

Next Exit 0.5 Million Kilometers

by Douglas L. Smith

Left: The planets are connected by a vast network of low-energy “tunnels” in space that run between the solar system’s Lagrange points.

Above: The Lagrange points are the regions where the gravitational forces between a pair of massive bodies balance.

Right: Wheels within wheels—this diagram shows the five Lagrange points of the Earth-moon system (marked as LLs, for Lunar Lagrange) and the two closest sun-Earth Lagrange points, marked as ELs. Images courtesy of Khrysaundt “Cici” Koenig, Caltech department of computer sciences, graphics group.

The year is 2020. Under a crescent Earth, the assembly crew at the Lunar Gateway Service Area some 62,000 kilometers above the moon’s surface installs new electronics on an infrared telescope and sets it moving, at the speed of a Ford Pinto climbing an on-ramp, off to park itself deep in Earth’s shadow, where a little cryogenic coolant goes a long way and where Earth is always directly overhead for high-speed data downlinks. This spacecraft has, in fact, just entered what Martin Lo (BS ’75), a member of the technical staff at JPL, calls the Interplanetary Superhighway—“a vast network of winding tunnels in space” that connects the sun, the planets, their moons, and a host of other destinations as well. But unlike the wormholes beloved of science-fiction writers, these things are real. In fact, they are already being used.

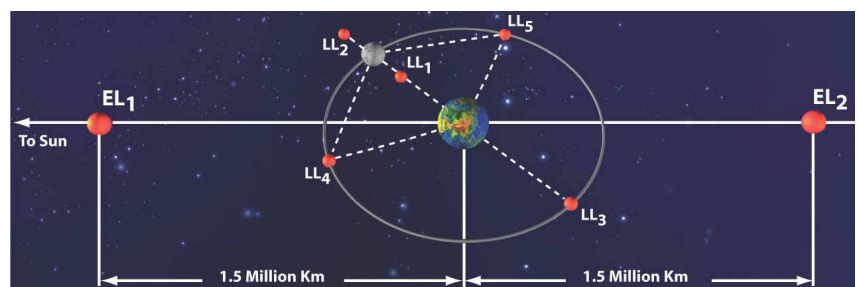
The Genesis mission, for example, is following a route that a team of scientists led by Lo plotted through the sun-Earth interchange of this freeway system. (In fact, this low-energy route helped JPL win the mission.) Genesis, of which Caltech professor of nuclear geochemistry Don Burnett is the principal investigator, is collecting samples of the solar wind—the torrent of charged particles that emanates from the sun, and whose makeup reflects that of the disk of gas and dust from which the sun and the planets condensed. In 2004, Genesis will bring its booty home the same way. (Well, technically not the *same* way, as it will return through the other side of the cloverleaf, as it were.)

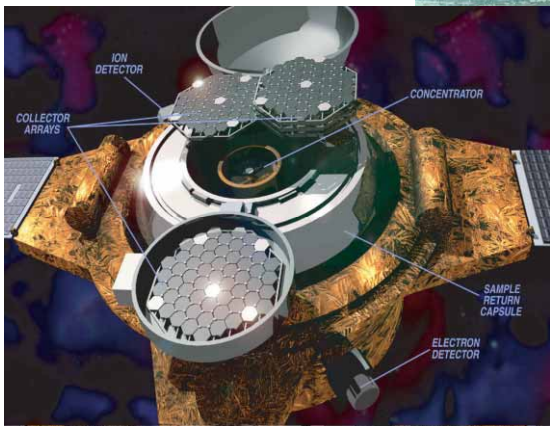
Lo, together with Caltech’s Jerrod Marsden, professor of control and dynamical systems (CDS), and their coworkers Shane Ross (BS ’98, a CDS graduate student) and Wang Sang Koon (a CDS senior postdoc)

have begun a systematic mapping effort of what is more properly known as the Interplanetary Transport Network. As a freeway system, the network is more akin to the Pacific Coast Highway and other scenic routes than to interstates like the I-5—a collection of meandering byways for leisurely travel, not the fastest, most direct routes between points. But the quickest paths in outer space are all toll roads (it costs a lot of rocket fuel to use them), while you can ride the Interplanetary Superhighway almost for free. Gravity does the driving, so the system is really more like an elaborate set of Hot Wheels tracks. All you have to do is let go of the car at the right place. (It’s a lot more complicated than this, because the tracks are in constant motion, but we’ll get to that later.)

Think of a planet as a bowling ball sitting on a taut rubber sheet; the depression the ball makes is its gravitational well. To hoick a marble (the spacecraft) up out of that well takes thrust—often quite a lot of it. But what if the marble was balanced on a cusp, such as where Earth’s well and the moon’s well meet? The gravitational (and sometimes rotational) forces would balance one another, and the slightest sneeze, a mere feather touch, would nudge the spacecraft in the right direction.

A set of five of these balance points, called Lagrange or libration points, exist between every pair of massive bodies—the sun and its planets, the planets and their moons, and so on. Joseph-





Launched on August 8, 2001, Genesis is sweeping up specks of the sun—individual atoms of the solar wind—on five collector arrays the size of bicycle tires and in an ion concentrator. The sample-return capsule will parachute onto the Air Force's Utah Testing and Training Range in September 2004. Because the samples are incredibly fragile, the capsule will be snagged in midair. The sample, consisting of several milligrams of hydrogen and helium and some 20 millionths of a gram of heavier elements, will be the only extra-terrestrial material brought back to Earth from deep space since the Apollo landings, and the first to be collected from beyond the moon's orbit.

Louis Lagrange (1736–1813) discovered the existence of the two points now known as L_4 and L_5 , each of which is located in the orbital plane at the third vertex of an equilateral triangle with, say, Earth at one vertex and the moon at the other. So L_4 is 60° in advance of the moon, and L_5 60° behind it. Ideally, a spacecraft at L_4 or L_5 will remain there indefinitely because when it falls off the cusp, the Coriolis effect—which makes it hard for you to walk on a moving merry-go-round—will swirl it into a long-lived orbit around that point. Comet debris and other space junk tends to collect there, and Jupiter has accumulated an impressive set of asteroids that way.

Leonhard Euler (1707–1783) rounded out the assortment with L_1 , L_2 , and L_3 , where the rotational forces don't balance out and nothing stays put for long. (Euler actually discovered L_1 , L_2 , and L_3 first, but Lagrange had a better press agent.) For the sun-Earth pair, L_1 lies on a line between them, about 1.5 million kilometers from Earth. Genesis is parked in a halo orbit around L_1 , so called because, as seen from Earth, the flight path follows a halo around the sun. (Sitting right on L_1 isn't a good idea, as the spacecraft's radio signals would be lost in the sun's glare.) Since orbits around L_1 are unstable, Genesis needs a small boost every 60 days or so to keep it on station. Because of its unobstructed view of the sun, L_1 is a popular place these days—the Solar and Heliospheric Observatory (SOHO), a joint project of the European Space Agency and NASA, and NASA's WIND and the Advanced Composition Explorer (ACE) are also there. (Ed Stone, the Morrisroe Professor of Physics, is ACE's principal investigator, and two of its nine instruments were built on campus.) L_2 is a similar distance from Earth as L_1 , but in the opposite direction, and L_2 orbits are also unstable on a 60-day scale. NASA's MAP, the Microwave Anisotropy Probe, has been in orbit around L_2 since October 2001, mapping the variations in the leftover heat from the Big

Bang. And finally, L_3 lies hidden from our view directly behind the sun. We haven't found any reason to keep a spacecraft there, but it's proven a dandy place for science-fiction writers to conceal inhabited, Earthlike planets—despite the fact that all these anti-Earths and Planet Xs would, if left unattended, fall out of orbit in 150 days or so.

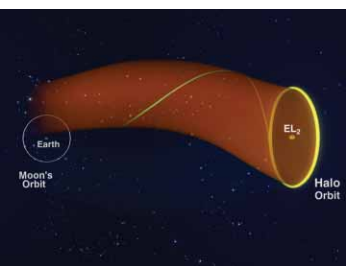
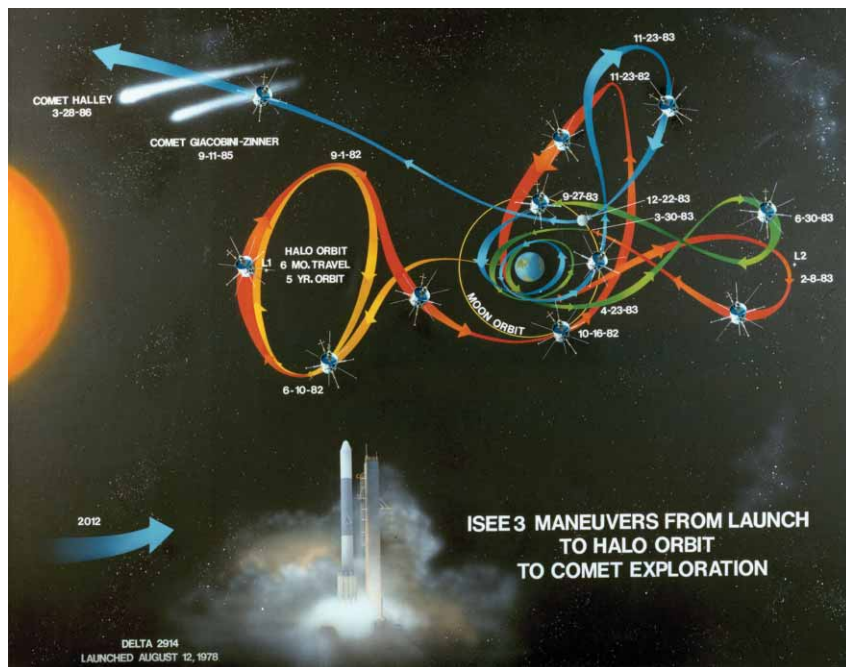
"Historically, L_1 and L_2 were not interesting," says Koon. "People were interested in L_4 and L_5 , because they are stable. But instability can be a good thing, because a little force achieves a big result." Adds Ross, "You, walking around, are dynamically unstable. You keep falling forward." Without your inner-ear balance system constantly sending signals for your body to right itself, you'd be flat on your face in an instant. (Watch a toddler learning to walk some time.) But this imbalance is good, as it allows a little forward impetus to move your mass.

The tools to deal with this celestial instability were developed by Jules-Henri Poincaré (1854–1912) in the 1890s. Poincaré was working on the infamous three-body problem, which has bedeviled mathematicians since the days of Isaac Newton. The problem is simplicity itself: calculate the orbits of three masses whose only interaction with one another is through their gravitational pulls. Building on Kepler's foundational work, Newton solved the two-body version—Earth going around the sun, for example—but throwing a third mass into the mix gives a complex interplay of constantly shifting forces. Says Marsden, "You can fit the equations for the three-body problem in a corner of the blackboard somewhere, but the subtleties in it are very interesting. Many computational scientists like working on it because it's one of the simplest problems that's complicated enough to test out computational theories." And trying to do all nine planets at once? Fuggedaboutit.

Poincaré simplified the mess by organizing similar orbits into "manifolds." A manifold is any nice, smooth surface: a sheet of paper is a manifold, as is the surface of a sphere; or the crust of a donut, also called a torus. Poincaré saw that families of orbits lay on "invariant" manifolds—"No matter where it goes, a particle that starts on that surface will remain on that surface forever unless you give it a knock," Lo explains. "So that surface is invariant." These manifolds sit inside what is called six-dimensional phase space, because it includes the three dimensions of normal space plus a dimension for the particle's velocity in each direction. Thus particles that have the same location but different velocities will appear at different points in the 6-D phase space. (Marsden and Lo are working with Alan Barr, professor of computer science, on ways to visualize such higher-dimensional objects, but, fortunately, there's a lot that's easy to see in two dimensions.)

Poincaré noticed that if an unstable orbit is periodic—that is, if it returns to its starting point,

A well-traveled spacecraft. The International Sun-Earth Explorer 3 (ISEE3) was put in a halo orbit around L_1 to study solar flares and cosmic gamma-ray bursts. It was later renamed the International Comet Explorer (ICE) and dispatched to comet Giacobini-Zimmer by way of L_2 . (The wildly looping orbits, like the bow on a Christmas package, are typical of a route through the tubes.) ISEE3/ICE flew through Giacobini-Zimmer's tail on September 11, 1985, and went on to become the only U.S. representative in the international fleet of spacecraft that greeted Halley's comet in 1986. It is now plying its original trade in a 355-day sun-centered orbit, and will return to Earth's vicinity in August 2014. Image courtesy of the Goddard Space Flight Center and the National Space Science Data Center.



A spacecraft falling out of orbit around Earth's L_2 point would describe a path on this tube. Image courtesy of Cici Koenig.

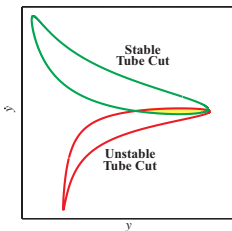
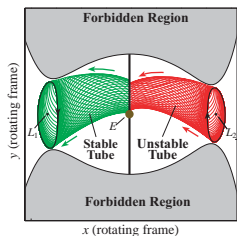
like a circle or an ellipse does—it generates a tube-shaped manifold containing all the paths one could take to fall out of that orbit with no change in energy. So if you were to plot the path of a spacecraft drifting out of orbit around Earth's L_2 point, for example, you'd see it slowly unwind into a spiral wrapped along the surface of the tube. This tube is called the “unstable” manifold of that orbit. Furthermore, another manifold contains all the paths that wind *onto* the original orbit—the movie can run backward as well as forward. Just to muddy the waters, the manifold leading to the unstable orbit is called the “stable” manifold.

And there the matter stood for nearly 100 years, waiting for computers to plot the manifolds and for spacecraft to ride them. In the late 1960s, Charles Conley and Richard McGehee (BS '64) noticed that the sun-Earth system has, for any energy level in a fairly broad range, only one periodic orbit about L_1 (and another about L_2) that lies entirely in the plane of Earth's orbit. Called a Lyapunov orbit after its discoverer, it is unstable. Conley and McGehee were able to classify all the orbits winding onto and off of the Lyapunov orbit, as well as all the orbits that entered its vicinity, and found that these orbits completely controlled the paths of bodies near Earth's L_1 and L_2 points. In other words, a slow-moving asteroid near L_1 or L_2 can only approach or leave Earth via a Lyapunov tube.

But the pioneers of spaceflight weren't particularly interested in the Lagrange points—as Gertrude Stein once said in another context, there is no there there. And there's a much more straightforward way to get spacecraft into low-Earth orbit, or to the moon, or to the other planets. Newton and Kepler left behind the tools for constructing flight paths from simple conic

sections—bits of parabolas, hyperbolas, ellipses, and the ubiquitous circle—and their use is now a highly developed art. However, a visionary named Robert Farquhar persuaded NASA to fly the first mission to a Lagrange point—the International Sun-Earth Explorer 3, launched to orbit Earth's L_1 in 1978. (Farquhar also coined the term “halo orbit.”) Farquhar's team found a path to the halo orbit with the aid of numerical searches. Says Lo, “Because the dynamics of the tubes are so strong, when you search around the halo orbits for a transfer trajectory from Earth, your path will invariably be controlled by the halo orbit's stable manifold.” (In fact, it takes a prohibitive amount of thrust to avoid the manifold.) Then a group led by Carles Simó at the University of Barcelona explicitly proposed riding the stable manifold as a cheap and easy way to get SOHO out to Earth's L_1 , and developed software tools to compute the flight path by using segments of trajectories on the manifold to “seed” the calculation. SOHO wound up taking another path, but people started dusting off their Poincaré. “Farquhar's pioneering methods required lots of human interaction,” Lo recalls. “On the Genesis team, we used his group's tools to compute the initial trajectory” back in the mid-'90s when the mission was first proposed.

“So we've known about halo orbits since the '60s,” says Lo, “and in the '80s the Spaniards reintroduced Poincaré's tube theory. And the question I began to ask was: If you continue these orbits out, where do they go? Is it possible to go from one planet to the next?” In other words, would the unstable outbound tube from, say, Earth's L_2 point intersect the stable inbound one to Mars's L_1 point? If so, you wouldn't need a big engine and a massive fuel tank to get there and a great deal of money might be saved.



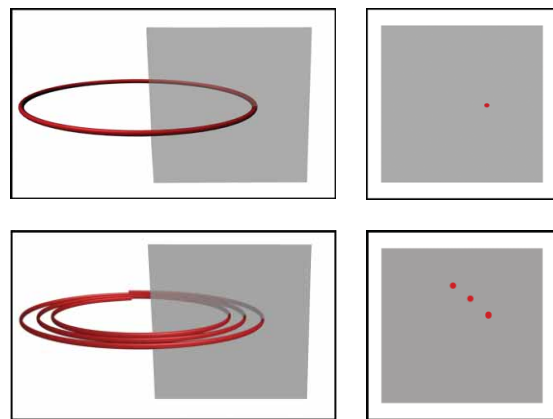
Top: A close-up of the plane of Earth's orbit in our immediate neighborhood, looking down from above. The sun is way over to the left. The plot rotates around the sun at the same speed that we do, keeping Earth frozen in the center. A spacecraft at a given energy can make a halo orbit (black arrows) around L_1 or L_2 . A portion of some of the paths winding onto the L_1 orbit is shown in green, and the corresponding portion of the paths leaving L_2 is in red. The gray "forbidden region" is inaccessible to a spacecraft at the given energy—you can't get there from here without firing the rocket.

Bottom: A Poincaré cut passing through Earth in the y -direction. The top plot's vertical axis is now this plot's horizontal one, and this plot's vertical axis is the velocity in the y -direction. You can cross from one manifold to the other at the points where they intersect.

1995 was the summer of Shane Ross's freshman year. A physics major, he had signed up for a SURF (Summer Undergraduate Research Fellowship) with Andrew Lange, a physics professor who was working on a competitor of MAP, the Big-Bang heat-mapper currently at L_2 . Recalls Ross, "When I saw the trajectory that the spacecraft was going to do, I was more interested in that than the physics. So Andrew directed me towards Martin." Says Lo, "I was trying to find out whether the invariant manifolds of different planets intersected, but I was also interested in the behavior of the regions around Earth's L_1 and L_2 because of the Genesis trajectory-design project. As a freshman, Shane didn't have the math to develop the tools to compute the manifolds of halo orbits, so I thought maybe the manifolds of the Lagrange points themselves might tell us something. If you think of the tubes of a set of concentric periodic orbits as the layers of a leek, the manifold of the Lagrange point is a line down the leek's middle. So if the manifolds of two Lagrange points intersect—or even just come very close—then the tubes probably intersect as well. And if the manifolds intersect in space, even if they're at different energies, they're useful. You can bridge the energy difference by firing a rocket, so long as the paths connect." The exercise bore fruit almost immediately. Says Ross, "In July, I noted [in my lab book] that the invariant stable and unstable manifolds for the sun-Earth L_1 and L_2 visually appeared 'close to intersecting.'"

As Earth orbits the sun, the tubes are lashing through space like water from a demented lawn sprinkler. So when you plot a tube, you do it in a rotating reference frame, meaning that the two massive bodies are plotted at fixed points on the x axis. Now the manifolds are frozen in space, and the only thing moving is the spacecraft. (See the drawing at upper left.) You also hold the spacecraft's velocity constant, because you'd need a fresh sheet of paper for each new velocity. Drawing a pair of halo orbits, or rather drawing their two-dimensional projections in the (x, y) plane, gives the bean shapes around L_1 and L_2 . They aren't ellipses, explains Ross, because "in 3-D, a halo orbit looks like the edge of a potato chip." But they're periodic, and that's the important thing. The unstable manifold coming off L_2 is shown in red, and the stable manifold leading to L_1 is green. Now, if you take a cross section through the manifolds parallel to the y axis, as shown by the heavy line, you can make a second plot of the spacecraft's position (y) versus its velocity in the y -direction (y -dot), as shown in the lower left drawing. This is called a Poincaré section (above right), and the intersections of the red and green lines mark the locations and speeds at which one can segue from manifold to manifold for free.

But if you're trying to get from, say, Earth's L_2 point to Mars's L_1 point, you suddenly have a four-body problem, which is truly a computational

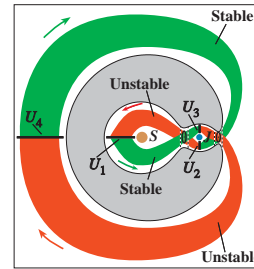
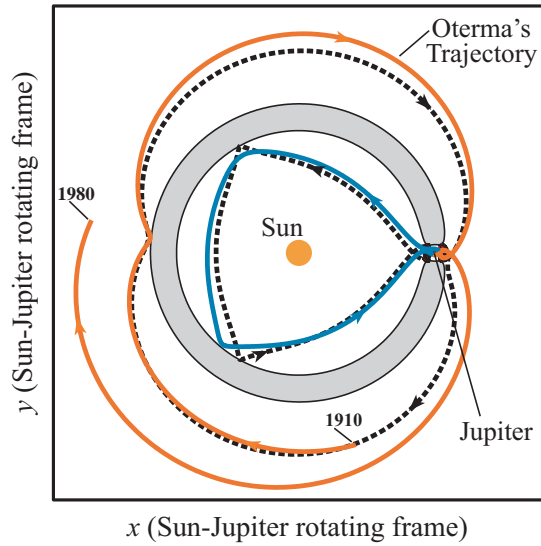
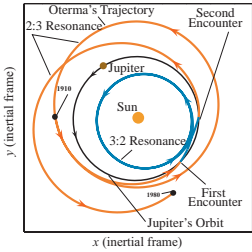


A Poincaré primer. Poincaré invented a general method for classifying orbits by plotting any two parameters against each other—in this case position versus velocity. If you have an elliptical orbit (top left), a Poincaré section taken perpendicular to it (gray) will plot as a single point (top right) because the orbit always returns to the same spot with the same velocity. But if you have an orbit that is gradually spiraling inward (bottom left), the Poincaré cut will show a set of points trending to the left as the radius decreases, and up because objects in lower orbits move faster. Courtesy of Cici Koenig.

swamp. So you make two simplifying assumptions: everything lies in one plane (from Earth on out, all the planets except Pluto are within 2.5°) and all planetary orbits are circular (again pretty much true except for poor old Pluto). Now you can treat the system as two three-body problems (sun-Earth-spacecraft and sun-Mars-spacecraft) coupled together through their common members.

Says Lo, "Originally, I wanted to find the intersections of the manifolds between Mars and Earth. When that proved too difficult, we switched to Jupiter-Saturn, which yielded an immediate result" the summer of Ross's sophomore year. Adds Ross, "The manifolds of Jupiter's L_2 and Saturn's L_1 intersected in position within a short time—a few decades." Further work showed that you could go between any of the outer planets for free. It might take a few hundred years, however—possibly a couple of thousand. Earth to Mars just happened to be the worst-case example, taking tens of thousands of years.

Still, the Voyagers took a mere two years to get from Jupiter to Saturn using conic sections and gravity assists, so a free tube ride that "only" takes a few decades may not seem like an exciting prospect. Comets and asteroids, however, have all the time in the world. Could they be roaming the freeways from planet to planet, like celestial retirees in their motor homes? Yes, they could. For example, at the time of its discovery in 1943, a comet named Oterma was in a 3:2 resonance



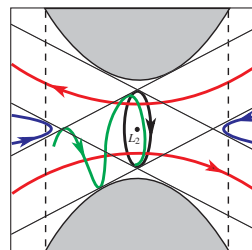
Above: The path of comet Oterma from 1910 to 1980, plotted in a non-rotating reference frame.

Above, right: When plotted in a sun-Jupiter rotating frame, Oterma's orbit matches the tube trajectories (dashed line) almost perfectly. Note that in the rotating frame, Oterma moves clockwise relative to a stationary Jupiter, even though the planets—and Oterma—orbit the sun counter-clockwise.

Far right: The patched-together sections of Jupiter's tubes that Oterma used. U_1 through U_4 are the Poincaré cuts.

inside Jupiter's orbit—that is, Oterma made three laps around the sun for every two that Jupiter did. But it hadn't been there long—in 1958 Liisi Oterma back-calculated the orbit of her find and discovered that it had been in a 2:3 resonance *outside* Jupiter's orbit until 1937, when a close encounter with the giant planet had flung it onto the inside track. Then in 1963 Oterma suddenly changed lanes again without signaling, nose-diving past Jupiter like the driver in the left lane who suddenly realizes he's about to pass his exit, to once more lie in a 2:3 resonance outside Jupiter. The two nearly traded paint in the process—Oterma only missed Jupiter by 14.7 million kilometers. It turns out that the 3:2 resonant orbit passes very close to the sun-Jupiter L_1 point, while the 2:3 orbit (surprise!) sideswipes L_2 . Says Lo, "Oterma's trajectory almost followed a cookie-cutter" path along the manifolds.

Recalls Lo, "This was before we went to Jerry [Marsden], so I didn't have the full theory of *how* this transport occurred, and Jerry suggested the possibilities that he knew about." It turns out that just missing the manifold was the key, as Conley had discovered for the sun-Earth combo some 40 years earlier: if you ride the manifold in, you get trapped in an orbit around the Lagrange point, but if you pick a point in the Poincaré cut that lies inside the tube, you'll plunge toward the planet. What happens next gets very complicated—chaotic, in fact—but you can emerge at either Lagrange point, or you can wind up in orbit around the planet. (Conversely, if you skirt the tube's mouth without entering it, you'll head back out where you came from without crossing over.) Says Marsden, "Back in the



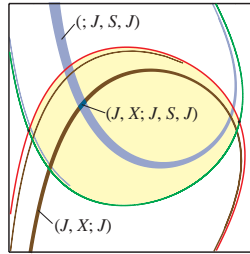
'60s, Conley and McGehee discovered a lot of the bits and pieces of this that were very important, but we developed the mathematical underpinnings that established the big-picture framework."

Koon, Lo, Marsden, and Ross proved that you could pick any itinerary you liked for *any* set of Lagrange points, and a trajectory existed that would follow it. So, in principle, you could set a course for a comet that would swoop in from the outside, whirl around Jupiter three times, cross to the inside track, make fifteen orbits around the sun, cross over to the outside track again, make three more orbits around the sun, and then get permanently captured by Jupiter to take up a new life as a Jovian moonlet. The software Ross adapted to do these computations was provided by Gerard Gómez of Barcelona University and Josep Masdemont of the Polytechnic University of Catalunya, also in Barcelona, members of Simó's SOHO team.

Koon et al. also devised a notation system for plotting the itinerary and keeping track of where you were in it, allowing them to classify paths based on what points were visited, even if the details of course and speed were wildly different. Koon calls this "the most amazing thing. Dynamical systems theory—symbolic dynamics—allows us not only to prove the existence of all these complicated trajectories, but also to keep

Conley and McGehee's bestiary of orbits is based on the periodic (black) orbits, in this case a Lyapunov orbit—an ellipse that lies in the plane of Earth's orbit and is easy to deal with mathematically. Asymptotic (green) orbits wind onto or off of the Lyapunov orbit, and collectively define the stable and unstable manifolds. Transit (red) ones pass inside it and enter one of its tubes. Nontransit (blue) ones shy away from the tube's mouth and return to the region of space from whence they came.

Right: A Poincaré cut in Jupiter's plane, like the one on page 10. The green and red lines are the stable and unstable manifolds. The region where they intersect (light yellow) contains the crossover trajectories. The blue strip labeled $(; J, S, J)$ winds around and around, approaching the green line but never actually touching it, and contains particles that are in the Jupiter region and will slip inside to visit the sun before returning. The brown strip $(J, X; J)$ approaches the red line in a similar manner and contains trajectories that have returned to Jupiter after having been out beyond its orbit. The strips' intersection $(J, X; J, S, J)$ gives the paths where a particle began near Jupiter, ventured outside, and is now back at the planet, but will plunge toward the sun and later return to Jupiter.



track of them. Otherwise there would be almost no way to think about them.” The notation system divides the space in the neighborhood of any moon or planet into three regions—the region beyond its orbit, abbreviated X for exterior; the region between the L_1 and L_2 points, abbreviated E for Earth, J for Jupiter, and so on; and the region within the body's orbit, abbreviated S for sunward. So the itinerary described in the previous paragraph would be written as $(X, J, S, J; X, J)$, with the semicolon showing that the comet is currently in its brief venture back outside Jupiter's orbit.

All this work, beginning with the Earth L_1 - L_2 analysis, was published in a massive paper in *Chaos* on April 12, 2000. Says Marsden, “The real understanding of the tubes—using the ideas of transport through the tubes or bouncing back, as well as how the tubes navigate the neck region—was motivated by the Genesis return orbit and is a critical ingredient to understanding the whole picture. It is an enabling set of ideas that builds on what Conley and McGehee did, but goes well beyond it. This was the most important contribution of the *Chaos* paper.”

Poincaré had, in fact, been dabbling in chaos theory, although he didn't know it because he was in the process of inventing the field. Chaos theory, a branch of dynamical systems theory, studies systems like the three-body problem that can be completely described by a few simple equations, but whose behavior alters dramatically depending on very small changes in one's choice of initial conditions. The various tubes emerging from a set of periodic orbits around L_1 and L_2 wrap around one another in a very complex way. The Poincaré cut winds up looking like the chocolate swirls in marble fudge ice cream, and adjoining points will take you to wildly different destinations. And that's why the theory is so powerful—it doesn't take much exertion to move a smidgeon in any direction along the cut, making a large number of tubes accessible at a very low energy cost. Less

fuel means less mass, which means more payload—more bang for the buck, and fewer bucks for the mission.

Says Lo, “I grew up with the picture of the solar system we inherited from Kepler and Copernicus—a series of planets isolated in stately, concentric, nearly circular orbits. And you're always surprised when things like comets intrude.” But the tube model provides an easy way for comets and asteroids to get into the inner solar system. As a tube sweeps through the outer reaches, every now and then some debris will fall into it and be whisked in toward the center. In fact, it's almost a circulatory system, with space junk for blood cells and the tubes acting as blood vessels. And while the sun pumps the system, it's not built to the mammalian plan but rather, with the planets, is more like an earthworm, which has multiple hearts.

Seen from this cardiac point of view, the dinosaurs could be considered to be victims of a blood clot—the asteroid that whacked them was traveling down an artery that happened to be obstructed by the earth. (Yes, the tubes can pass through planets as well as going around them.) Marsden once gave a presentation to an audience that included physicist Richard Muller and geologist Walter Alvarez, who have brought the asteroid extinction theory from crackpot sci-fi to mainstream science over the last 20 years. Says Lo, “They told Jerry that they wondered if that asteroid used our orbits, because it had a very low impact velocity. You can infer that from the huge deposit of iridium it created. If it was a hot, fast impact, the iridium would have been vaporized and destroyed.”

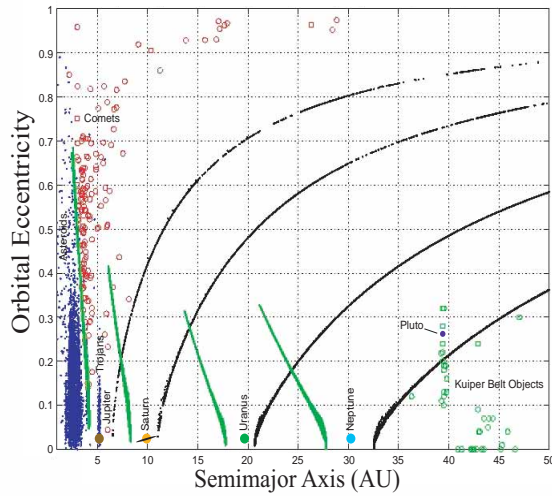
But an even better circulatory analogy might be the world's wildest water park. “Imagine a set of water slides winding downhill every which way,” says Marsden. “Now, imagine that the slides are in constant motion, moving up and down and passing through one another unhindered. You can go anywhere in the park by hopping on the nearest slide and then switching from slide to slide as they rise and fall.” Eventually, a slide will pass by your destination, and you hop off. Says Marsden, “Sliding is naturally dynamic and requires very little active control, unlike a car whose motor is always running and which has to be steered. And proper timing is critical, which is a key ingredient to the whole methodology—you have to jump from one tube to another at just the right moment, while a freeway interchange is static.”

The fluidic analogies are not far off the mark. Says Marsden, “I inherited postdoc Chad Coulliette and grad student François Lekien from Steve Wiggins [a Caltech professor from 1987 to 2001]. They are working on fluid mixing and transport of materials in the ocean, which turn out to have basically the same mathematical infrastructure as dynamical astronomy.” In other words, whether you're dropping dye overboard in



This sliver of Mars probably rode a tube to Earth.

A different kind of Poincaré section of the outer solar system, plotting orbital eccentricity—how far out of round the orbit is, with 0 being a circle—versus the orbit's semimajor axis, i.e., the maximum distance it gets from the sun. (The average sun-Earth distance is defined to be one AU, or astronomical unit.) The green lines are the L_1 manifolds and the black lines are the L_2 manifolds. The Trojans are the family of asteroids caught at Jupiter's L_4 and L_5 points.



Monterey Bay and letting the submarine currents carry it out to sea, or tracing a family of asteroids back to a single source, the rules are the same. “The fluids people are a little bit ahead of the dynamical-astronomy people, and François and Chad came up with some very clever tricks that allow us to compute transport quantities for much longer times than were known before. That leap would not be possible unless you have a mathematical framework, in this case dynamical systems theory, that encompasses both fields.”

In yet another leap, the collaboration includes a pack of chemists—Charles Jaffé of West Virginia University, Turgay Uzer from Georgia Tech, and Utah State’s David Farrelly. “Suppose an asteroid hits Mars and throws up a bunch of debris,” Marsden asks. “What’s the probability of some of it reaching Earth? Being bound in Mars orbit and then escaping is mathematically analogous to a molecule breaking apart. Jaffé, Uzer, and Farrelly brought in all sorts of techniques from chemistry, because chemists have really been worrying about those problems.”

It’s useful to know how material sloshes through the solar system, drifting on gravity’s currents, but no man will wait for that tide. A mission a grad student would consider a good career move needs to get where it’s going in only a few years at most. You can do this if you stay in the vicinity of one planet—taking the beltway rather than the interstate, as it were—and a Japanese spacecraft named Hiten was the first to do just that. Launched in January 1990, it was placed into a highly elliptical orbit around Earth, from which it was supposed to release a small probe named Hagoromo into lunar orbit. Hagoromo’s radio failed before deployment, however, and Hiten didn’t have enough fuel to get to the moon itself on a conventional path. So JPL’s Edward Belbruno (now affiliated with Princeton) and James Miller proposed gradually nudging Hiten’s orbit into a very long ellipse extending some 1.4 million kilometers from Earth. (You

may recall that the sun-Earth L_1 point is about 1.5 million kilometers out.) In this region, which Belbruno and Miller called the Weak Stability Boundary, a carefully timed rocket burst sent Hiten looping into lunar orbit.

The same trick will work in other planetary neighborhoods. The gulf between Jupiter’s moons Ganymede and Europa is only about 400,000 kilometers—about the distance from Earth to the moon. So a mission could fly to Jupiter on conventional conic sections, then slip into a “petit grand tour” of the Jovian system. Based on the method developed in the *Chaos* paper, Koon, Lo, Marsden, and Ross created a proof-of-concept flight plan in which a spacecraft took one loop around Ganymede before settling into a permanent orbit around Europa—a moon that planetary scientists are dying for a long look at, as it may have oceans of liquid water beneath its frozen surface. “Most previous applications of dynamical systems theory to mission design focused on the surfaces of the invariant manifolds,” says Koon. “We showed that the regions inside and outside the manifolds can be used to advantage as well as the manifolds themselves.” Recent, more ambitious itineraries include Callisto and Io, and go on for a hundred or so orbits. Says Ross, “I think 100 is sort of the limit of predictability. Like you can only predict the weather for so many days into the future, we can only predict what’s going to happen up to some finite horizon in any chaotic environment.” So you still need to fire the maneuvering thrusters every now and then, just as Genesis needs a nudge every couple of months to keep it in orbit around L_1 . But you’ll use a lot less fuel if you work with the manifolds rather than trying to punch through them.

Nowadays, these calculations for space missions are done using a software package called LTool, which can actually handle an N -body problem using positional data from JPL’s elaborate model of the solar system, called an ephemeris. LTool grew out of a set of computational tools developed by astrodynamists at Purdue over the last 20 years, and which Lo borrowed from longtime collaborator Kathleen Howell, a halo-orbit expert there. Lo, Howell, and her grad student Belinda Marchand used these tools on a Jupiter-resonant comet called Helin-Roman-Crockett (codiscovered by JPL’s Eleanor Helin), meticulously matching the comet’s orbit with the appropriate segments of Jupiter’s tubes—the model’s first use of the real motions of actual celestial bodies instead of idealized circular orbits. In 1998, Lo assembled a JPL team to develop and expand the software, calling on Larry Romans (PhD ’85), George Hockney, Brian Barden and Roby Wilson (both Howell alums), Min-Kun Chung (BS ’81), and James Evans.

LTool got its first real workout on the Genesis mission—the spacecraft that’s now sampling the solar wind at L_1 —for which Lo served as mission-

Jupiter’s four biggest moons, from the outermost in: Callisto (top), Ganymede, Europa, and Io.



design manager. Lo, Howell, Barden, and Wilson came up with a trajectory that carries the spacecraft from liftoff through five orbits around L_1 before the craft drops out to make a single pass around L_2 in order to touch down in Utah's Great Salt Desert during daylight. When the launch date slipped from February to August 2001, the team was able to completely redesign the working orbit in a week—a task that would normally have taken months.

More recently, LTool calculated the first formation flight around a Lagrange point, an option being considered for the proposed Terrestrial Planet Finder (TPF), which will look for Earthlike planets around other stars. A fleet of several small spacecraft flying in formation could act as a single large telescope, and one convenient place to deploy such a thing is L_2 . Lo, Gómez and Masdemont (the boys from Barcelona), Romans, and Caltech's Ken Museth showed that the gravitational balance at L_2 would make it very easy for such a fleet to stay in formation even as the individual ships maneuvered to point the virtual dish in any direction.

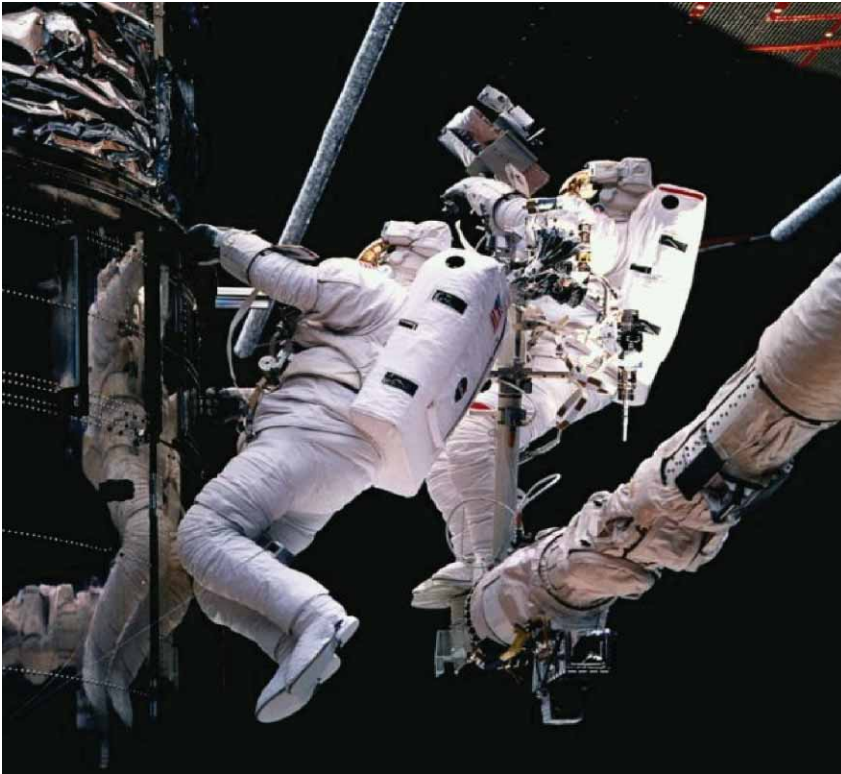
And refinements to the transport theory could narrow the list of stars for TPF to look at. Our solar system is filled with a diffuse cloud of "zodiacal dust" fed by cometary tails and the dandruff from asteroid collisions. As Earth and its sister planets swim through the dust, they leave wakes. Similar exozodiacal clouds have been seen around other stars, and "as telescopes get sharper and sharper, you can identify more and more features in them," says Lo. Many astronomers hope to be the first to spot an Earthlike planet by its wake. Looking for signs of life, of course, is much more difficult—you'd need to keep the telescope focused on the planet for some time in order to try to discover, say, oxygen and methane spectroscopically. You could cull some duds in advance by calculating whether the planet was in the habitable zone—the proper distance from its sun for water to be liquid. But it's generally agreed that many of the carbon-rich molecular building blocks of life get delivered to newborn planets by asteroid and comet impacts, and the rate of this fusillade changes with time as the planets sweep up the debris. "If you have too many bombardments, life never takes hold, but if you have too few you don't get enough raw materials. What is that happy medium, and when does it occur?" Lo asks. Says Marsden, "We are hoping that our transport theory will enable us to better understand the process, and do the calculations more efficiently." You could plug various star and planet combinations into the model and find out which ones seem conducive to life, and then predict how their wake patterns would look at various stages.

So. The Lunar Gateway Service Area we began with—is it pie in the sky, or perhaps castles at the Lagrange point? Well, the long-term vision put

together by the NASA Exploration Team (NExT) sees the Earth-moon L_1 point as a way to get humanity beyond the low-Earth orbit occupied by the International Space Station. No timetable and budget has been set, however, and operations at the space station are set to run through at least 2017. But the lunar L_1 point would be "a very attractive location if [NASA] decides to send advanced robots or even humans to the surface of the moon. The entire surface of the moon is accessible with moderate ease. It's an excellent staging area for deep-space missions, human or robotic, and maybe for one day sending humans to Mars," Harley Thronson, the NExT team science chief, told the *Houston Chronicle* at the 2002 World Space Congress in October, where the plan was widely discussed. "This is really where you learn to drive around the neighborhood and develop your capabilities."

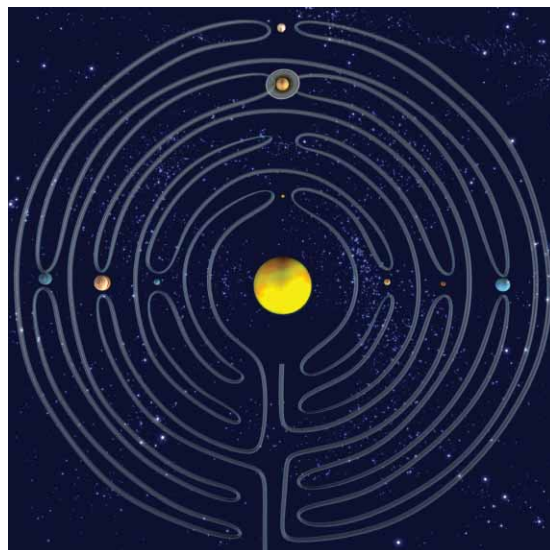
"You don't have to fight against Earth's gravity," says Lo. "When you lift off from Earth, you have to build your spacecraft to withstand a lot of G forces. But if you were launching it from space, you could have a much lighter structure." And you could build things at the Gateway that couldn't be built on Earth at all—thin-film mirrors, for example, incapable of supporting their own weight. The same holds true at the space station, but it would cost a lot more fuel to launch from there, and there's no cheap path back if the telescope needs service. And the Gateway would also widen the very narrow launch windows for some planetary missions. You could keep the spacecraft hanging around in the neighborhood until the time is ripe, revving it up in the meantime with multiple Earth flybys like David whirling his sling around his head before letting fly at Goliath. It could take weeks to get to the moon's L_1 from Earth by tube, which might prove a bit too leisurely for impatient humans but would be fine for freight—food, toilet paper, and spacecraft parts. People would probably prefer to hop a fast rocket on a conic section. It would be like crossing the Pacific today—cargo goes by ship, but most people take a plane.

But right now Lo, Marsden, and company are a long way from compiling a Rand McNally-esque road atlas of the entire system, much less a detailed street map. The work they've done so far is more of a point-to-point nature—the equivalent of having a bunch of people just hop into cars and drive, and seeing where everyone winds up. To do this, they pick a large number of initial conditions and let the computer run the paths out to wherever they go. They can make informed choices of the starting points to set the machine off in the right direction, but the final destinations remain the luck of the draw. What they need is enough computer power to make the equivalent of time-lapse aerial photographs of all those flailing tubes. Furthermore, there are whole families of periodic and quasi-periodic orbits that haven't even been



Above: Space Shuttle astronauts replace a gyroscope on the Hubble Space Telescope in low-Earth orbit. Future telescopes at L_1 and L_2 could be brought into the Lunar Gateway Service Area for repairs.

Below: The planets' stately, isolated orbits are really connected parts of a larger whole, like the labyrinth in Chartres Cathedral. Courtesy of Cici Koenig.



PICTURE CREDITS:
8 – Genesis; 10-13 –
Shane Ross; 12 – Doug
Cummings; 13, 15 –
NASA

catalogued yet, much less explored—you can have an orbit that lies on the surface of a torus, for example, so that it looks as if you had soldered the ends of a Slinky together. The manifolds winding onto and off of this orbit look like hairy donuts. And Randy Paffenroth, staff scientist in applied and computational math; Eusebius Doedel, visiting associate in applied math; Herb Keller, professor emeritus of applied math; and Don Dichmann of the Aerospace Corporation have discovered even weirder orbits that are impossible to describe in simple terms.

But what if you aren't anywhere near the tube you want to take? Lo, Marsden, and company, in collaboration with a group headed by Michael Dellnitz at the University of Paderborn in Germany, are also working on an extension of the theory they call "lobe dynamics." Lobe dynamics allows you to begin at a distant point on the Poincaré cut and, over many successive passes, hop chaotically between orbital resonances until you arrive in the vicinity of the proper tube and fall in. It's kind of like starting in the middle of a grassy paddock and bouncing your way over the rough ground to reach a paved road.

"Exactly how three-body dynamics can be used to help solve Galileo-type trajectories is a real-life research problem," says Lo. "How this connects up to conventional conic orbits is not entirely clear. They're not separate, not clearly distinct, and we would like to be able to use elements of both. Ultimately, this will enable us to do missions we can't even conceive of now. The rational numbers didn't replace the integers, they just increased the number of things we could do. And who knows what lies ahead? Beyond the rationals are the irrational numbers, and beyond them the imaginaries." Says Marsden, "Since the foundation of the Interplanetary Transport Network we have laid is so broad and fundamental, it helps us understand many different, otherwise disparate phenomena at multiple scales, from the trajectories of spacecraft to the chaotic motion of comets and the transport of zodiacal dust particles."

Says Marsden, "We'd really like to establish a formal joint Caltech/JPL center to carry on this research, perhaps in collaboration with Caltech's Center for Integrative Multiscale Modeling and Simulation. But we need a donor—someone who thinks this stuff is really cool." Adds Lo, "It's a large-scale project. Maybe not quite as horrendous as, say, the human genome, but a lot like those star catalogs. It's going to take some time, because there are a lot of theoretical underpinnings that are still not really understood." The center would take advantage of Caltech's supercomputer facility and draw faculty from a wide range of disciplines. And the work done at the center would redound to other fields in return—perhaps the celestial dynamicists will ultimately teach the chemists a thing or two. □