Optimal control of structural dampers

Shibabrat Naik, Nicholas Sharp, Shane Ross

Engineering Science and Mechanics, Virginia Tech

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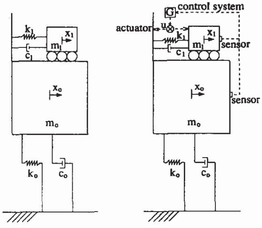
Motivation

- High rise structures are susceptible to dynamic excitations like earthquakes and wind gusts.
- Typically active tuned mass dampers, semi-active TMD, and MR dampers are engineered for controlling response.
- Trajectory based control uses information from phase space and optimal sets to reach or move a solution subset.
- For our interests, objective is to reduce inhabitant discomfort by reducing excessive sway in top floors and adverse structural stresses.

Modeling challenges

- Efforts in structutal health monitoring has been focussed on controlling response under earthquake as base excitations.

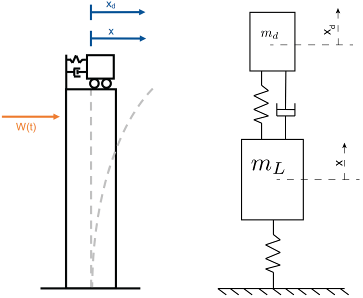


(a) Tuned mass damper (passive)
(b) Active mass damper (hybrid)

- Damper characteristics are controlled based on the feedback from the structure to optimize cost function of very high dimensional (100 or so) state vector.

- Response to wind load is calculated from empirical gust formulae and designed using probabilistic approach towards worst gust over the life span of the structure.

Reduced order model



- Model the structure as a cantilever beam with simple oscillation
- Assumptions:
  - Oscillation only in first mode
  - No damping coefficient for structure
  - Apply wind force entirely at top floor
- This system has just **two** degrees of freedom
- Use an *equivalent lumped mass*, $m_L = \frac{33}{140} mL$ and $c_d = \alpha m_d + \beta k_d$

Equations of motion and state-space model

Mass-spring model with a damper: $\mathbf{M}\ddot{\vec{x}} + \mathbf{C}\dot{\vec{x}} + \mathbf{K}\vec{x} = \vec{F}$ with $\vec{x} = \begin{bmatrix} x \\ x_d \end{bmatrix}$

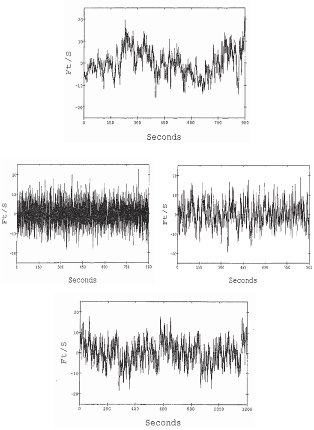
$$\mathbf{M} = \begin{bmatrix} m_L & 0 \\ 0 & m_d \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} c_d & -c_d \\ -c_d & c_d \end{bmatrix},\ \mathbf{K} = \begin{bmatrix} k + k_d & -k_d \\ -k_d & k_d \end{bmatrix},\ \vec{F} = \begin{bmatrix} W(t) \\ 0 \end{bmatrix}$$

$$\dot{s} = \mathbf{A}s + \mathbf{B}w$$
$$z = \mathbf{P}s + \mathbf{Q}w$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},\ \mathbf{Q} = \begin{bmatrix} \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} \\ -\mathbf{M}^{-1} \end{bmatrix}$$

Wind gust and related statistics

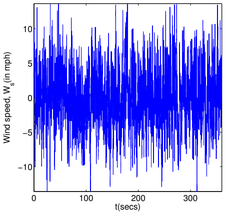
- Wind velocity is a **stochastic** variable which simulates physical wind effects
- Observations of wind speed follow a **gaussian** distribution
- Must consider **continuity** of wind, represented by the **autocorrelation**
- Simulate wind using **Markov chains** trained on experimental measurements



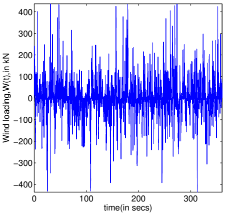
Time series of measured and synthetic wind speed [1]

Synthetic wind using Markov chains

- Constructing state transition matrix(or transition probability matrix,**P**) for all the available wind states.
- Compute the cumulative probability matrix(**C**) for the state transition matrix as:
  $C_{ij} = \sum_{k=1}^{j} P_{ik}$
- Initialize the state(s) with a random integer in $(1, N_s)$, where N_s is the number of wind states.
- The current state s corresponds to a row in **C** and generate a random number R from the uniform random distribution over $(0, 1)$ such that next state K is $C_{ik-1} \leq R \leq C_{ik}$.

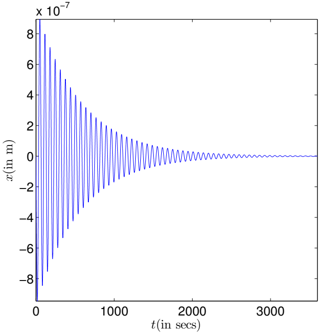


(a)

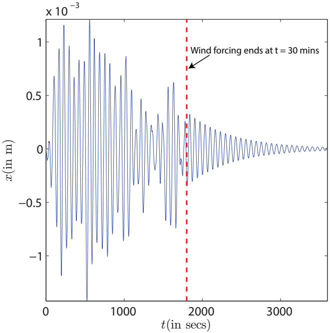


Uncontrolled dynamic response

- Impulse and wind loading to test the validation of heavy mass



(a)

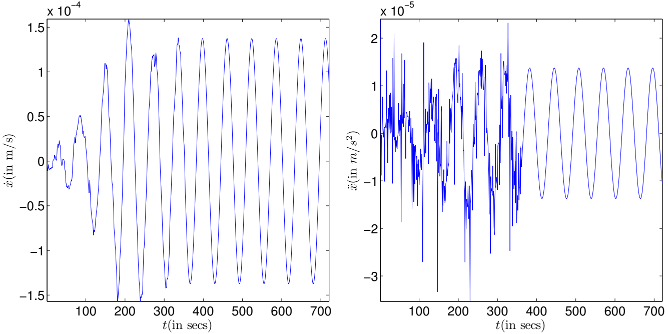


(b)

Displacement responses for impulse forcing at $t = 0$ and wind loading for 30 mins. $m_d/m_L = 1/10$

Uncontrolled dynamic response

- Wind loading with a light damper, indicating metric for objective function



(a) (b)

Displacement responses for impulse forcing at $t = 0$ and wind loading for 30 mins. $m_d/m_L = 1/1000$

Optimal Control - Why is it hard?

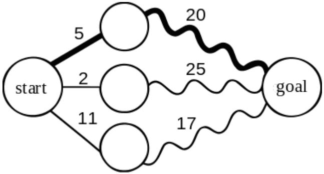
- **State includes wind**
  The wind must be included in the state because it affects future state probabilities
- **Stochastic Wind is not a distribution**
  Normal methods from stochastic optimal control do not apply
- **Optimal trajectory only followed for one timestep**
  Must take new wind information in to account and re-plan

Markov Decision Process

- This problem perfectly meets the definition of a Markov Decision Process (MDP):
  - System transitions from state to state according to a random input and user action
  - Each state has an associated cost
- These problems are studied in economics, robotics, and operations research
- There is not single method for finding solutions, but common techniques include **dynamic programming** and **reinforcement learning**

Dynamic Programming

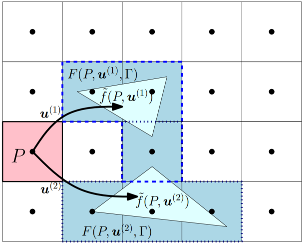
- We design a dynamic programming algorithm to solve the problem
- Process the set of all possible states backwards from the horizon time T
- Appears to work, but $\mathcal{O}(N^5)$ complexity inhibits practical utility



A general example of overlapping substructure in dynamic programming (Wikimedia Commons)

Future approaches

- Investigate more techniques from literature on Markov Decision Processes
- Algorithms from machine learning could yield very good approximate solutions
- State space search on a weighted hypergraph, but might require a different cost metric



A hypergraph formulation of a similar problem (Courtesy of Oliver Junge)

Thanks

- Thanks to Dr. Singh, Dr. Bisht, Dr. Junge for their advice
- **Questions?**

References

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