The Interplanetary Transport Network

Some mathematical sophistication allows spacecraft to be maneuvered over large distances using little or no fuel

Shane D. Ross

People often picture the solar system as a cosmic clockwork. And why not? With few exceptions, the planets orbit the Sun in near-perfect circles, and the moons orbit their planets in the same manner, all moving with the famous regularity of the heavens. One imagines the gravitational field created by this orderly mechanism to be equally regular. Drop a rock or spacecraft somewhere close to the Sun, and the object should plummet into the huge solar mass; release it somewhere near our planet, and it ought to drift, perhaps more slowly, back to Earth.

Nature, alas, is not so simple. The underlying complication, of course, is that the Earth is orbiting the Sun, not just hovering fixed in space. As a result, some very unintuitive things can happen. A rock let go near our planet could find itself following a complex and chaotic path, perhaps orbiting first the Earth, then the Sun, and back again, over and over for years. Add in the tugging of all the other planets and moons, and the possible routes through space can get enormously complicated—and quite interesting.

Investigators from fields as diverse as mathematics, chemistry and fluid dynamics have recently revealed the existence of a complex set of allowable trajectories for such objects—an interplanetary transport network of crisscrossing pathways. These invisible highway lanes, originating near a planet or moon, guide traffic through the solar system. But unlike the thoroughfares one finds on the ground, the space highways and their interchanges are dynamic, with lanes moving past one another according to the varying geometrical relations between planets and moons. Staggering through this tangled web, comets and asteroids find themselves jumping from one lane to another willy-nilly, getting handed off between planets—or sometimes running into them. For such pieces of cosmic flotsam, the solar system turns out to be more like a turbulent sea than a clockwork.

Although people cannot influence the fates of speeding comets or asteroids (not yet, anyway), mission planners do maneuver spacecraft and can direct them to jump from lane to lane on the interplanetary highway in such a way that they can travel vast distances using practically no fuel. Like an island castaway who throws a message-laden bottle into the right current at the right time, the controller can send a spacecraft to gravitational sweet spots that provide natural gateways to more distant destinations. Harnessing this effect to good purpose, specialists can plan fuel-efficient routes, ones that would not otherwise be imaginable or technically feasible.

Buck Rogering Through Space

Most interplanetary travel doesn't exploit subtle gravitational effects. Instead, spacecraft race quickly to their destinations Buck Rogers style, using chemical rockets. Blasting around in this way is a straightforward exercise. The person planning the trajectory need only to consider the influence of one celestial body at a time. That is, you can treat the departure from Earth and the arrival at a distant planet each to be interactions between the spacecraft and one massive body. Similarly, the transfer from Earth's general neighborhood to that of a faraway planet may be worked out by considering the spacecraft and the Sun alone. Hence one only has to deal with two bodies at a time. The overall path can then be approximated using an appropriately linked series of simple curves: ellipses, hyperbolas and parabolas—the two-body problem's well-known "conic solutions" (these being the curves one gets after slicing a cone with a plane), which were discovered by Johannes Kepler in the 16th century. NASA's spectacular multiple flyby missions of the outer solar system, such as Voyager 1 and 2, were based on such a patched-conic approach, which was used to provide an initial guess for a numerical procedure that produced a more precise solution by taking into account all gravitational and nongravitational influences on the spacecraft.

For missions sent out to fly past multiple bodies—say, Jupiter, Saturn, Uranus and Neptune in the case of the Voyager 2—the speed of the spacecraft relative to these planets is high, and the time during which the Sun and a planet produce comparable accelerations on the spacecraft is very short. So the patched-conic approach works very well. One drawback with this approach, though, is that planetary flybys end up being very brief. Another is that fuel becomes a major factor limiting the spacecraft's itinerary. A prohibitively large amount of propellant would be needed,

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Figure 1. Spaceflight typically requires the expenditure of considerable quantities of propellant. But after it blasted off from Earth, the Genesis probe was able to travel 1.5 million kilometers toward the Sun *(green portion of the trajectory)*, which is some four times farther than the Moon's orbit *(gray circle)*. Genesis then orbited the Earth's L_1 Lagrange point *(white cross in foreground)* collecting particles of the solar wind for two and a half years before traveling millions of kilometers along a circuitous path *(blue)* that looped by another Lagrange point, L_2 *(second white cross)*, before returning to Earth in September 2004. Amazingly, Genesis completed this vast trek using hardly any fuel. The probe did so by following one of the many possible low-energy paths through the solar system, routes that have long served as natural conduits between planets for asteroids and comets. Some of these conduits lead to collision with Earth, as the Genesis probe's path did by design.

for example, to put on the brakes and insert a craft into orbit around some distant planet or moon, observe for a while and then blast off to the next destination. And taking extra fuel for maneuvering means that the scientific payload must be made smaller than would otherwise be possible. Thus mission planners have to strike a balance between the proposed trajectory and the amount of instrumentation that can be carried. The Jupiter-bound Galileo probe and the Apollo lunar lander, for example, began their journeys away from Earth with about half of their masses being made up of fuel.

In another category entirely was the Genesis Discovery Mission to sample the solar wind, which used only 5 percent of its mass for fuel. Launched in 2001, the Genesis spacecraft flew 1.5 million kilometers toward the Sun, where it loitered for two and a half years gathering individual atoms of the solar wind, ultimately bringing them back to Earth in 2004. In an unfortunate mishap, the parachute failed to deploy after re-entry, and the sample-return canister was badly damaged when it struck the ground at high speed. Thankfully, scientists were able salvage some of what was collected, the first extraterrestrial material brought back to Earth from deep space since the last of the Apollo landings in 1972 and the first to be collected from beyond the Moon's orbit. Genesis completed its journey of more than 30 million kilometers using only a minimal amount of propellant. For comparison: A car with a full tank of gas (also about 5 percent of the vehicle's total mass) can go only about 500 kilometers before it's time for a refill.

Voyages like that of the Genesis spacecraft would have been inconceivable not long ago, but they are now possible thanks to the better appreciation of the low-energy passageways that wind between planets and moons. Conceptually, the approach needed to mount such a journey through space is similar to what sailors have long done—taking advantage of ocean currents to speed them where they want to go. Ancient mariners often discovered natural currents by noting the motion of driftwood or seaweed being carried with them. To some extent modern space navigators can do the same, observing the movement of natural objects, namely comets and asteroids.

A comet called Oterma is particularly interesting in this regard. Early in the 20th century, this icy body circled the Sun outside Jupiter's orbit. Then, after passing close to that planet in 1937, Oterma began to orbit inside Jupiter. The two bodies met up again in 1963, at which point the comet moved back to the outside of Jupiter, where it remains today. During each of its encounters with Jupiter, the comet loosely orbited the planet. That is, for a time Oterma was a captured moon.

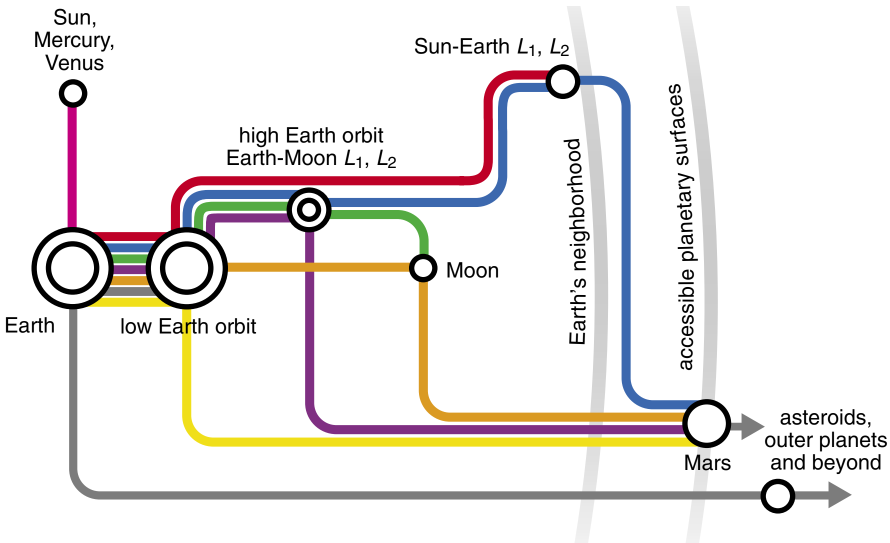

Figure 2. Mission planners have come to appreciate that in certain cases the best routes for spacecraft are not always the most direct ones. In some instances it may be smarter to take advantage of the low-energy pathways connecting key points in space. For example, a spacecraft destined for the surface of the Moon might get there via one of the lunar L_1 or L_2 Lagrange points (*green path*). Such Lagrange points may also serve as way stations for trips to other planets, as shown in this space "subway map." (Courtesy of NASA.)

What made this comet move along such a strange path? The best way to get a sense of the answer is to simplify the problem and consider the motion of the comet (a relatively small object) as it is being acted on by the gravitational tug of two massive bodies: in this case, the Sun and Jupiter. In the study of celestial mechanics, this situation is referred to as the *restricted three-body problem* (restricted by the requirement that the third body have negligible mass compared with the other two). Although the full solution to this problem is rather hard to fathom, the key elements can be understood with just a little physical insight.

Jimmy Dorff

Figure 3. The existence of low-energy passageways through space can be understood on an intuitive level by considering the physics displayed by a "gravity well." These funnel-shaped devices (like the one shown here, located at the Morehead Planetarium in Chapel Hill, North Carolina) allow coins to circle stably much the same way that a planet orbits the Sun.

Tubes and Funnels

Building on Kepler's work of the previous century, Isaac Newton solved the gravitational two-body problem. The result is a relatively simple formula, which can be used to compute elliptical orbits or hyperbolic spacecraft flybys. But it turns out that it is much more difficult to determine the path a comet or space probe will follow when it is under the gravitational influence of two bodies in orbit about each other. Allowing that one of the three bodies is much smaller than the other two helps to make the problem more tractable, but it still remains a thorny one to solve. Many scientists have worked hard on it through the years. Newton tried and got fed up. He wanted to write down an equation that describes the motion of the third body for all time. But he failed, and the three-body problem was declared unsolvable.

Let's not give up so easily. The trick is to not seek a tidy little equation to describe the motion. Instead, one can proceed by considering the problem from a geometrical point of view, seeking intuitive insight into what the solutions might look like. For this, it is helpful to think of one of those funnel-shaped contraptions one sees every now and then at science museums, often called "a gravity well," by virtue of the physical analogy with gravitation. A little chute on the side allows you to roll a coin into the device in such a way that it will roll around the inside of the funnel for quite some time. A marble would travel in the same way.

In a frictionless world, a coin or marble would just keep circling around such a funnel, mimicking the orbit of, say, Earth around the Sun (if one assumes that the funnel is standing in for the Sun's gravitational well). But in the real world, a circling coin or marble suffers some frictional loss. So over time its orbit around the funnel decays, causing the coin or marble to spiral inward and downward. This tendency is easily observed and indeed forms the economic basis for many of these devices: They eventually take your money. But in watching your change disappear, you'll learn some things. In particular, you'll notice that the smaller the size of the orbit, the more times the coin goes around in a given period. That is to say, smaller orbits have higher angular frequencies. The same is true for objects orbiting in space.

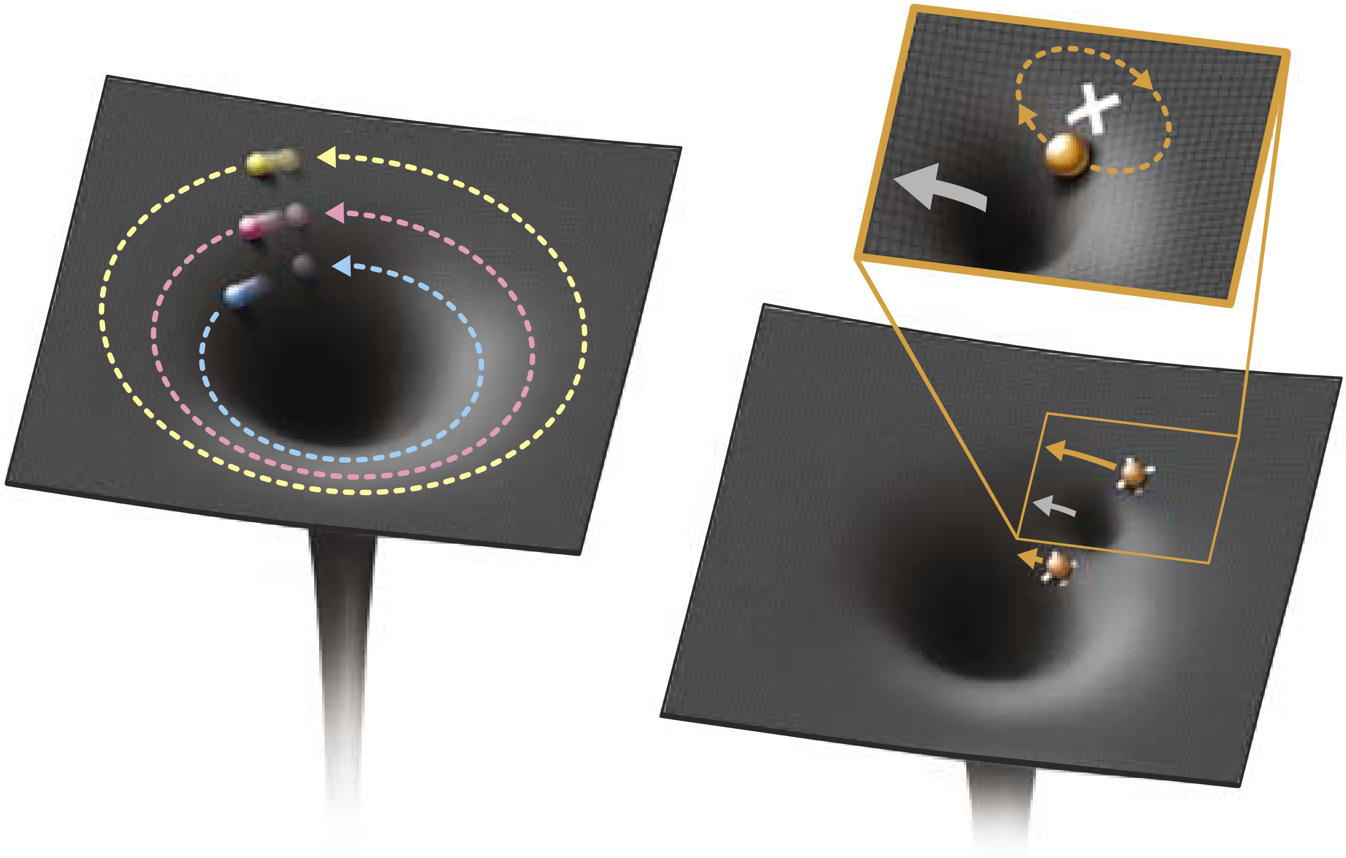

Figure 4. Marbles circling in a gravity well—or planets circling the Sun—do so at a rate that depends on the size of their orbits: The smaller the orbit, the larger the angular frequency required to achieve balance *(left)*. A demonstration device engineered to mimic the gravitational field that arises from a pair of massive objects, say the Sun and Earth, would be shaped like a large funnel with a small funnel embedded in it *(lower right)*. Here the small funnel would have to orbit the large one, just as Earth orbits the Sun. A marble traveling around the large funnel at the same angular frequency could balance at two spots that straddle the small funnel *(white crosses beneath marbles)*—corresponding to Earth's L_1 and L_2 Lagrange points. With care, a marble could be positioned and given an initial velocity such that it would then "orbit" such special locations (at least for a limited time), just as Genesis was made to orbit Earth's L_1 Lagrange point *(upper right)*.

Imagine now a funnel with three marbles circling in three closely spaced, parallel orbits, one just a bit farther out than the next. Compared with the one in the middle, the marble in the outer orbit would have to travel at a slower angular frequency to be stable; the one in the inner orbit would go around at a higher angular frequency. The same is true in space, say, for three asteroids orbiting the Sun. If the middle one were traveling at one astronomical unit from the Sun (150 million kilometers, the Earth-Sun distance), it would take 365 days to make one revolution. The outer asteroid would take slightly longer than 365 days to make a full circle; the inner asteroid would complete its orbit faster than 365 days.

This pattern is easy enough to understand when you recall what happens with one of those coin-and-funnel gizmos. But now imagine that one of those science-museum devices has a small funnel shape embedded in the main funnel. What is more, let that small funnel circle around the larger one at the same rate that a coin or marble would have moved around at that position. This arrangement mimics the gravitational wells of, say, the Earth (the small funnel) and Sun (the large funnel) combined.

Consider now what would happen if you shot a marble around in such a way that it circled the central depression at the same distance from the center and at the same rate as the small (Earth) funnel—but not too close by. The marble would just circle around nicely for a long while. If it were moving at the same angular velocity but positioned farther out (where the surface has a gentler slope), this marble would be going too fast to circle around stably and would be flung outward. Conversely, if it were inside of the Earth-funnel's orbit, it would be moving too slowly to support itself against the steep walls of the main funnel and would be drawn inward. The only place a marble with that particular angular frequency circles properly is at the radius of the Earth-funnel's orbit—or is it?

Closer scrutiny of this weird surface will reveal two very special points. One lies close to the ridge that connects the little Earth-funnel to the main Sun-funnel. Let's get there by starting near the center (deep in the Sun-funnel) and moving in the direction of the Earth-funnel. The surface first rises with its usual steepness, but then it rolls over—that is, the slope first becomes less steep, then things flatten out, then you drop into the Earth-funnel. Before getting to the top of the intervening ridge, you'll pass a point where the slope is just right for balancing a marble that is circling around at the same rate as the Earth-funnel.

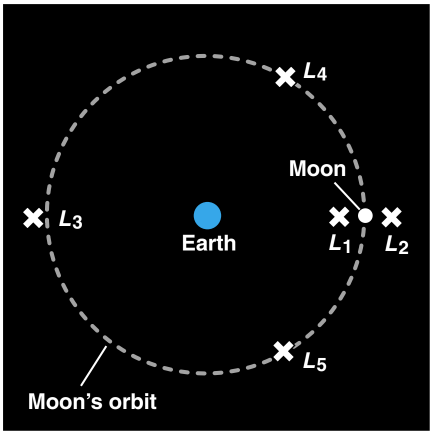

Figure 5. Five points of gravitational equilibrium exist in the restricted three-body problem, as shown here for the Earth-Moon system. All five move with the Moon as it orbits Earth. Lagrange points L_1, L_2 and L_3 are considered unstable, because an object placed at one of these three points will tend to drift slowly away from it over time. The L_4 and L_5 Lagrange points are stable in the sense that objects placed in their vicinity will naturally tend to remain close by.

Remember, you'd normally expect this marble to have to circle around faster to stay balanced. But because the slope here is somewhat less steeply inclined than is typically the case for this orbital radius, the marble can circle around just fine. This point of balance for the marble has an equivalent in space. It's located 1.5 million kilometers from Earth in the direction of the Sun.

Getting back to the funnel realm, on the outside of the Earth-funnel there is another special balance point. Recall that on this side, a marble would normally orbit at an angular frequency that is less than that of the Earth-funnel. A marble that moved with the angular frequency of the Earth-funnel but positioned farther out, where the walls of the main funnel slope less steeply, would normally be expected to fly outward. But there is one spot where it won't do that: just on the outside of the Earth-funnel, where the surface is somewhat steeper than normal. Again, there is an equivalent balance point in space, located 1.5 million kilometers from Earth in the direction opposite the Sun.

The 18th-century mathematician Leonhard Euler discovered these two special points (along with a third). His contemporary Joseph-Louis Lagrange discovered two others, and the five are now known as Lagrange points. Although each represents a special orbit around the Sun, they are called "points" because they appear as fixed locations when viewed in a reference frame that rotates at the same rate that the Earth and Sun orbit around their center of mass (a point deep inside the Sun). Five such special spots, designated L_1 through L_5, exist for every pair of massive bodies—the Sun and a planet, a planet and one of its moons, and so on. L_1 corresponds to the inner balance point for the marble described above (the one located between the Earth-funnel and Sun-funnel); L_2 corresponds to the outer balance point. L_1 and L_2 are of direct interest for understanding the interplanetary transport network, because they form key gateways to faraway destinations.

Although L_1 and L_2 are classified as unstable points, that categorization can be misleading, because spacecraft can stick around these points for long periods of time. Indeed, the delicate interplay of gravitational and rotational forces allows a spacecraft to move about these points, "orbiting" L_1 or L_2 in the rotating frame of reference, even though there is no material object there. Although such orbits around a mere point in space appear very bizarre, they are, in fact, nothing more than near misses to being exactly on L_1 or L_2 and moving at just the right velocity for perfect balance.

To understand better, imagine that a space probe was orbiting around the Sun close to Earth's L_2 point but just a little bit to the inside of it. Assume too that it was moving just a little bit faster than it needed to go had it been positioned right on L_2. What's it going to do? Again, visualizing a marble circling around in a double-funnel arrangement helps. A marble with the corresponding position and velocity to this space probe would start to move ahead in its orbit around the Sun-funnel (ahead compared to L_2); it would also tend to be flung outward slightly. But the surface around here has a strange shape. So as the marble moves slightly ahead and outward, the surface in front of it rises, causing the marble to slow. Soon L_2 catches up with it (on the inside). The marble then begins to trail L_2 and encounters another rising surface behind, which acts to speed the marble up and to scoot it toward the Sun, just as a wave propels a surfer toward the beach (and often a little sideways). So the marble ends up pretty much where it began and with about the same velocity. It might successfully "orbit" L_2 a few times in this manner before either being flung outward or falling off into the nearby Earth-funnel.

A spacecraft positioned near L_1 can act similarly. Viewed from the perspective of someone on Earth, the craft would appear to orbit Earth's L_1 Lagrange point for a while and then go shooting off toward Earth or around the Sun—all without expending any fuel. Some of the possible orbits about these two Lagrange points lie in the plane of Earth's orbit. Others, like the one followed by the Genesis probe, are

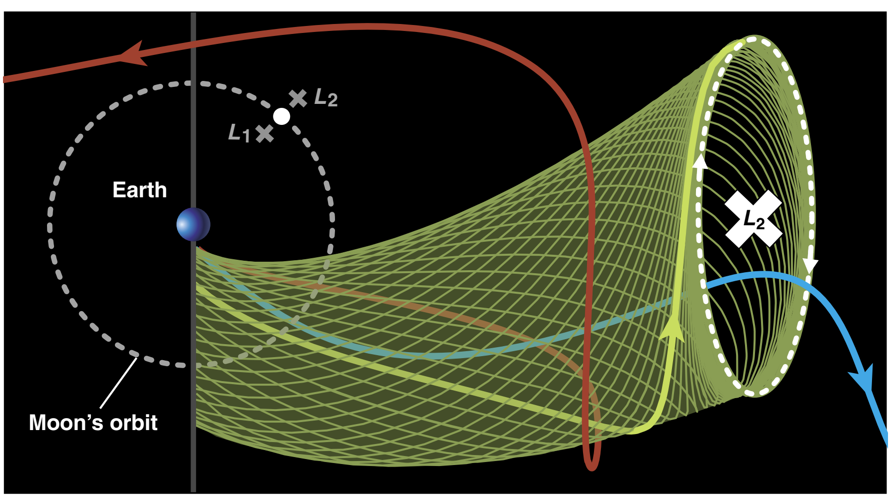

Figure 6. A spacecraft given the proper initial velocity can be sent along a trajectory that would then carry it into orbit around, for example, Earth's L_2 Lagrange point *(pale green line)*. A collection of similar trajectories (any point on which the spacecraft would have a specific position and velocity) constitutes one "tube" of the interplanetary transport network *(green mesh)*. A spacecraft on a trajectory inside this tube will pass L_2 and head toward the outer solar system *(blue line)*, whereas one on a trajectory to the outside will fly back toward the Sun *(red line)*.

three-dimensional and have a variety of spiraling shapes, dipping into and out of the orbital plane of the two massive bodies.

Surfing Between Planets

At the end of the 19th century, the French mathematician Henri Poincaré made significant strides in understanding of celestial mechanics at work here. Poincaré was the first to appreciate the complicated motion of the third body that could result. The geometric methods he used to come to this conclusion laid the foundation for what is now known as nonlinear dynamics, more generally called chaos theory. It is important to keep in mind that "chaotic" does not mean random. Chaotic-looking paths exist in this problem, but they are nevertheless predictable, at least for a while into the future. So a mission designer with sufficient understanding can take full advantage of them to work out various low-energy routes through space. Poincaré brought order to the chaos by organizing similar paths into special collections of surfaces, which exist in what mathematicians refer to as a "six-dimensional phase space," one that includes the three dimensions of normal space (say, x, y and z) and three dimensions for an object's velocity in each direction.

Building on Poincaré's work, in the late 1960s Charles C. Conley (a mathematician then at the University of Wisconsin) discovered a collection of tube-shaped surfaces for objects under the gravitational influence of two mutually orbiting bodies, a result later pursued by Robert P. McGehee, then Conley's student and now at the University of Minnesota. An object located on one of these six-dimensional tubes (which is to say, having just the right position and velocity) will naturally be carried toward or away from a trajectory that orbits about the L_1 or L_2 Lagrange points as seen in the rotating frame of reference. Trajectories on the inside of such a tube snake past the Lagrange point, whereas those on the outside end up with the object bouncing back.

Starting in the mid-1990s, I have worked with Martin W. Lo of NASA's Jet Propulsion Laboratory (JPL) and Wang Sang Koon and Jerrold E. Marsden of the California Institute of Technology to extend this approach. We've shown that the important physical property of these tubes is that anything that shifts from an orbit that is inside a planet's orbit to an orbit that lies outside must pass along them. Like water directed by a hose, the set of possible planet-passing objects is imagined to flow along these tubes, but in six dimensions instead of just three. The comparison with fluids is more than just analogy. Indeed, computational tools originally designed by Francois Lekien of Princeton University and his co-workers for computing dynamical channels in the ocean have been used to ascertain low-energy trajectories in the celestial context as well.

Computing the configuration of these tubes out farther than Conley or McGehee were able to do, my colleagues and I found that they extend far from their region of origin (the vicinity of L_1 or L_2) and wind around whatever two massive bodies are being considered, stretching and twisting along the way. One can think of there being a gateway region around L_1 and another around L_2, with the tubes being the passageways in and out of the domain of the planet or moon. Another property of objects traveling along such a tube is that they will move at their slowest relative to the nearby planet or moon when in the gateway, which can be thought of as a region of near-equilibrium, the top of an energetic hill that objects must climb and overcome.

It turns out that Oterma's strange path lies along the tubular passageways connected to Jupiter's L_1 and L_2 Lagrange points—almost as if the com-

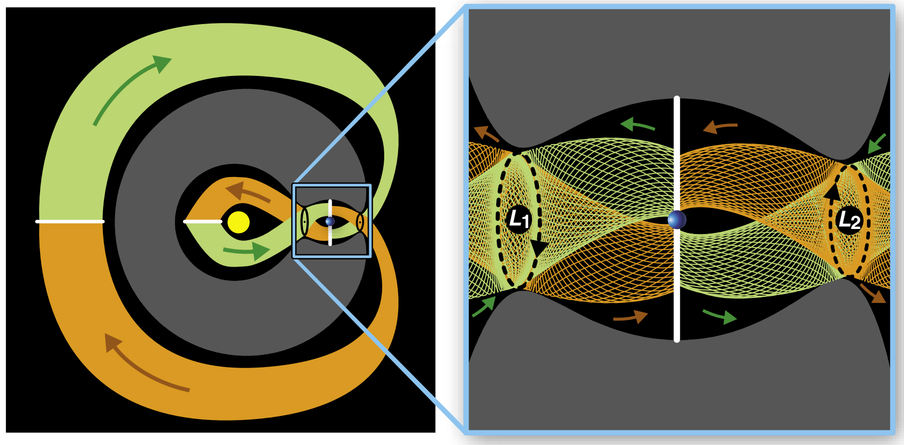

Figure 7. Some tubes of the interplanetary transport network lead objects into orbit around Earth's L_1 and L_2 Lagrange points *(green trajectories, right)*, whereas others lead objects away from such orbits *(orange)*. Mission planners can make use of the intersection of these incoming and outgoing tubes, directing a spacecraft to hop from one tube to another in a way that allows it to travel between L_1 and L_2 or into orbits around the Sun that can be smaller or larger than that of Earth *(left)*. However, a spacecraft with limited kinetic energy (say, just the amount needed to orbit L_1 or L_2) cannot visit certain regions *(gray)* no matter what tube pathway it then follows.

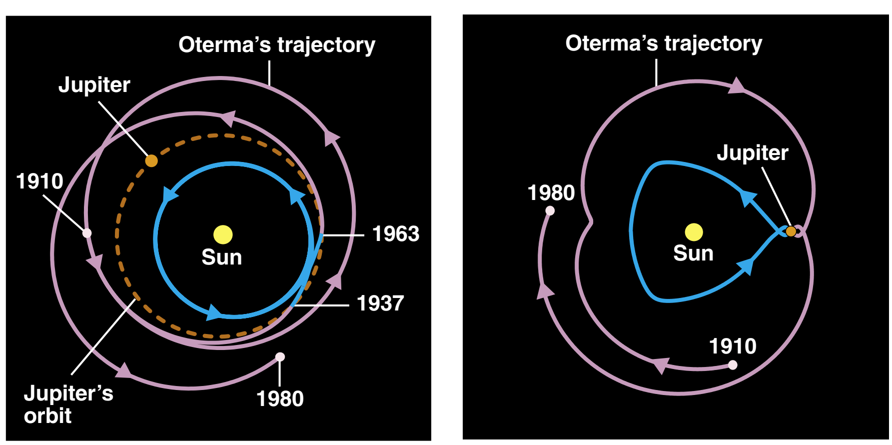

Figure 8. Comet Oterma followed the interplanetary transport network from an orbit that was outside Jupiter's in 1910 to an orbit that was inside Jupiter's between 1937 and 1963, when it once again shifted to an outside track. Its curious route through space is shown here in both fixed *(left)* and rotating *(right)* reference frames.

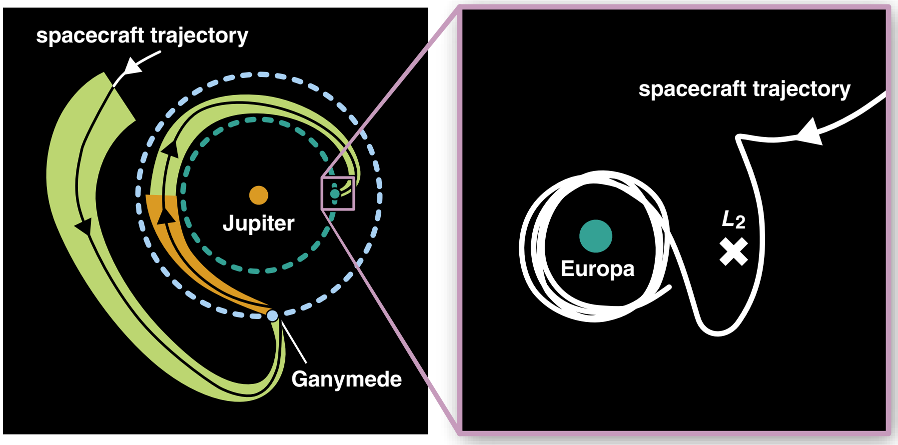

Figure 9. Exploration of Jupiter's icy moons could benefit from a cleverly designed trajectory. A probe could, for example, enter the Jovian system along an inbound tube *(outer green swath at left)* that carried it toward Jupiter's moon Ganymede, which it would orbit briefly before following an outbound tube *(orange)* that conveyed it into an orbit around Jupiter that was smaller than Ganymede's. The probe would then hop to an inbound tube toward Jupiter's moon Europa *(inner green swath)*, which it would then orbit for a significant time *(right)*.

et had followed an interplanetary subway tunnel linking distantly separated regions of space. Portions of these passageways can run into the planet itself. Oterma's cousin comet Shoemaker-Levy 9 may have been traveling in just such a tube when it broke up and collided with Jupiter in 1994.

Hitchhiker's Guide

If you could hitch a ride on Oterma or one of the other natural objects that travels the tube-highway between the planets, you could get around the solar system for free. But why wait for the right asteroid to come by? All you need to do is direct your spacecraft into one of these celestial conduits. Traveling in these passageways would slash the amount of fuel required to explore the solar system. The place to start to look for such opportunities is right around Earth.

Because it has an open view of the cosmos, Earth's L_2 Lagrange point is well suited for deep-space telescopes, whereas the region around L_1, because of its unobstructed view of the Sun, is a good place to put instruments for doing solar science. Indeed, part of the reason that the Genesis mission was feasible was its special "halo orbit" around Earth's L_1 Lagrange point. As viewed from the vantage point of someone on Earth, Genesis moved in a halo around the Sun. Such orbits were originally named for lunar halo orbits by their discoverer in the 1960s, Robert Farquhar of Johns Hopkins University's Applied Physics Laboratory, who was the driving force behind the first Lagrange-point mission, the International Sun-Earth Explorer 3.

Genesis took a low-energy passageway to its halo orbit, stayed there while collecting samples and then found its way home on another low-energy path that looped by L_2. Using their knowledge of the tubes, lead mission designer Lo, along with Purdue University's Kathleen C. Howell and Brian Barden (who was then Howell's student), found a way for Genesis to achieve this exotic trajectory using hardly any fuel. That feat created a great deal of interest in both the astronautical and mathematical communities.

In particular, the work on Genesis inspired Lo and me to explore the dynamics of Earth's neighborhood in a deeper way. We recognized that Lagrange points L_1 and L_2 in both the Sun-Earth and Earth-Moon systems are important hubs and destinations. Fortunately, the tubes connecting the neighborhoods of these four Lagrange points are such that they sometimes intersect one another. Once each month or so, halo orbits around the Moon's L_1 and L_2 Lagrange points connect to halo orbits around the Earth's L_1 or L_2 points via low-fuel, or even fuel-free, pathways. The implications of this fortuitous arrangement for the exploration and development of the solar system are enormous.

Lo and I, along with colleagues at NASA, have championed the idea that a permanent space station be established at the lunar L_1 Lagrange point to serve as a transportation hub, one that could help considerably in advancing spacefaring activities beyond low-Earth orbit. The station would be the closest rest stop on the interplanetary superhighway. From there cargo could be sent in slow but energy-efficient, low-thrust freighters, whereas astronauts would travel in higher-speed vehicles. Spacecraft leaving the facility could reach any point on the lunar surface within hours, making it a perfect way station for the return of people to the Moon. This gateway would also be an excellent point of departure and arrival for conventional interplanetary flights to Mars, the asteroids and the outer solar system. Natural paths for journeying between planets without using fuel exist too, but they require thousands of years to get you to your destination. Only asteroids, comets and Martian meteorites (rocks blasted off Mars that later landed on Earth) have the patience for that.

Future space telescopes destined for deployment near Earth's L_1 or L_2 points could be assembled at this station and conveyed to their final destinations using very little fuel. And when these instruments require servicing, they could be returned to the vicinity of the station, again without costing much fuel.

But the exploitation of low-energy passageways is in no way limited to near-Earth space. I'm part of an international team (one that includes Koon, Marsden, Lo, Gerard Gómez of the University of Barcelona and Josep Masdemont of the Technical University of Catalonia, also in Barcelona) that has proposed a new class of space missions. Our idea is that a single spacecraft could orbit several of the moons of any one of the outer planets, allowing for long-duration observations. For example, a multi-moon orbiter could explore Jupiter's planet-sized and likely water-bearing moons—Callisto, Ganymede and Europa—one after the other, taking a path that uses a technologically feasible amount of fuel. NASA had been considering just such a project, dubbed the Jupiter Icy Moons Orbiter, which would exploit linkages among the low-energy tubes of Jupiter and its moons, but funding for that mission was slashed last year, and its prospects are in doubt.

From Atoms to Galaxies

The growing understanding of the restricted three-body problem and the dynamics associated with Lagrange points will surely aid in the exploration and development of space. But it turns out that the idea of low-energy passageways has broader application. That realization began in 2000, when Charles Jaffé, a chemist at West Virginia University, observed that under proper experimental conditions, the paths taken by valence electrons in *Rydberg atoms* (whose valence electrons orbit far from an ionized atomic core) look a lot like the trajectory of the Genesis probe. And it indeed turns out that when subjected to electric and magnetic fields that are perpendicular, Rydberg electrons also follow tubular pathways. Jaffé teamed up with me, Marsden, Lo, Turgay Uzer of the Georgia Institute of Technology and David Farrelly of Utah State University to apply techniques from statistical chemistry to study the fate of Martian material shot into space as a result of an impact on that planet. This work was the first application of a well-known technique from chemistry to celestial mechanics.

This cross-fertilization has gone in the other direction as well. In collaboration with Koon, Marsden, Tomohiro Yanao of Caltech, Frederic Gabern of the University of Barcelona and a group headed by Michael Dellnitz of the University of Paderborn in Germany and Oliver Junge at the Munich University of Technology, I've been working to develop mathematical and computational foundations of a reaction-rate theory that overcomes some of the classical difficulties encountered in chemistry. This work was inspired by computations of transport in the solar system along tubes and related geometrical techniques. It is the underlying mathematics, of course, that provides the link between chemistry and planetary-system dynamics.

Tubes are known to govern structure and motion over galactic scales too, as Toshi Fukushige of the University of Tokyo and Douglas Heggie of the University of Edinburgh have shown that tubes related to Lagrange points lead to the "evaporation" of small star clusters in orbit around some galaxies.

Even more dramatic examples of tube-like structures occur when two galaxies interact strongly. About 420 million light-years away, the galaxy Arp 188, otherwise known as the Tadpole galaxy, reveals evidence of a brief but violent episode in its past. The huge tail stretching out of the Tadpole marks where stars slipped into tubes connecting it with an intruder galaxy, one that has since moved and is now mostly hidden from view. The Tadpole's tail is thus a 280,000-light-year bridge to nowhere, but some other galaxy pairs (such as the "Mice" galaxies) show tubelike conduits connecting them.

Although they have not been charted yet, one would expect that similar tubes connect the solar system with neighboring stars. Imagine if one of the two Voyager probes, which have now left the solar system, has entered a tube heading toward a region of force balance between the Sun and, say, Alpha Centauri, which is several light-years distant. That spacecraft might get a free ride all the way to another star. Even so, at the rate the Voyagers are going, they wouldn't reach that destination for thousands of years. However, in the distant past other stars have come much closer to the Sun than our current nearest neighbors. It is likely that exchange of material between our solar system and such wandering stellar systems has occurred, the tubes being the invisible channels of exchange. Fans of Douglas Adams should thus take heart: Although it might take a very long time, hitchhiking around the galaxy may indeed be possible.

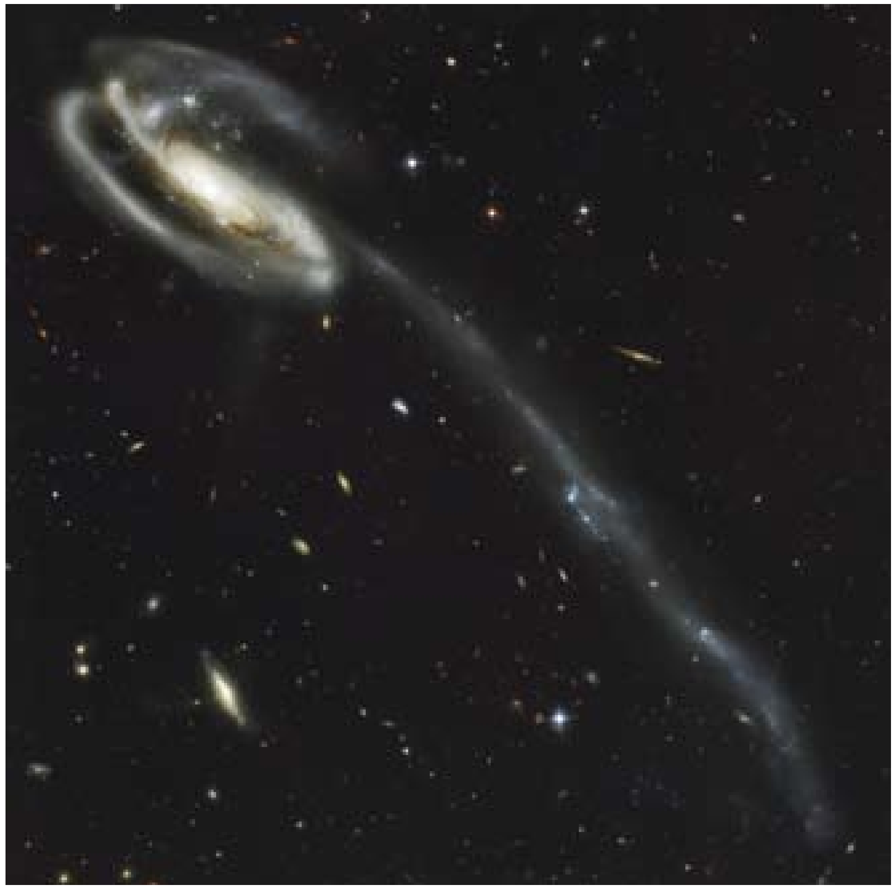

Figure 10. Stars stream outward from the Tadpole Galaxy (Arp 188) along a tubelike channel that stretches for some 280,000 light-years. This conduit (the galactic equivalent of the tubes making up the interplanetary transport network) arose through gravitational interaction with a compact galaxy that can now be seen lurking behind one of the Tadpole's spiral arms. (Courtesy of ACS Science & Engineering Team and NASA.)

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