NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CONTRACT NO. NAS 7-918

TECHNICAL SUPPORT PACKAGE

On

LOW-ENERGY TRANSFER FROM NEAR-EARTH TO NEAR-MOON ORBIT

for 03/01/2002

NASA TECH BRIEF Vol. 26, No. 3

from

JPL NEW TECHNOLOGY REPORT NPO- 20936

Inventor(s):
Jerold Marsden
Martin W Lo
Shane Ross
Wang S Koon

TSP assembled by:
JPL Intellectual Assets office

pp. i, 1-2, 1a-3a

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

03/01/2002

Low-Energy Transfer From Near-Earth to Near-Moon Orbit

A report presents a theoretical approach to designing a low-energy transfer of a spacecraft from an orbit around the Earth to ballistic capture into an orbit around the Moon. The approach is based partly on the one presented in "Low-Energy Interplanetary Transfers Using Lagrangian Points" (NPO-20377), *NASA Tech Briefs*, Vol. 23, No. 11 (November 1999), page 22. The approach involves consideration of the stable and unstable manifolds of the periodic orbits around the Lagrangian points L1 and L2 of the Sun/Earth and Earth/Moon systems. (The Lagrangian points are five points, located in the orbital plane of two massive bodies, where a much less massive body can remain in equilibrium relative to the massive bodies.) To generate a transfer trajectory, one uses the intersection of (1) the unstable manifold of a periodic orbit about the Sun-Earth L1 or L2 with (2) the stable manifold of a periodic orbit about the Earth-Moon L2. This intersection is generated by a Poincaré section. The different regions within the Poincaré section all have different dynamical properties. By picking points in the correct region, one can generate a transfer from orbit around the Earth to capture into a highly elliptical orbit around the Moon.

This work was done by Martin Lo, Jerrold Marsden, Wang S. Koon, and Shane Ross of Caltech for **NASA's Jet Propulsion Laboratory**.

NPO-20936

NTR INVENTOR'S REPORT
NTR: 20936

PLEASE BE AS CLEAR AND SPECIFIC AS POSSIBLE, AS THIS REPORT MAY BE MADE AVAILABLE THROUGH TECH BRIEFS

Section 1 (Novelty), 2A (Problem), and 2B (Solution) must be completely fully. Your published paper may be attached to satisfy Section 2C (Description and Explanation).

1. Novelty- Describe what is new and different about your work and its improvements over the prior art. Attach supporting material if necessary.

   There are two previous work to consider. The first is the WSB (Weak Stability Boundary) work of Belbruno and Miller; the second is the work of Lo and Ross "Low Energy Interplanetary Transfers Using Langrangian Points", NASA Tech Brief NPO 20377. Inspired by the WSB work, we set out to find an alternate method for computing low energy transfers such as used by Hiten mission. Our methods are completely different. We do not use Mather sets in any way. Combining the work of Lo and Ross [1998] and Koon et al [1999], we came up with a low energy lunar transfer and capture which uses the dynamical channels provided by the invariant manifolds of the periodic orbits around L1 and L2 of the 3 body systems. This provides a systematic approach using well known mathematical concepts without the use of new concepts like the WSB.

2. Technical Disclosure

   A. Problem-Motivation that led to development or problem that was solved.

   Find a systematic approach to the construction and design of a transfer trajectory from the Earth to the Moon with ballistic capture at the Moon.

   B. Solution

   To generate the transfer trajectory, use the intersection of the unstable manifold of a periodic orbit about the Sun-Earth LaGrange Point (L1 or L2) with the stable manifold of a periodic orbit about the Earth-Moon L2. This intersection is generated by a Poincare section. The different regions within the Poincare section all have different dynamical properties. By picking points in the correct region, a transfer from the Earth to the Moon can be generated which automatically is captured by the Moon into a highly elliptical orbit.

   C. Detailed Description and Explanation

   See Exhibit A.

*REFERENCES

Belbruno, E. [Aug. 1990], "Examples of Nonlinear Dynamics of Ballistic Capture and Excape in the Earth-Moon System," AIAA Paper #90-2896

Belbruno, E., J. Miller, [1993], "Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture," Journal of Guidance, Control, and Dynamics, Vol. 16, No. 4, July-August 1993, Pages 770-775

Belbruno, E. [Nov 1994], "The Dynamical Mechanism of Ballistic Lunar Capture Transfers in the Four-Body Problem from the Perspective of Invariant Manifolds and Hill's Regions," Research Report no 270, Centre de Recerca Matematica, Institut D'Estudis Catalans

Lo, M., S. Ross, [1998], "Low Energy Interplanetary Transfers Using Invariant Manifolds of L_1 and L_2 and Halo Orbits," AAS/AIAA Space Flight Mechanics Meeting, Monterey, CA Feb 9-11, 1998

Koon, W.S., M. Lo, S. Ross, J. Marsden, [May 1999], "Heteroclinic Connections between Liapunov Orbits and Resonance Transition in Celetial Mechanics," draft

Koon, W.S., M. Lo, S. Ross, J. Marsden, [Aug. 1999], "The Genesis Trajectory and Heteroclinic Connections," AAS/AIAA Astrodynamics Specialist Conference, Girdwood, Alaska, AAS99-451

Lo, M., S. Ross [Nov. 1999] "Low Energy Interplanetary Transfers Using Lagrangian Points," NASA Tech Brief, Vol. 23, No. 11, November 1999, NPO-20377

***Please obtain references from sources listed.**

Exhibit A

Our goal is to construct a trajectory which gets ballistically captured by the Moon and uses less fuel than the standard Hohmann transfer. Our model will incorporate the Earth, Moon, Sun, and spacecraft (SC). We will attempt to take full advantage of the dynamics of this 4-body system by modeling it as two coupled planar circular restricted 3-body systems. In this approach, we will utilize the libration point dynamics of both the Earth-Moon-x and Sun-Earth-sc systems.

As discussed in Koon et al. [1999], the phase space in the vicinity of the stable and unstable manifolds of libration point orbits is complicated. The stable and unstable manifolds are 2-dimensional "tubes" in a 3-dimensional energy surface. As separatrices, they form the boundary between transit and non-transit regimes of motion, where the transit is between two of the three energetically accessible regions: interior, capture, and exterior. For example by targeting the region enclosed by the stable manifold tube (exterior branch) of the L_2 point in the Earth-Moon-sc system, we can construct an orbit which will get ballistically captured by the Moon.

Near the manifolds (the edge of the tubes), orbits exhibit a "twist" after visiting the equilibrium region. The degree of twisting during an equilibrium region encounter increases without limit as one approaches the manifold. See Figure 0.1. In position space projections

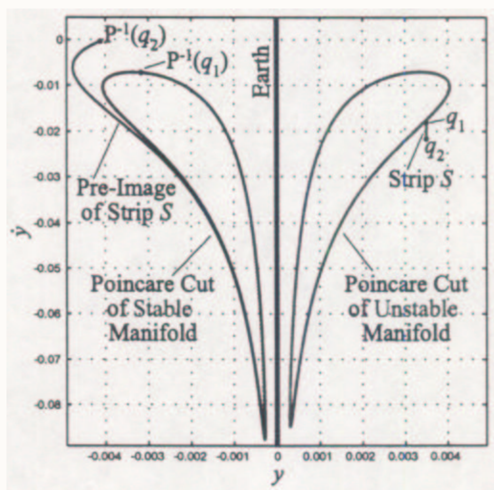

Figure 0.1: The "twist" near libration point orbit manifolds. The example shown is in the Sun-Earth system with a Poincaré surface-of-section at $x = 1 \mu$ in the Sun-Earth rotating frame. The first Poincaré cuts of the L_2 orbit manifolds are shown (unstable in red, stable in green). The small strip S new the unstable manifold and just outside of it, with endpoints q_1 and q_2, has a pre-image $P^{-1}(S)$ under the Poincaré map P. The position of the Earth is indicated by the blue vertical strip in the middle.

of such orbits, the degree of twisting appears to correspond to the amount of time spent wrapping around the libration point orbit before leaving the equilibrium region. The amount of twisting a particular orbit will undergo depends very sensitively on its distance from the manifold and therefore can change dramatically with a very minute thrust (AV). This is best visualized in terms of Poincaré sections, to be shown later. We will exploit this property of the manifolds' vicinity in the Earth-Sun-sc system to generate our final trajectory.

Key to our method is the visualization of orbits within a rotating frame, where patterns are made plain which are not otherwise discernable in an inertial frame. The circular restricted 3-body problem equations are formulated in a rotating frame, which co-rotates with the two primary masses in their periodic orbit about their common center of mass. Therefore, the structure and geometry of the solution space is most easily seen in a rotating frame. We have two rotating frames in our coupled 3-body problem, the Sun-Earth *(SE)* rotating frame and the Earth-Moon *(EM)* rotating frame, which we will use when appropriate.

The equations of the planar circular restricted 3-body problem permit a constant of motion in the rotating frame known as the Jacobi constant (-2xHamiltonian energy). For certain ranges of values of the Jacobi constant, the position space is partitioned into three regions (interior, capture, and exterior) which are connected only by two narrow "necks", one each around L_1 and L_2. These necks are also known as equilibrium regions and contain a periodic orbit (p.o.) around each libration point. This energy regime, known as Case 3, is the one we will be in for both the launch from Earth in the *SE-sc* system and the capture at the Moon in the *EM -sc* system.

We begin our construction by choosing an angle within the SE rotating frame (θ_{SE}) at which to take a Poincaré section of the SE L_2 p.o.'s unstable manifold (capture region branch). See Figure 0.2(a). We will restrict our study to the first intersection (first Poincaré

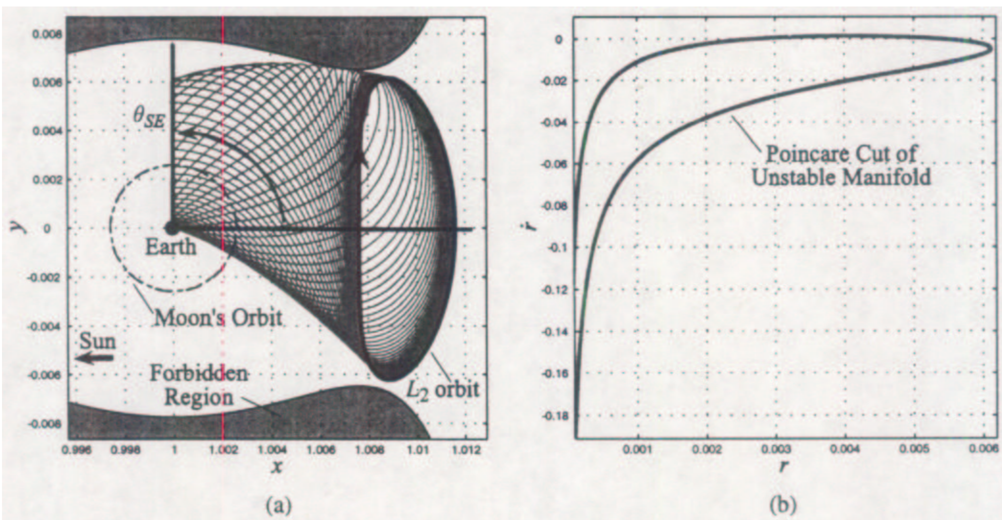

Figure 0.2: (a) Position space projection of unstable manifold (Earth branch, red) of Sun-Earth L_2 periodic orbit. Shown in Sun-Earth rotating frame. Manifold integrated backward in time until it reached the angle θ_{SE} in the Sun-Earth rotating frame. Forbidden region shown in gray for this energy (Jacobi constant). (b) Poincaré section of unstable manifold at θ_{SE} =constant. Shown in polar coordinates coordinates $(r, \dot{r})$ centered on the Earth, which for $\theta_{SE} = 90$ coincide with $(y, \dot{y})$.

cut) of the manifold with the surface $\theta_{SE} = 90°$, which is equivalent to the surface specified by $x = 1 - \mu$. The plot of the Poincaré cut in the SE rotating frame variables $(y, \dot{y})$ is shown in Figure 0.2(b).

We also choose an angle within the EM rotating frame (θ_{EM}) at which to take a Poincaré section of the EM L_2 p.o.'s stable manifold (exterior region branch). See Figure 0.3(a). We will restrict our study to the first Poincaré cut of the manifold with the surface $\theta_{EM} = 110°$. We choose this surface to coincide with the $\theta_{SE} = 90°$ surface in the SE rotating frame. We can then plot this cut in the SE rotating frame variables $(y, \dot{y})$. See Figure 0.3(b). This fixes the Moon's position in the SE rotating frame at $\theta_{Moon} = \theta_{SE} - \theta_{EM}$. Assuming the Sun is a small enough perturbation to the *EM-sc* 3-body dynamics, any sc with initial conditions within this closed loop (with the appropriate EM Jacobi constant) will be ballistically

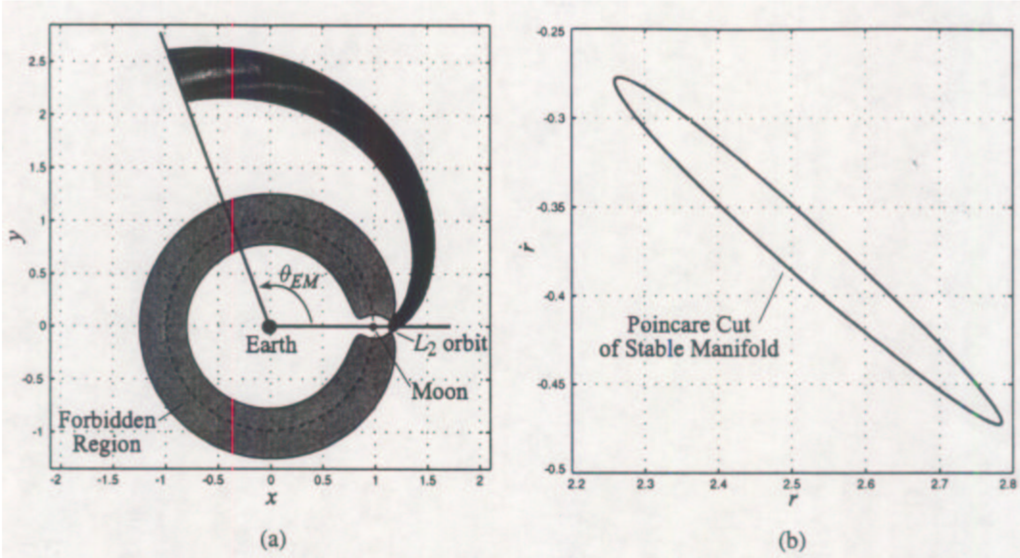

Figure 0.3: (a) Position space projection of stable manifold (exterior branch, green) of Earth-Moon L_2 periodic orbit. Shown in Earth-Moon rotating frame. Manifold integrated backward in time until it reached the angle θ_{EM} in the Earth-Moon rotating frame. Forbidden region shown in gray for this energy (Jacobi constant). (b) Poincaré section of stable manifold at θ_{EM} = constant. Shown in polar coordinates coordinates $(r, \dot{r})$ centered on the Earth.

captured by the Moon.

If we choose such an initial condition, this will fix the position (x, y) at this point for our trajectory, which we will call "time zero" (t = 0). For fixed y on this Poincaré section, there is a $\dot{y}$ value close to, but just outside, the loop of the SE L_2 p.o.'s unstable manifold which will backward integrate to a 200 km altitude perigee. This is ensured because of the twisting in the vicinity of the manifolds and the particular choice of Jacobi constant for this L_2 p.o. for which the first Poincaré cut of both the stable and unstable manifolds comes within 200 km of the Earth's surface.

"The work described here was carried out at the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration."