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Application of Dynamical Systems Theory to a Very Low Energy Transfer

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APPLICATION OF DYNAMICAL SYSTEMS THEORY TO A VERY LOW ENERGY TRANSFER*

S.D. Ross†, W.S. Koon‡, M.W. Lo§, J.E. Marsden¶

Abstract

We use lobe dynamics in the restricted three-body problem to design orbits with prescribed itineraries with respect to the resonance regions within a Hill's region. The application we envision is the design of a low energy trajectory to orbit three of Jupiter's moons using the patched three-body approximation (P3BA). We introduce the "switching region," the P3BA analogue to the "sphere of influence." Numerical results are given for the problem of finding the fastest trajectory from an initial region of phase space (escape orbits from moon A) to a target region (orbits captured by moon B) using small controls.

INTRODUCTION

Low energy trajectories have been increasingly investigated, due to the possibility of large savings in fuel cost (as compared to classical approaches) by using the natural dynamics arising from the presence of a third body. Recent work by our group gives a rigorous explanation of these phenomena by applying some techniques from dynamical systems theory to systems of n bodies considered three at a time.[1–3] We obtain a systematic way of designing trajectories with a predetermined future and past, in terms of transfer from one Hill's region to another. One of the examples we have considered is an extension of the Europa Orbiter mission[4–6] to include an orbit around Ganymede.[7] More recently, we have considered a mission in which a single spacecraft orbits *three* of Jupiter's planet-size moons—Callisto, Ganymede and Europa—one after the other, using very little fuel.[8] Using our approach, which we have dubbed the "Multi-Moon Orbiter" (MMO), a scientific spacecraft can orbit several moons for any desired duration, instead of flybys lasting only seconds. Our approach

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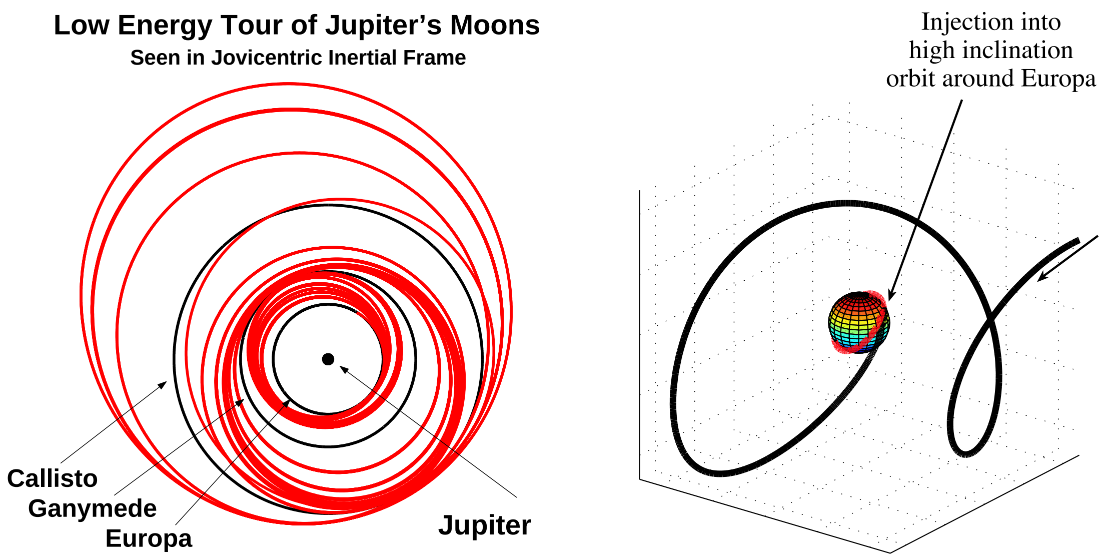
should work well with existing techniques, enhancing NASA's trajectory design capabilities for missions such as the Jupiter Icy Moons Orbiter.

The main concern of this study is not to construct flight-ready end-to-end trajectories, but rather to determine the fuel consumption versus time of flight trade-off for a MMO mission using models which provide dynamical insight and are computationally tractable. The fuel requirements in terms of the sum of all velocity changes (ΔV) are greatly reduced by including multiple gravity assist (GA) maneuvers with the jovian moons. For instance, by using multiple GAs, we have found tours with a deterministic ΔV as low as $\sim$20 m/s as compared to $\sim$1500 m/s using previous methods.[4,5] In fact, this extremely low ΔV is on the order of statistical navigation errors. The lowest energy MMO tour is shown in Figure 1. By using small impulsive maneuvers totaling only 22 m/s, a spacecraft initially injected into a jovian orbit can be directed into an inclined, elliptical capture orbit around Europa. Enroute, the spacecraft orbits both Callisto and Ganymede for long duration using a ballistic capture and escape methodology developed previously.[7] This way of designing missions is called the *patched three-body approximation* (P3BA) and will be elaborated upon further in this paper.



(a) (b)

Figure 1: **The Multi-Moon Orbiter space mission concept for the jovian moons** involves long duration orbits of Callisto, Ganymede, and Europa, allowing for extensive observation. Starting in an elliptical jovian orbit with perijove near Callisto's orbit, the spacecraft trajectory gets successively reduced in jovicentric energy by resonant gravity assists with the various moons, effectively jumping to lower resonances at each close approach, as shown in (a). The trajectory has its jovicentric energy reduced by Callisto, Ganymede, and Europa, in sequence. As the orbit converges upon the orbit of Europa, it will get ballistically captured by Europa. Small corrections during the tour add up to a total ΔV of about 20 m/s, on the order of statistical navigation errors. At the end of the tour phase, the spacecraft is at a 100 km altitude periapse with respect to Europa. A ΔV of approximately 450 m/s is then needed to get into a 100 km altitude circular orbit about Europa, with an inclination of about 45°, as shown in (b).

Trade-Off Between Fuel and Time Optimization

The dramatically low ΔV needed for the tour of Figure 1 is achieved at the expense of time—the present trajectory has a time of flight (TOF) of about four years, mostly spent in the inter-moon transfer phase. This is likely too long to be acceptable for an actual mission. With refinement, we believe the method could be applied to an actual mission, maintaining both a low ΔV for the tour and low accumulated radiation dose (a concern for an actual mission in the jovian system). Therefore, in this paper we explore the ΔV vs TOF trade-off for the inter-moon transfer between Ganymede and Europa. We find a roughly linear relationship between ΔV vs TOF, and that a reasonable TOF for a MMO can be achieved using a feasible ΔV.

THE BUILDING BLOCKS FOR DETERMINING THE ΔV VS TIME OF FLIGHT TRADE-OFF

In order to make this trade-off study computationally tractable, one needs to use simplified models. The forward-backward method in the restricted three-body problem phase space is used.[7,9] The influence of only one moon at a time is considered. Criteria are established for determining when the switch from one moon's influence to another occurs.

Much evidence[1–3] suggests that the use of invariant manifold structures related to L_1 and L_2 Lagrange points (e.g., "tubes") yields fuel efficient impulsive trajectories. Using the planar circular restricted three-body problem as our baseline model, we will compute tubes over a range of three-body energies (i.e., Jacobi constants). The tubes are the passageways leading toward or away from the vicinity of L_1 and L_2, and therefore toward or away from the Hill's region around the smaller primary. The tubes have the numerically observed property that the larger the energy, the further the tube travels from its associated Lagrange point in a fixed amount of time. We will use this property to find the TOF between Ganymede and Europa as a function of the energy in their respective three-body systems. These energies can be used to calculate the ΔV of escape from each moon under certain assumptions.

First, we review the P3BA and the dynamics in the circular restricted three-body problem.

The Patched Three-Body Approximation (P3BA)

The P3BA discussed by Ross et al.[8] considers the motion of a particle (or spacecraft, if controls are permitted) in the field of n bodies, considered two at a time, e.g., Jupiter and its ith moon, M_i. When the trajectory of a spacecraft comes close to the orbit of M_i, the perturbation of the spacecraft's motion away from purely Keplerian motion about Jupiter is dominated by M_i. In this situation, we say that the spacecraft's motion is well modeled by the Jupiter-M_i-spacecraft restricted three-body problem. Within the three-body problem, we can take advantage of phase space structures such as tubes of capture and escape, as well as lobes associated with movement between orbital resonances. Both tubes and lobes, and the dynamics associate with them, are important for the design of a MMO trajectory.

The design of a MMO of the jovian system is guided by three main ideas.[7,8]

1. The motion of the spacecraft in the gravitational field of the three bodies Jupiter, Ganymede, and Europa is approximated by two segments of purely three body motion

in the circular, restricted three-body model. The trajectory segment in the first three body system, Jupiter-Ganymede-spacecraft, is appropriately patched to the segment in the Jupiter-Europa-spacecraft three-body system.

2. For each segment of purely three body motion, the invariant manifolds tubes of L_1 and L_2 bound orbits (including periodic orbits) leading toward or away from temporary capture around a moon, are used to construct an orbit with the desired behaviors. Portions of these tubes are "carried" by the lobes mediating movement between orbital resonances. Directed movement between orbital resonances is what allows a spacecraft to achieve large changes in its orbit. When the spacecraft's motion, as modeled in one three-body system, reaches an orbit whereby it can switch to another three-body system, we switch or "patch" the three-body model to the new system.

3. This initial guess solution is then refined to obtain a trajectory in a more accurate four-body model. Evidence suggests that these initial guesses are very good,[8] even in the full n-body model and considering the orbital eccentricity of the moons.[10]

Tube Dynamics: Ballistic Capture and Escape

The tubes referred to above are cylindrical stable and unstable invariant manifolds associated to bounded orbits around L_1 and L_2. They are the phase space structures that mediate motion to and from the smaller primary body, e.g., mediating spacecraft motion to and from Europa in the Jupiter-Europa-spacecraft system. They also mediate motion between primary bodies for separate three-body systems, e.g., spacecraft motion between Europa and Ganymede in the Jupiter-Europa-spacecraft and the Jupiter-Ganymede-spacecraft systems. Details are discussed extensively in Koon et al.[1,2] and Gómez et al.[11]

Inter-Moon Transfer and the Switching Orbit

During the inter-moon transfer—where one wants to leave a moon and transfer to another moon, closer in to Jupiter—we consider the transfer in two portions, shown schematically in Figure 2, with M_1 as the outer moon and M_2 as the inner moon. In the first portion, the transfer determination problem becomes one of finding an appropriate solution of the Jupiter-M_1-spacecraft problem which jumps between orbital resonances with M_1, i.e., performing resonant GA's to decrease the perijove.[8] M_1's perturbation is only significant over a small portion of the spacecraft trajectory near apojove (A in Figure 2(a)). The effect of M_1 is to impart an impulse to the spacecraft, equivalent to a ΔV in the absence of M_1.

The perijove is decreased until it has a value close to M_2's orbit, in fact, close to the orbit of M_2's L_2. We can then assume that a GA can be achieved with M_2 with an appropriate geometry such that M_2 becomes the dominant perturber and all subsequent GA's will be with M_2 only. The arc of the spacecraft's trajectory at which the spacecraft's perturbation switches from being dominated by moon M_1 to being dominated by M_2 is called the "switching orbit." A rocket burn maneuver need not be necessary to effect this switch. The set of possible switching orbits is the "switching region" of the P3BA. It is the analogue of the "sphere of influence" concept used in the patched-conic approximation, which guides a mission designer regarding when to switch the central body for the model of the spacecraft's Keplerian motion.

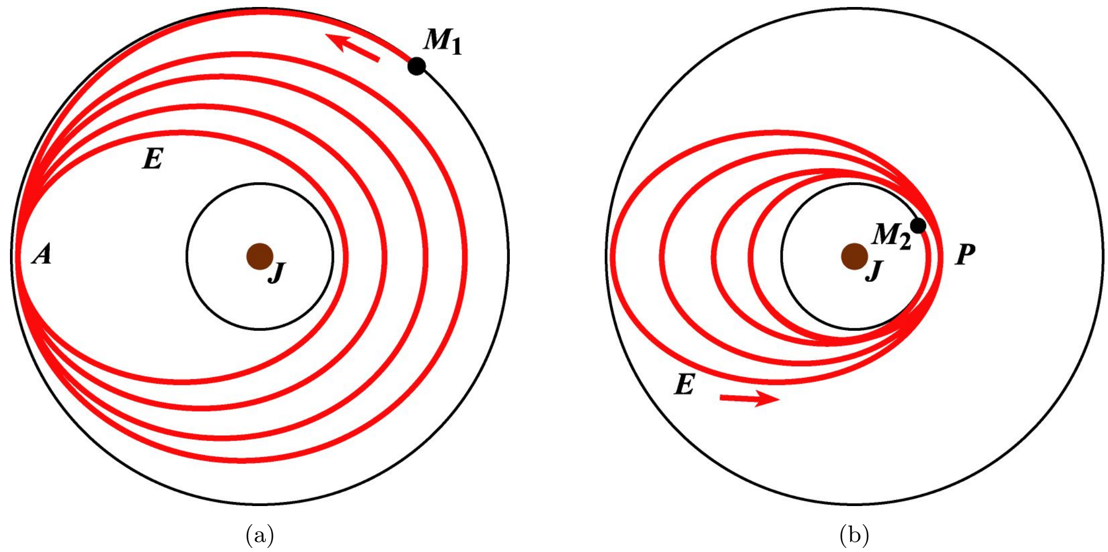


Figure 2: **Inter-moon transfer via resonant gravity assists.** (a) The orbits of two Jovian moons are shown as circles. Upon exiting the outer moon's (M_1's) sphere-of-influence, the spacecraft proceeds under third body effects onto an elliptical orbit about Jupiter. The spacecraft gets a gravity assist from the outer moon when it passes through apojove (denoted A). The several flybys exhibit roughly the same spacecraft/moon geometry because the spacecraft orbit is in near-resonance with the moon's orbital period and therefore must encounter the moon at about the same point in its orbit each time. Once the spacecraft orbit comes close to grazing the orbit of the inner moon, M_2 (in fact, grazing the orbit of M_2's L_2 point), the inner moon becomes the dominant perturber. The spacecraft orbit where this occurs is denoted E. (b) The spacecraft now receives gravity assists from M_2 at perijove (P), where the near-resonance condition also applies. The spacecraft is then ballistically captured by M_2.

The spacecraft orbit where M_2 takes over as the perturbing moon is denoted E in Figures 2(a) and 2(b). The spacecraft now gets GA's from the inner moon at perijove (P). One can then search for solutions of the Jupiter-M_2-spacecraft problem which cause the apojove to decrease at every close encounter with M_2, causing the spacecraft's orbit to get more and more circular, as in Figure 2(b). When a particular resonance is reached, the spacecraft can then be ballistically captured by the inner moon at M_2.[1] We note that a similar phenomenon has been observed in previous studies of Earth to lunar transfer trajectories.[9,12]

Resonant Structure of Phase Space and Lobe Dynamics

Solutions to the four-body problem which lead to the behavior shown schematically in Figure 2 have been found numerically and the phenomena partially explained in terms of the P3BA.[8] The switching region between neighboring pairs of moons can only be accessed by traversing several subregions of the three-body problem phase space, known as "resonance regions," where the resonance is between the spacecraft orbital period and the dominant moon's orbit period around Jupiter, respectively.

Early investigation into the phase space of the restricted three-body problem using Poincaré sections has revealed a phase space consisting overlapping resonance regions.[9,13]

This means that movement amongst resonances is possible.

Lobe dynamics provides a general theoretical framework, based on invariant manifold ideas from dynamical systems theory, for discovering, describing and quantifying the transport "alleyways" connecting resonances.[14] A resonance region and the lobes of phase space associated with movement around it are shown in Figure 3 on a Poincaré section in quasi-action-angle coordinates. The lobes are defined using the stable and unstable manifolds associated to unstable resonant orbits. Starting in one of the lobes above the resonance, an initial condition can get transported to below the resonance, and vice versa. This corresponds decrease or increase in the spacecraft's semimajor axis for zero fuel cost.

Lobe dynamics tells us the most important spacecraft trajectories, i.e., the uncontrolled trajectories which traverse the resonance regions in the shortest time. Consequently, it proves useful for designing low energy spacecraft trajectories, such as shown in Figure 4(a). An initial condition in the upper right hand side of Figure 4(a) moves through the phase space as shown, jumping between resonance regions under the natural dynamics of the three-body problem, i.e., at zero fuel cost. Figure 4(b) shows a schematic of the corresponding trajectory in inertial space.

In this study, we will use tube dynamics along with lobe dynamics to find uncontrolled trajectories which quickly traverse the space between moons during the inter-moon transfer phase. Essentially, the lobes act as templates, guiding pieces of the tube across resonance

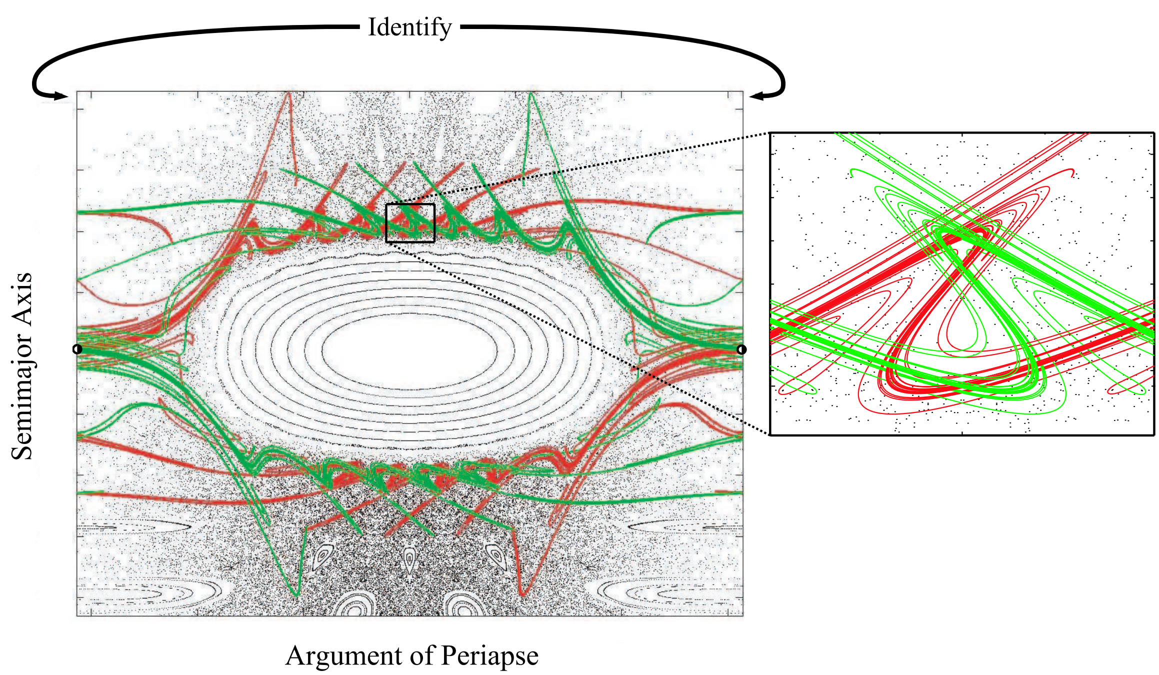
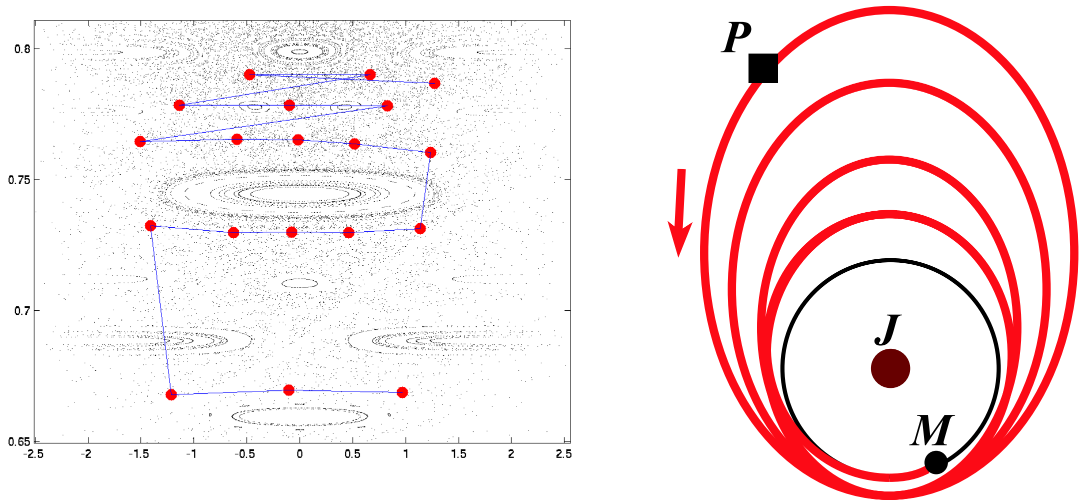


Figure 3: **Movement across an orbital resonance using lobe dynamics.** An unstable resonant orbit appears as a hyperbolic fixed point (the half-filled circles, identified as the same point) on this Poincaré section. The orbit's stable and unstable manifolds define both the resonance region (the central region with the target pattern) and "lobes" which transport phase points from above the resonance to below, and vice versa. An enlargement of the boxed region is shown at right.



(a)

(b)

Figure 4: **Jumping between resonance regions leads to large orbit changes at zero cost.** An initial condition in the upper right hand side of (a) moves through the phase space as shown, jumping between resonance regions. In (b), a schematic of the corresponding trajectory of a spacecraft P in inertial space is shown. Jupiter (J) and one of its moons (M) are also shown schematically.

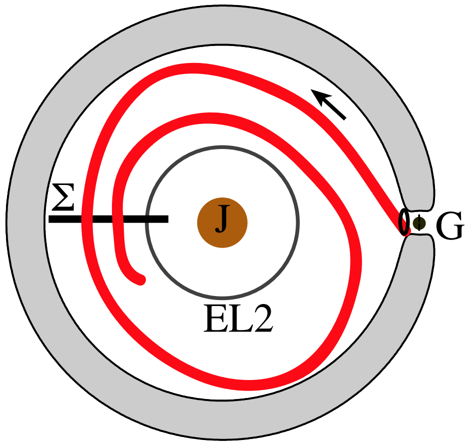
regions. We can numerically determine the fastest trajectory from an initial region of phase space (e.g., orbits which have just escaped from moon M_2) to a target region (e.g., orbits which will soon be captured by a neighboring moon M_2). This yields the ΔV vs. TOF trade-off for the inter-moon transfer between Ganymede and Europa.

NUMERICAL RESULTS: THE ΔV VS TIME OF FLIGHT TRADE-OFF

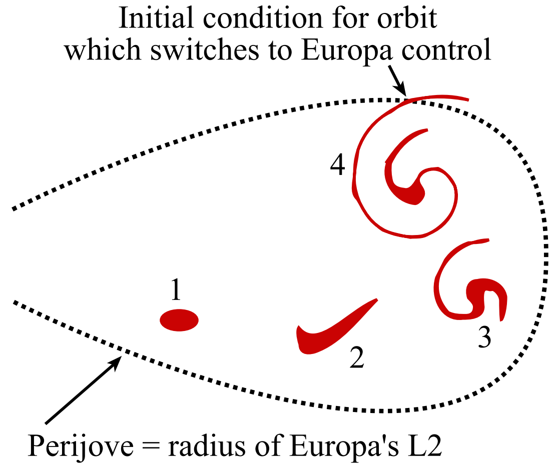
Method Description

In order to do a trade study of transfers between orbits around Ganymede and Europa, we can initially consider an impulsive transfer from a Ganymede L_1 orbit (denoted Ga_{L1}) to a Europa L_2 orbit (Eu_{L2}). If we find such a transfer, we know that a transfer between orbits around Ganymede and Europa is nearby in phase space.[1] We can break the transfer into two pieces.

1. In the first piece, we consider the transfer along the unstable manifold tube of a Ga_{L1}, which we denote $U(\mathrm{Ga}_{L1})$. The object $U(\mathrm{Ga}_{L1})$ has two branches, but we consider the one, as shown in Figure 5(a), heading initially in the direction of Europa's orbit. The set of all Ga_{L1}'s is parameterized by the energy E_{Ga}, one of our tunable parameters. For each E_{Ga}, one can compute the Ga_{L1} and $U(\mathrm{Ga}_{L1})$. By taking a Poincaré section such as Σ in the figure, one can determine the trajectory within $U(\mathrm{Ga}_{L1})$ which takes the least time to transfer to a perijove distance r_p, equal to the approximate radial distance from Jupiter of Europa's L_2 point, labeled EL2 in the figure. This is performed numerically by determining the minimum tube crossing number on Σ which



(a)



(b)

Figure 5: **Numerical construction of natural trajectory arcs which will switch control from Ganymede to Europa.** Suppose we want to find trajectories which begin near Ganymede (G in the figure) and escape toward Europa, finally getting naturally captured by Europa. The first step is to numerically construct the Ganymede L2 tube heading toward Europa, or $U(\mathrm{Ga}_{L1})$ in the terminology of the text. We take a Poincaré section, Σ, at the position shown in (a). We show only two crossings of Σ, but there are an infinite number. We also show the radial distance of Europa's L_2, labeled EL2, and the forbidden region at this energy; the gray "C" shape. In (b), we show a schematic of the initial cross-section of the tube on Σ, labeled 1. The successive crossings are labeled 2, 3, ... In this schematic, we also show the dotted line corresponding to a perijove equal to the radial distance of Europa's L_2. The first three crossings are entirely within the zone of perijoves greater than the radial distance of Europa's L_2. Any spacecraft trajectory in the Jupiter-Ganymede-spacecraft system which crosses this line can be assumed to "switch" control to Europa, meaning the Jupiter-Europa-spacecraft system becomes a good approximation from then on. The coordinates represented here are not quasi-action-angle coordinates as in Figures 3 and 4(a), but are related to cartesian coordinates, which are easier to handle numerically.

crosses the aforementioned perijove distance, the dotted line shown in Figure 5(b). Thus we find the natural trajectory arc which will switch "control" from Ganymede to Europa as the main perturber from jovicentric motion. The time of flight of this portion of the inter-moon transfer trajectory, T_{Ga}, is seen numerically to be a function of E_{Ga}.

2. For the second piece, we consider the transfer along the branch of the stable manifold tube of a Eu_{L2}, denoted $S(\mathrm{Eu}_{L2})$, heading initially in the direction of Ganymede's orbit. The set of all Eu_{L2}'s is parameterized by the energy E_{Eu}, another tunable parameter. For each E_{Eu}, one can compute the Eu_{L2} and $S(\mathrm{Eu}_{L2})$. One can determine the trajectory within $S(\mathrm{Eu}_{L2})$ which takes the least time to transfer to an apojove distance r_a, equal to the approximate radial distance from Jupiter of Ganymede's L_1 point. The time of flight of this trajectory, T_{Eu}, is found numerically to be a function of E_{Eu}.

The sum, TOF $= T_{\text{Ga}} + T_{\text{Eu}}$, is an approximate inter-moon transfer time. The total fuel expenditure, ΔV_{tot}, needed to perform the transfer can be estimated as follows. We assume only two impulsive maneuvers, ΔV_{Ga} and ΔV_{Eu}.

$\Delta V_{\text{Ga}} =$ the ΔV to escape from the scientific orbit around Ganymede, which can be estimated using the energies of the transfer away from Ganymede, E_{Ga}, and of the scientific orbit at Ganymede, E_{GaO}.

$\Delta V_{\text{Eu}} =$ the ΔV to enter the scientific orbit around Europa, which can be estimated using energies of the transfer toward Europa, E_{Eu}, and of the scientific orbit at Europa, $E_{\text{Eu}}O$.

The total fuel expenditure is the sum, $\Delta V_{\text{tot}} = \Delta V_{\text{Ga}} + \Delta V_{\text{Eu}}$. We suppose that E_{GaO} and E_{EuO} are given. We can then perform this procedure for a range of tunable parameters (E_{Ga} and E_{Eu}), to determine the fuel consumption (ΔV_{tot}) versus time of flight (TOF) trade-off.

Computing the ΔV's

We assume that portions of each tube quickly reach a periapse of 100 km altitude above each moon, and that the solutions which do this are close in phase space to the transfer solutions found, assumptions justified by earlier work.[1,11] Given these assumptions, we can estimate ΔV_{Ga} and ΔV_{Eu} as follows. In the rotating frame of a Jupiter-moon-spacecraft three-body system, a spacecraft with a velocity magnitude v has a three-body energy

$$E = \frac{1}{2}v^2 + \bar{U}, \tag{1}$$

where the effective potential, a function of position, is

$$\bar{U} = -\frac{1}{2}r^2 - \frac{1-\mu}{r_J} - \frac{\mu}{r_M}, \tag{2}$$

where μ is the mass ratio $\frac{m_M}{m_J+m_M}$, r_J is the spacecraft's distance from Jupiter's center, r_M the spacecraft's distance from the moon's center, and r the spacecraft's distance from the Jupiter-moon center of mass, which is very close to Jupiter. At a distance of 100 km altitude above the moon, we are very close to the moon. Therefore, using the standard non-dimensional units, $r \approx r_J \approx 1$, and we can approximate Eq. (2) as

$$\begin{aligned} \bar{U} &\approx -\frac{1}{2}(1)^2 - \frac{1-\mu}{1} - \frac{\mu}{r_M}, \\ \bar{U} &\approx -\frac{1}{2} - 1 - \frac{\mu}{r_M}, \\ \bar{U} &\approx -\frac{3}{2} - \frac{\mu}{r_M}. \end{aligned} \tag{3}$$

Using Eq. (1), the velocity can then be approximated as

$$v \approx \sqrt{2(\frac{\mu}{r_M} + \frac{3}{2} + E)}. \tag{4}$$

Therefore, the approximate ΔV to go between energies E_1 and E_2 while at the same distance $r_M \ll 1$

$$\Delta V \approx \left| \sqrt{2(\frac{\mu}{r_M} + \frac{3}{2} + E_1)} - \sqrt{2(\frac{\mu}{r_M} + \frac{3}{2} + E_2)} \right|. \tag{5}$$

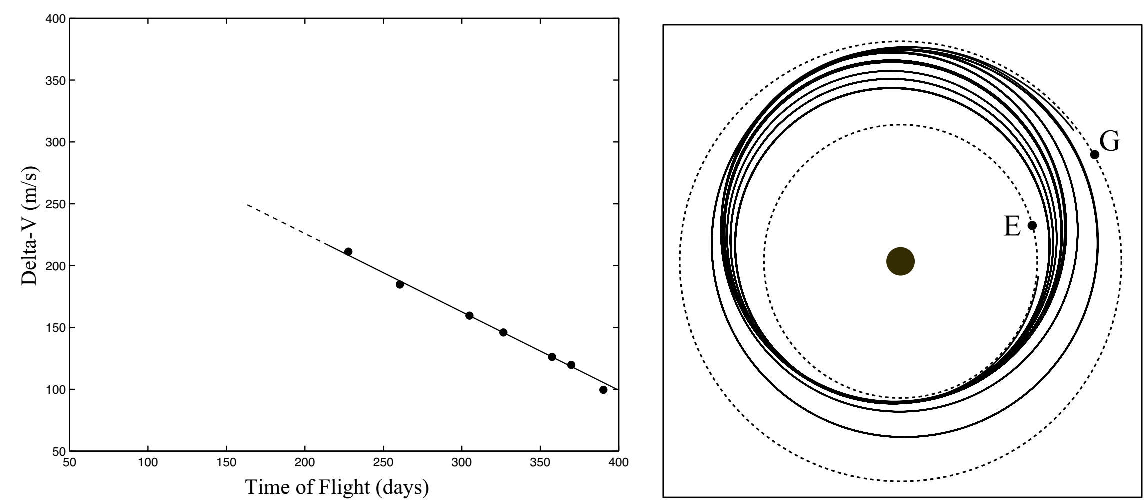
We can use the above equation to compute ΔV_{Ga} given $E_1 = E_{\text{Ga}}$ and $E_2 = E_{\text{GaO}}$. For this study, we take E_{GaO} to be the energy of L_1 in the Jupiter-Ganymede-spacecraft system. This corresponds to a bound elliptical orbit around Ganymede which is at the energy threshold of escape, and therefore cannot escape the Hill region around Ganymede. We can perform similar calculations for ΔV_{Eu}.

The result of tabulating $\Delta V_{\text{tot}} = \Delta V_{\text{Ga}} + \Delta V_{\text{Eu}}$ for each TOF $= T_{\text{Ga}} + T_{\text{Eu}}$ is given in Figure 6(a). We find a near linear relationship between ΔV_{tot} and TOF, given roughly by

$$\Delta V_{\text{tot}} = 340 - 0.60 \times \text{TOF}, \tag{6}$$

where ΔV_{tot} is given in m/s and TOF is given in days.

For this study we looked at a range of energies in both three-body systems. The highest energy (and lowest TOF) transfer we computed is shown in Figure 6(b) in inertial coordinates, where G labels Ganymede's orbit and E labels Europa's. This transfer has a TOF of 227 days and a ΔV of 211 m/s. Beyond this lower TOF limit to our computations, we speculate that the linearity will continue for a while, indicated by the dashed line. Further



(a) (b)

Figure 6: **Fuel consumption versus flight time trade-off for the inter-moon transfer phase of a multi-moon orbiter mission.** (a) The ΔV vs. time of flight plot for several transfer trajectories from Ganymede to Europa are shown. For the several cases run, we find a near linear relationship between ΔV and time of flight. For this study we looked at a range of energies in both three-body systems. The highest energy (and lowest TOF) transfer we computed is shown in (b) in inertial coordinates, where G labels Ganymede's orbit and E labels Europa's. This transfer had a TOF of 227 days and a ΔV of 211 m/s. Beyond this lower TOF limit to our computations, we speculate that the linearity will continue for a while, indicated by the dashed line. Further computations are needed to settle this matter.

computations are needed to settle this matter. Furthermore, future studies addressing the important compromise between time and fuel costs for a MMO mission will need to address the three-dimensionality of the problem. A possible requirement for a MMO may be the necessity to go from an inclined orbit about one moon to an inclined orbit about another, such as shown in Figure 1(b). We speculate that this may lead a nearby curve, possibly linear, in the ΔV vs. TOF plot.

Transfers between low altitude, circular orbits. In the above, we have computed only the minimum necessary to go between bound orbits around each moon. It is instructive to note the additional ΔV which would be necessary to effect a transfer between low altitude circular orbits of zero inclination. According to Villac et al.,[15] who used the Hill three-body problem for their model, the minimal ΔV to escape a low altitude (around 100 km) Ganymede would be at most 669.5 m/s, and to inject into a low altitude (around 100 km) Europa orbit would be at most 451.2 m/s. Therefore, an additional $\Delta V_{\text{circ}} = 1120.7$ m/s can be added to each point in the curve of Figure 6(a) in order to approximate a transfer between zero inclination, low altitude circular orbits. For example, the transfer with a 227 day TOF requires a total ΔV of about 1332 m/s to transfer between circular orbits.

CONCLUSIONS AND FUTURE WORK

Minimum time, two-impulse transfers from Ganymede to Europa in the framework of the patched three-body approximation have been investigated as a function of the three-body energy (Jacobi integral). The transfers are between orbits bound each moon, respectively. Tube dynamics have been used along with lobe dynamics to find uncontrolled trajectories which quickly traverse the space between the moons. The lobes act as templates, guiding pieces of the tube across resonance regions. The tubes have the numerically observed property that the larger the energy, the further the tube travels from its associated Lagrange point in a fixed amount of time. This property has been exploited to find the time of flight between Ganymede and Europa as a function of the energy in their respective three-body systems. The energies have been used to calculate the ΔV of escape, or get captured, from each moon, respectively.

Our results show that in the range of energies studied, the ΔV vs. time of flight relationship is nearly linear and that a reasonable inter-moon time of flight for a multi-moon orbiter can be achieved using a feasible ΔV.

Future work will try to determine this relationship for a larger range of energies and for the three-dimensional case for purposes of transferring between large inclinations orbits about moons of Jupiter. Furthermore, we will incorporate low thrust into future models. There is evidence that optimal trajectories using multiple low thrust burns are "geometrically similar" to impulsive solutions.[12,16] Thus, multiple burn impulsive trajectories that we construct for a multi-moon orbiter trajectory can be good first guesses for an optimization scheme which uses low thrust propulsion to produce a fuel efficient mission. We have found that a good first guess is often vital for numerical optimization algorithms, especially for an n-body problem which is numerically very sensitive.[17]

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