process using a series of numerical computations. In the same way, the *Genesis* Earth return trajectory was computed using heteroclinic-like orbits as initial models. Therefore, advances in the theory and computation of these orbits are essential to the simplification and eventual automation of this complex process of continuation. It is remarkable to think how Poincaré was able to see all of these complex issues and actually perform continuation calculations of orbits without the benefit of modern computers. The discussion of their importance to new mission concepts occupies the rest of this paper.

The Three Body Problem

We start with the PCR3BP as our first model of the mission design space, the equations of motion for which in rotating frame with normalized coordinates are:

$$\ddot{x} - 2\dot{y} = \Omega_x, \qquad \ddot{y} + 2\dot{x} = \Omega_y,$$

where

$$\Omega = \frac{x^2 + y^2}{2} + \frac{1-\mu}{r_S} + \frac{\mu}{r_J} + \frac{\mu(1-\mu)}{2}.$$

The subscripts of Ω denote partial differentiation in the variable and dots over the variables are time derivatives. The variables r_S, r_J, are the distances from (x, y) to the two primary bodies, which we refer to generically as the Sun and Jupiter, respectively.

The coordinates of the equations use standard PCR3BP conventions: the sum of the mass of the Sun and Jupiter is normalized to 1 with the mass of Jupiter set to μ; the distance between the Sun and Jupiter is normalized to 1; and the angular velocity of Jupiter around the Sun is normalized to 1. Hence in this model, Jupiter is moving around the Sun in a circular orbit with period 2π. The rotating coordinates, following standard astrodynamic conventions, are defined as follows: the origin is set at the Sun-Jupiter barycenter; the x-axis is defined by the Sun-Jupiter line with Jupiter on the positive x-axis; the (x, y)-plane is the plane of the orbit of Jupiter around the Sun (see Figure 2).

Although the PCR3BP has 3 collinear libration points which are unstable, for the cases of interest to mission design, we examine only L_1 and L_2 in this paper. These equations are autonomous and can be put into Hamiltonian form with 2 degrees of freedom. It has an energy integral called the Jacobi constant which provides 3-dimensional constant energy surfaces:

$$C = -(\dot{x}^2 + \dot{y}^2) + 2\Omega(x, y).$$

The power of dynamical systems theory is that it is able to provide additional structures within the energy surface and characterize the different regimes of motions.

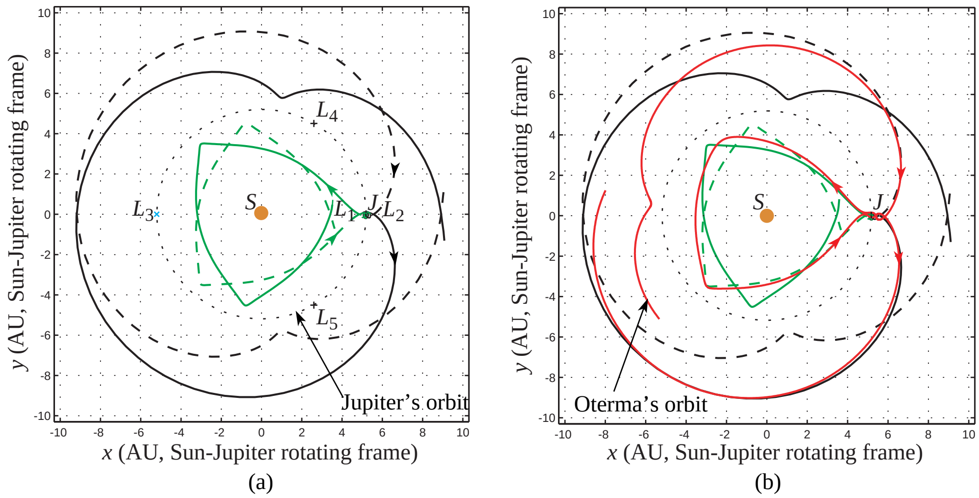


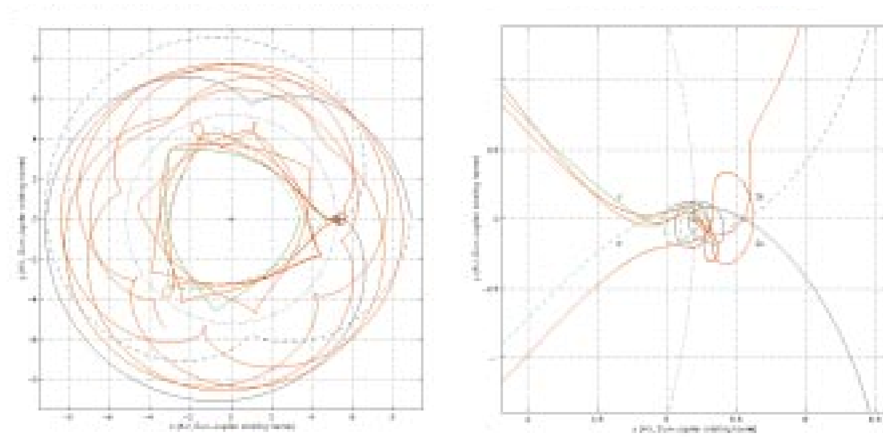
Figure 2: (a) Stable (dashed curves) and unstable (solid curves) manifolds of L_1 and L_2 projected to position space in the Sun-Jupiter rotating frame. (b) The orbit of comet *Oterma* (AD 1915–1980) in the Sun-Jupiter barycentered rotating frame follows closely the invariant manifolds of L_1 and L_2.

Examples from Nature: The Motion of Comets

The Jupiter family of comets exhibit many puzzling phenomena the most interesting of which to mission design are the temporary capture phenomenon and resonance transition. Lo and Ross[2] proposed an explanation based on observations of the stable and unstable manifolds of L_1 and L_2. Figure 2a shows the stable manifolds as dashed curves, the unstable manifolds as solid curves for L_1 and L_2 of the Sun-Jupiter system in rotating frame. The Sun is labeled S, Jupiter is labeled J. In Figure 2b, the orbit of the comet *Oterma* is overlaying the manifolds. Notice how well *Oterma*'s orbit fits with that of the manifolds of L_1 and L_2.

Lo and Ross[2] argued that this suggests that the comet's orbit is under the control of the invariant manifold structure of the Lagrange points. The term invariant manifold structure is a catch-all phrase for the entire structure of periodic and quasiperiodic orbits around the Lagrange points and all of their associated invariant manifolds, such as the stable and unstable manifolds of the periodic orbits. Recall a manifold is simply a mathematical term for higher dimensional surfaces. An invariant manifold in dynamical systems theory is a special manifold consisting of orbits; hence a point on the invariant manifold will forever remain on the manifold under the flow of the equations of motion. The Lagrange points are examples of 0-dimenisonal invariant manifolds. A periodic orbit is an exmaple of a 1-dimensional invariant manifold. The stable manifold of a Lyapunov orbit is an example of a 2-dimensional manifold. Its energy surface in the PCR3BP is an example of a 3-dimensional invariant manifold.

In Figure 3a the orbit of comet *Gehrels 3* is overlaid against the manifolds of Jupiter's L_1 and L_2. In Figure 3b, a close-up in the Jupiter region shows how *Gehrels 3* nearly goes into a halo orbit for one revolution around L_2 before capturing into Jupiter orbit for several revolutions. Once again, the manifolds match closely with the comet orbit. Furthermore,



(a) (b)

Figure 3: (a) The orbit of *Gehrels 3* overlaying the manifolds of Jupiter's L_1 and L_2. Stable manifolds are dashed lines, unstable manifolds are solid lines. (b) The orbit of *Gehrels 3* in the Jupiter region showing temporary captures and halo orbits.

the temporary capture of the comet by Jupiter suggests the possibilities of low-energy capture for interplanetary missions.

Based on the invariant manifold approach suggested by Lo and Ross[2], Koon *et al.*[1] gives a systematic and mathematically rigorous explanation of this dynamics. In addition to a more complete global qualitative picture, it also provides computational and predictive capabilities. It provides an algorithm based on standard Poincaré section methods to compute heteroclinic orbits. It provides the rudiments for the calculation of transport probabilities based on lobe dynamics theory (see Wiggins[7], Meiss[8]). Previous mission design work using heteroclinic dynamics were based on ad hoc numerical search and exploration. But the new computation tools of Koon *et al.*[1] enable the mission designer to construct heteroclinic trajectories in a systematic fashion instead of using a blind and brute force search. The results are promising and more research and development work remains before this process may be fully automated.

The Orbit Classes Near L_1 and L_2.

The work of Lo and Ross[2] demonstrated that the dynamics of the L_1 and L_2 region is extremely important for the understanding of many disparate dynamical phenomena in the Solar System and also for space mission design. In order to better understand the dynamics of this region, we now review the work of Conley[9] and McGehee[10] which provide an essential characterization of the orbital structure near L_1 and L_2. McGehee also proved the existence of homoclinic orbits in the interior and exterior regions. Llibre, Martinez, and Simó[11] showed that the homoclinic orbits of L_1 in the interior region are transversal. They used symbolic dynamics to prove a theorem for orbital motions in the interior and Jupiter region. One of the key results in Koon *et al.*[1] is the completion of this picture with the computation of heteroclinic cycles in the Jupiter region between L_1 and L_2. We will refer to the various regions by the following short hand: S for the interior region which contains the Sun, J for the Jupiter region which contains the planet, X for the exterior region outside the planet's orbit.

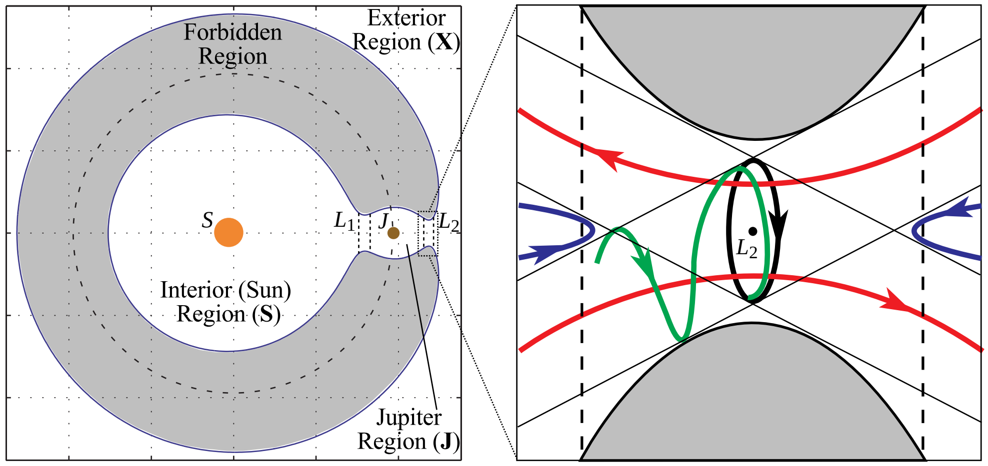


Figure 4: (a) Hill's region (schematic, the region in white) connecting the interior region, the capture region, and the exterior region. (b) Expanded view of the L_2 region showing a periodic orbit, a typical asymptotic orbit, two transit orbits and two non-transit orbits.

Figure 4 schematically summarizes the key results of Conley and McGehee. For an energy value just about that of L_2, the Hill's region is the projection of the energy surface from the phase space onto the configuration space, i.e. the xy-plane. This is represented by the white space in Figure 4a. The grey region is energetically forbidden. In other words, with the given energy, our spacecraft can only explore the white region. More energy is required to enter the grey Forbidden region.

Figure 4b is a blow-up of the L_2 region to indicate the existence of four different classes of orbits. The first class is a single periodic orbit with the given energy, the planar Lyapunov orbit around L_2. The second class represented by a spiral is an asymptotic orbit winding onto the periodic orbit. This is an orbit on the stable manifold of the Lyapunov orbit. Similarly, although not shown, are orbits which wind off the Lyapunov orbit to form the unstable manifold. The third class are transit orbits which pass through the Jupiter region between the S and X regions. Lastly, the fourth class consists of orbits which are temporarily trapped in the S or X regions.

Let us examine the stable and unstable manifold of a Lyapunov orbit as shown in Figure 5. Of course, only a very small portion of the manifolds are plotted. Notice the X-pattern typical of a saddle formed by the manifolds. This is reminiscent of the X-pattern of the manifolds of the Lagrange points in Figure 3b. It is precisely in this sense that we say the manifolds of the Lagange points are "genetic"; they characterize the essential shapes and dynamics of things to come when more complexity such as periodic orbits and their manifolds are introduced. Thus, by studying the simpler 1-dimensional manifolds of L_1 and L_2 , we are able to gain some understanding of the nature of the complex dynamics of the full invariant manifold structure.

Notice that the 2-dimensional tubes of the manifolds of the Lyapunov orbits are separatrices in the 3-dimensional energy surface! By this we mean the tubes separate regimes of qualitatively different motion within the energy surface. Referring back to the schematic

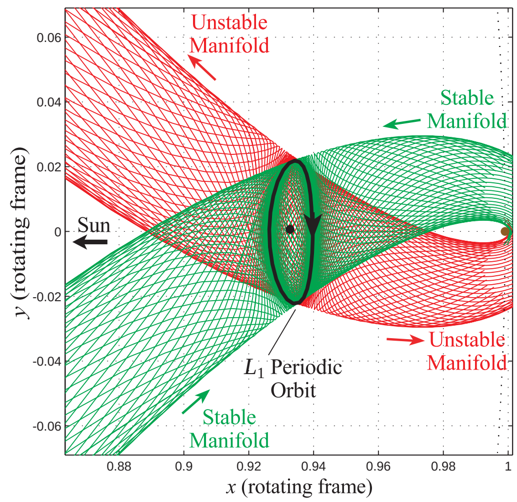


Figure 5: The stable and unstable manifolds of a Lyapunov orbit.

diagram, Figure 4b, we notice that the transit orbits pass through the oval of the Lyapunov orbit. This is no accident, but an essential feature of the dynamics on the energy surface. Lo and Ross[2] referred to L_1 and L_2 as gate keepers on the trajectories, since the Jupiter comets must transit between the X and S regions through the J region and always seem to pass by L_1 and L_2. Chodas and Yeomans[12] noticed that the comet *Shoemaker-Levy 9* passed by L_2 before it crashed into Jupiter. These tubes are the only means of transit between the different regions in the energy surface! In fact, this was already known to Conley and McGehee several decades ago.

The Homoclinic-Heteroclinic Chain

Putting all of these results together, we are able to construct a complete homoclinic-heteroclinic chain as shown in Figure 6: start with a homoclinic connection in the interior region, go to a heteroclinic cycle in the capture region, and finally end with a homoclinic connection in the exterior region. The pair of periodic orbits around L_1 and L_2 which generated this chain are Lyapunov orbits. The existence of this chain has many important implications for mathematics, astronomy, and astrodynamics. But, let us take a moment to see heuristically exactly what this chain means. We have essentially produced a series of asymptotic trajectories that connect the S, J, X regions. So what? Since these theoretical orbits take infinite time to complete their cycles, of what use are they?

Recall that large body of results in dynamical systems theory relating to heteroclinic orbits mentioned earlier? Here is where we cash in our chips after hitting the jack pot. It turns out one of the sources of deterministic chaos in a dynamical system is precisely the existence of homoclinic and heteroclinic cycles. This was known to Poincaré and gave him enormous difficulties. Basically, when these cycles exist, it implies that the stable and unstable manifolds have infinite number of intersections creating what is known as the homoclinic-heteroclinic tangle. It is truly a mess. The existence of this tangle means that very random transitions between the S, J, X set of regions can occur using the chain as the template for the transition. In other words, a comet could orbit the Sun in the X region

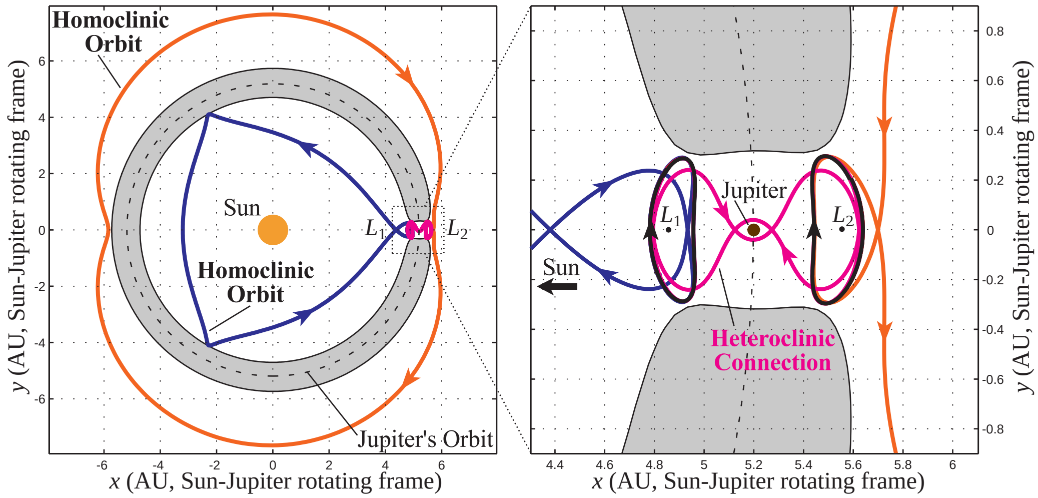


Figure 6: (a) Jupiter's homoclinic-heteroclinic chain. (b) The Lyapunov orbits and the heteroclinic cycle.

for many years, then suddenly change its orbit to the S region. Of course, to do this, it must transit through the J region, where it might get caught by Jupiter for a few orbits. It might also be caught by L_1 or L_2 doing a few revolutions near a halo orbit. Then it leaves via the L_1 region to enter the S region to orbit the Sun. This, of course, is exactly the itinerary of *Gehrels 3*, *Oterma*, and a host of other comets. This dynamics is completely explained by the tangle associated with this chain.

But actually an even more precise result is proved in Koon *et al.*[1] using symbolic dynamics (see Moser[13]), a standard technique in dynamical systems theory. The basic idea is as follows. We want to characterize the dynamics by following its motion in space. But, the detailed trajectory is too complicated to follow. Suppose we divide the space into three regions such as S, J, X in Figure 7. Let's just track the trajectory's séjour in each of the three regions. Thus a trajectory is characterized by an infinite sequence $(\ldots, X, J, S, J, X, \ldots)$ indicating its "itinerary". Certain sequences such as $(\ldots, X, S, \ldots)$ are impossible because as we know to go from X to S, the trajectory must pass through J. We call the set of all possible trajectories, *admissible* trajectories. A simplified sketch of the main theorem in Koon *et al.*[1] states that given any admissible itinerary, $(\ldots, X, J, S, J, X, \ldots)$, there exists a natural orbit whose whereabouts matches this itinerary. Here naturality implies no ΔV is required, a free energy transfer all the way! In fact, we can even specify the number of revolutions the trajectory makes around the Sun, Jupiter, L_1 or L_2! And this for an infinite sequence going back and forth between the S and X regions!

The Numerical Construction of Orbits with Prescribed Itinerary

At this point, skeptics will no doubt recall that mathematical existence proofs are worth very little for real engineering problems. This observation is quite mistaken. We use the *Genesis* trajectory design as an example. When we first studied the *Genesis* problem (Howell, Barden, and Lo[14]), what guided us in our mission design was the knowledge that there is heteroclinic behavior between the L_1 or L_2 regions. The knowledge of the conjectured

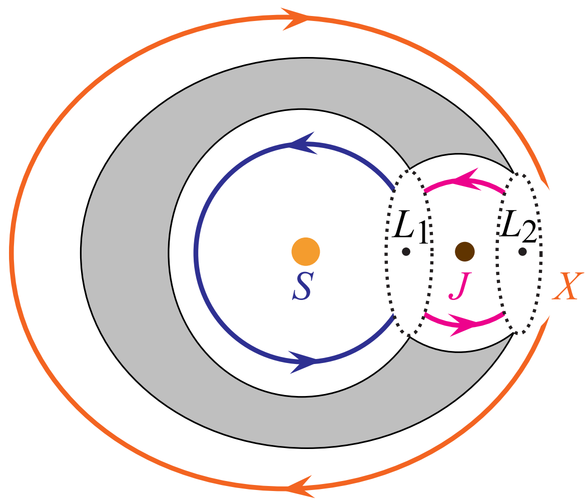


Figure 7: The symbolic dynamics of transitions between the S, J, X regions.

existence of these cycles provided the necessary insight for us to search in the design space to find the desired solution. Furthermore, our knowledge of heteroclinic orbit theory, though much less complete than it is today, provided the basic algorithms for the numerical search which produced the *Genesis* trajectory. We knew one had to compute periodic orbits at L_1 or L_2 and produce a transfer between them as a first step to find the return trajectory for *Genesis*. Once a heteroclinic shadow orbit is constructed, perhaps studded with ΔV's, this orbit provided the starting point for our differential correction process which numerically continued the orbit to eventually produce the 6 m/s ΔV mission! Hence existence proofs and theory do provide invaluable, necessary insight to solve very practical engineering problem even when the computational machinery associated with the theory has not been developed.

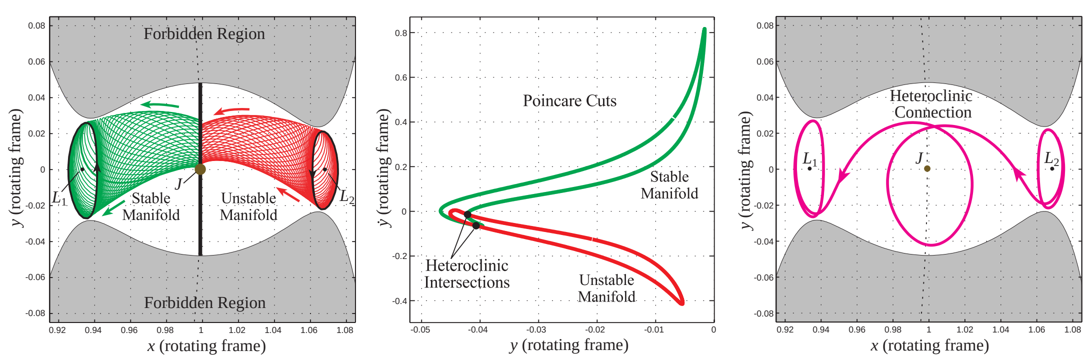


Figure 8: (a) The intersection of stable and unstable manifolds in the J region. (b) The Poincare cuts of the manifolds and their intersections. (c) The heteroclinic orbit generated from an intersection.

A second point is the fact that our theory actually provides a completely systematic method for the numerical construction of orbits with an arbitrary prescribed itinerary. Figure 8 below provides some details on how the heteroclinic orbits may be found. Suppose we wish to compute a heteroclinic orbit from L_2 to L_1. We start with two periodic orbits of the same energy around L_1 and L_2 (see Figure 2a). We compute the unstable manifold of the L_2 periodic orbi and the stable manifold of the L_1 periodic orbit. We find the intersection between the two manifolds at a convenient location such as the solid black line through Jupiter in Figure 2a. The solid black line actually represents a plane in phase space, say the $(y, \dot{y})$-plane. When we intersect the stable manifold with this plane, we expect to get a distorted circle; similarly, the unstable manifold will intersect this plane in another distorted circle. This is exactly what is shown in Figure 8b. This is called the Poincaré cut. It has reduced our manifold (surface) intersection problem by one dimension into a curve intersection problem. This is a much simpler problem. We see that there are two intersections. Taking one of these, integrating this state backwards and forwards towards the periodic orbits around L_1 and L_2 produces the heteroclinic orbit in Figure 8c.

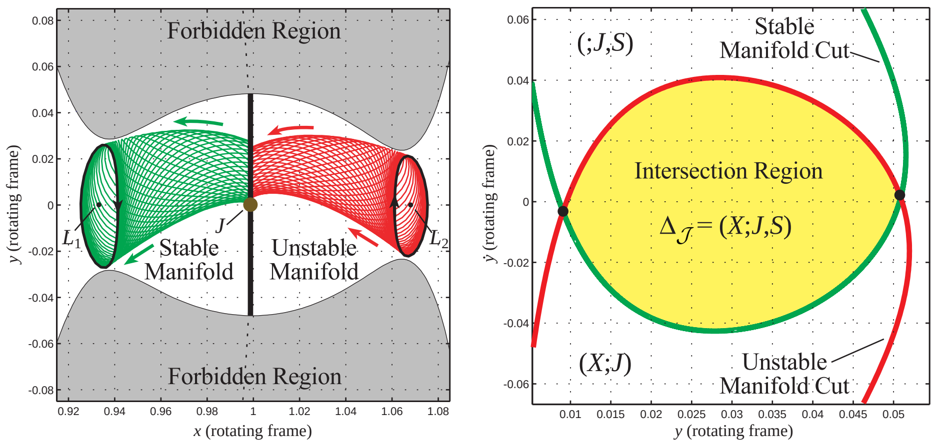


Figure 9: (a) The intersection of the stable and unstable manifolds of periodic orbits. (b) The Poincare section of the manifold intersection. The region in the middle depicts the $(X; J, S)$ sequence.

Notice, what we have constructed above is a portion of the chain controlling orbits belonging to the symbolic sequence $(X; J, S)$. The semi-colon divides the past from the present; we came from X region; we are currently at the J region and we will transfer to the S region. In Figure 9, we show the $(X; J, S)$ sequence graphically. Figure 9a depicts the manifold tubes as in Figure 8a. Figure 9b. magnifies the intersection of the manifolds in the Poincaré section. Recall that the invariant manifold tubes separate the transit orbits from the non-transit orbits. In other words, as we stated earlier, all orbits entering the J region from X at this energy level, must enter through the unstable manifold tube of the Lyapunov orbit. Hence, the lower curve in the Poincaré section shows all orbits of the sequence $(X; J)$. Similarly, the upper curve captures all orbits leaving the J region to enter the S region at this energy level, the $(J; S)$ sequence. Their intersection is exactly the $(X; J, S)$ sequence. And since Hamiltonian systems preserve area, by comparing the area of these curves and intersections in the appropriate coordinates, we can compute the transition probability from one region of phase space to another. For a little more complexity, in Figure 10 we show an orbit with the itinerary $(X, J; S, J, X)$.

These observations merely scratch the surface of the transition probability calculus which

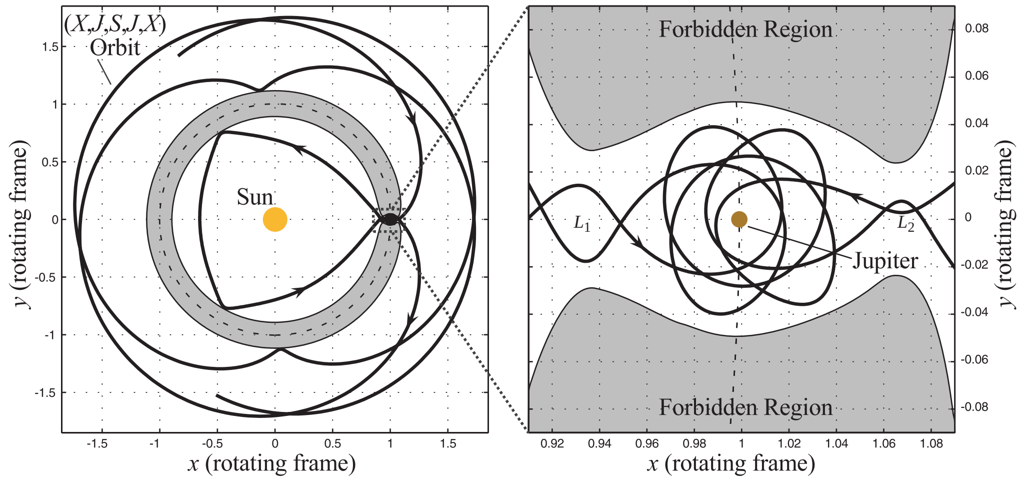


Figure 10: (a) The $(X, J; S, J, X)$ orbit. (b) The details of the $(X, J; S, J, X)$ orbit in the J region.

is possible using this technique. Thus, far from an esoteric mathematical curiosity, symbolic dynamics can be a very useful computational tool when viewed in this context. This remarkable theory is known as "lobe dynamics" in dynamical systems theory. It was developed about a decade ago and is currently an active area of research (Wiggins[7], Meiss[8]).

APPLICATIONS TO THE GENESIS MISSION

In Figure 11 we computed the chain for two Lyapunov orbits with the Jacobi energy of the *Genesis* halo orbit. Although the resulting heteroclinic orbit in the blow-up Figure 11b has an extra loop around the Earth, the general characteristics of the return orbit shadows the heteroclinic orbit in the gross details. As indicated in Bell, Lo, and Wilson[15], the influence of the Moon is crucial to the *Genesis* trajectory. Perhaps the role of the Moon is to pull up the trajectory closer into the Moon's orbit in order to produce the actual Earth return orbit. Also, since the *Genesis* orbit is fully 3-dimensional, our 2-dimensional theory may be missing important elements of the dynamics.

While the nominal trajectory for *Genesis* seems robust and malleable when subject to moderate disturbances, finding the initial orbit proved to be extremely difficult and time consuming (Howell *et al.*[16]). This suggests that given a sufficiently severe change in the orbit due to contingency problems, finding a new return trajectory will be very difficult. Our experience working on this trajectory for the past three years indicates that when this trajectory goes wrong, it is very hard to fix. Unlike conventional halo orbit missions where the specific halo orbit is of no concern, so long as the spacecraft remains in the general vicinity of the Lagrange point. The *Genesis Mission* requires the return of the solar wind samples precisely to UTTR in daylight. The combination of the UTTR target with daylight entry severely constrains the design problem. For example, the moon was originally not used in the *Genesis* orbit design (see Bell *et al.*[15], Howell *et al.*[16]). It was, in fact, purposely avoided to eliminate the difficulty of lunar phasing. In the end, it could not be avoided and now plays an essential role in the dynamics of the return trajectory. This indicates the

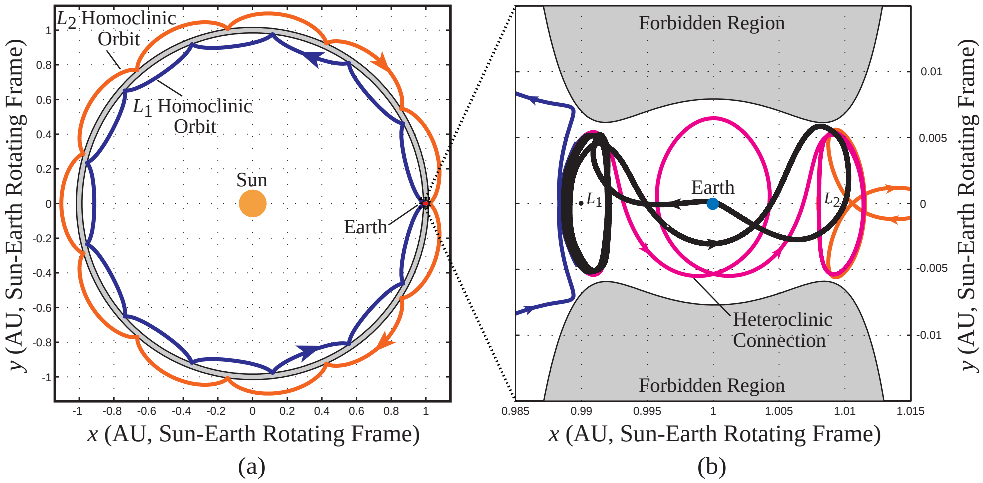


Figure 11: (a) The homoclinic/heteroclinic chain for the *Genesis* orbit. (b) Detail blow-up of the *Genesis* chain in the Earth region.

complexity of the dynamics of this trajectory.

A deeper understanding of the dynamics behind the *Genesis* return trajectory could greatly alleviate this problem. Clearly, Bell *et al.*[15] and this paper show that a thorough investigation of the theory of heteroclinic orbits with lunar perturbations in the Sun-Earth system is critical. But of even greater importance is to have the proper tools at hand which are responsive to the demands of the many potential contingency situations. The development of the JPL's proposed LTool (Libration Point Mission Design Tool) is a response to these challenges. With a deeper understanding of the fundamental dynamics, automation of the design process may be possible, since the invariant manifold structures and the various computation algorithms associated with them are well defined, at least theoretically.

It is a general rule of thumb for these highly nonlinear trajectory missions, and perhaps for all missions, that the role of contingency planning is critical to the success of the mission. Invariably, something does go wrong in a mission; and what goes wrong is rarely what you expect. In order to prepare for these challenges, the it is best to study the orbit design space thoroughly in order to understand what options are available. Having a good, flexible design tool to quickly implement various options will greatly enhance the mission and reduce the risk.

APPLICATIONS TO OTHER MISSIONS

Many potential applications of the dynamics of the homoclinic-heteroclinic chain are possible. The *Genesis Mission* is a prime example of an application of the heteroclinic connection between L_1 and L_2. The homoclinic orbits are very similar to the SIRTF-type heliocentric orbits. Clearly missions in the Earth region between L_1 and L_2, those going to the Moon, or the extended magnetosphere can all benefit from using this dynamics. We leave these mission applications to future papers. Instead, we introduce the "Petit Grand Tour" concept to conclude this paper.

The Petit Grand Tour is a tour of the Jovian satellites. But unlike previous flyby tours,

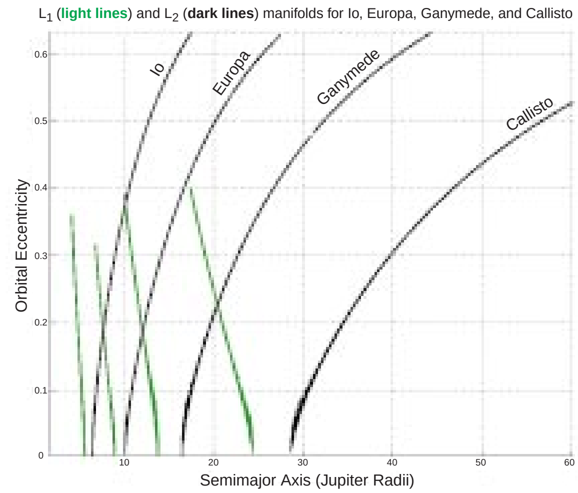


Figure 12: The network of interconnecting dynamical channels generate by the invariant manifolds of L_1 and L_2 for the Galilean satellites of Jupiter.

the concept here is to linger at each satellite in a temporary capture orbit for a prescribed number of orbits before moving on to the next satellite. The temporary capture orbits can be constructed using the heteroclinic cycles described above, but the intersatellite transfers requires a different mechanism. Figure 12 plots the L_1 and L_2 manifolds of the Galilean satellites, showing intersections between the L_1 manifold of an outer satellite with the L_2 manifold of the inner satellite. Depending on the relative phase of the orbits at the intersection point, a ΔV may be necessary to effect the transfer from the L_1 manifold of one satellite to the L_2 manifold of the next satellite. This network of dynamical channels was discovered by Lo and Ross[2]. Figure 14 depicts a similar network for the outer planets. Using this dynamical network to leap from satellite to satellite, and using the heteroclinic cycles to effect low-energy temporary captures at a satellite, the Petit Grand Tour concept is thereby accomplished.

In Figure 13 we illustrate a segment of the Petit Grand Tour of the Jovian moons. We prescribe one orbit around Ganymede, leave Ganymede via the L_1 unstable manifolds, transfer to the Europa L_2 stable manifolds using a ΔV, then almost get into a Lyapunov orbit around Europa's L_2, and finally capture into Europa orbit for four orbits. The trajectory is integrated using the planar restricted bicircular problem (PRBCP). In this model, both Europa and Ganymede orbit Jupiter in circular orbits with no gravitational effects on one another. The ΔV savings is a little more than half that required for a Hohmann transfer between Ganymede and Europa, although the trajectory was not optimized in any way. This preliminary design is illustrative of the types of mission which are possible using these techniques.

FUTURE WORK

The work presented in this paper represents an initial foray into the chaotic dynamics of the homoclinic-heteroclinic chain in the 3 body problem. There are many directions in

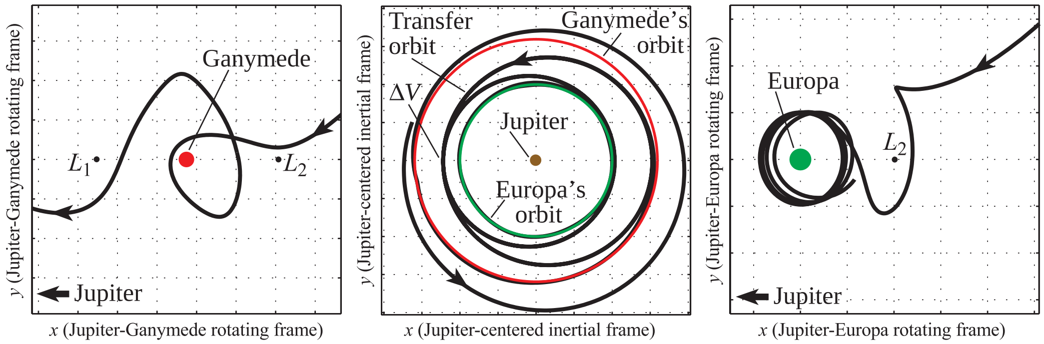


Figure 13: (a) One orbit around Ganymede in rotating coordinates. (b) The transfer from Ganymede to Europa via the invariant manifold intersections. (c) Temporary capture by Europa into 4 orbits.

which this work may be continued. We mention a few of the most important problems and applications in astrodynamics.

Extension of the 2-dimensional heteroclinic cycle to 3-dimension is the most important problem for astrodynamic applications, since halo orbits and not Lyapunov orbits are the ones of most interest to missions. The second problem is the systematic study of Earth-return/collision orbits. The *Genesis* orbit, for example, is an Earth collision orbit not all that different from the spectacular *Shoemaker-Levy 9* Jupiter collision orbit. The third problem is the interaction of three-body trajectories with the Moon. Here we speak of the interaction of the Sun-Earth Lagrange point dynamics with that of the Earth-Moon Lagrange point dynamics. This interaction helped to provided the low energy capture which rescued the Hitten mission (Belbruno and Miller [17]). Similarly, the recent Hughes geosychronous satellite rescue mission also used this dynamics. The fourth problem, a more technical one, is to combine dynamical systems theory with optimal control methods. It is hoped that the many difficulties which face optimal control problems may be alleviated if the dynamics of the problems are taken more into consideration. For example, we are currently working on targeting the stable manifold to compute an optimal transfer into a halo orbit. Lastly, perhaps the most important problem in this field currently, is the development of good software tools to perform the analysis. Work is continuing at JPL and elsewhere to develop an integrated suite of software to more capably deal with the complex dynamics of this most interesting region of space.

Acknowledgements: We thank Gerard Gómez and Josep Masdemont for many helpful discussions and for sharing their wonderful software tools with us. We thank Paul Chodas for discussions on the SL9 and NEO orbits. We also thank the following colleagues for helpful discussions and comments: Brian Barden, Julia Bell, Peter Goldreich, Kathleen Howell, Àngel Jorba, Andrew Lange, Jaume Llibre, Martínez, Richard McGehee, William McLaughlin, Linda Petzold, Nicole Rappaport, Ralph Roncoli, Carles Simó, Scott Tremaine, Stephen Wiggins, and Roby Wilson.

This work was carried out at the Jet Propulsion Laboratory and California Institute of Technology under a contract with National Aeronautics and Space Administration. In addtion, the work was partially supported by the Caltech President's fund, the NASA

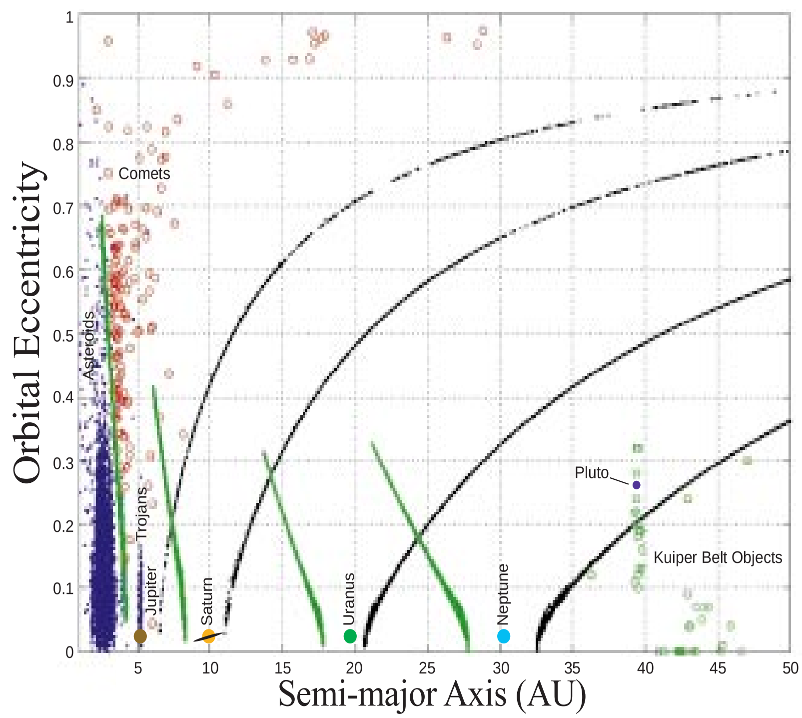


Figure 14: The network of interconnecting dynamical channels generate by the invariant manifolds of L_1 and L_2 for the outer planets of the Solar System.

Advanced Concepts Research Program, the Genesis Mission, and NSF grant KDI/ATM-9873133.

References:

1. Koon, W.S., M. Lo, J. Marsden and S. Ross [1999] Heteroclinic Connections between Liapunov Orbits and Resonance Transitions in Celestial Mechanics, *Chaos*, to appear.

2. Lo, M., and S. Ross [1997], SURFing the Solar System: Invariant Manifolds and the Dynamics of the Solar System, JPL IOM 312/97.

3. Lo, M., and S. Ross [1998], Low Energy Interplanetary Transfers Using Invariant Mnaifolds of L1 and L2 and Halo Orbits, AAS/AIAA Space Flight Mechanics Meeting, Monterey, CA, Feb 9-11, 1998.

4. Lo, M., *et al.* [1998] Genesis Mission Design, AAS/AIAA Astrodynamics Conference, Boston, MA, August 1998, Paper AIAA 98-4468.

5. Sharer, P., H. Franz and D. Folta [1995] WIND Trajectory Design and Control, Spaceflight Dynamics (Colloque 95), CEPAD, Toulouse, 1995, pp.859-870.

6. Howell, K., D. Mains and B. Barden [1994] Transfer Trajectories from Earth Parking Orbits to Sun-Earth Halo Orbits, AAS/AIAA Spaceflight Mechanics Meeting, Cocoa Beach, FL, Feb. 14-16, 1994.

7. Wiggins, S. [1992] *Chaotic Transport in Dynamical Systems*, Springer Verlag, Berlin.

8. Meiss, J. [1992] Symplectic Maps, Variational Principles, and Transport, *Review of Modern Physics*, Vol. 64, No. 3, July 1992.

9. Conley, C. [1968] Low Energy Transit Orbits in the Restricted Three-Body Problem, *SIAM J. Appl. Math.* 16, 732-746.

10. McGehee, R. [1969] Some Homoclinic Orbits for the Restricted Three Body Problem, Ph.D. Thesis, University of Wisconsin.

11. Llibre, J., R. Martinez and C. Simó [1985] Transversality of the Invariant Manifolds Associated to the Lyapunov Family of Periodic Orbits Near L2 in the Restricted Three-Body Problem, *Journal of Differential Equations* 58, 1985, 104-156.

12. Chodas, P. and D. Yeomans [1996] The Orbital Motion and Impact Circumstances of Comet Shoemaker-Levy 9, in *The Collision of Comet Shoemaker-Levy 9 and Jupiter*, edited by K.S. Noll, P.D. Feldman, and H.A. Weaver, Cambridge Unviersity Press, 1996.

13. Moser, J. [1973] *Stable and Random Motions in Dynamical Systems with Special Emphasis on Celestial Mechanics*, Princeton University Press.

14. Howell, K., B. Barden and M. Lo [1997] Application of Dynamical Systems Theory to Trajectory Design for a Libration Point Mission, *Journal of the Astronautical Sciences*, Vol. 45, No. 2, April-June 1997, pp. 161-178.

15. Bell, J., M. Lo and R. Wilson [1999] AAS/AIAA Astrodynamics Specialist Conference, Girdwood Alaska, August 16-20, 1999.

16. Howell, K., B. Barden and M. Lo [1997] Trajectory Design Using a Dynamical Systems Approach to GENESIS, AAS 97-709, AAS/AIAA Astrodynamics Specialist Conference, Sun Valley Idaho, August 4-7, 1997.

17. Belbruno, E. and J. Miller [1993] Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture, *Journal of Guidance, Control and Dynamics* 16, 770-775.