Trajectory based optimal control of swaying structures under wind gust

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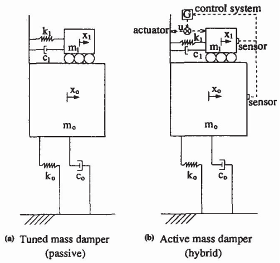
Motivation

- High rise structures are susceptible to dynamic excitations like earthquake and wind gust.
- Typically active tuned mass dampers, semi-active TMD, MR dampers are engineered for controlling response.
- Trajectory based control used information from phase space and optimal sets to reach or move a certain subset.
- For our interests, objective is to reduce inhabitant discomfort by reducing excessive sway in top floors and adverse structural stresses.

Modeling challenges

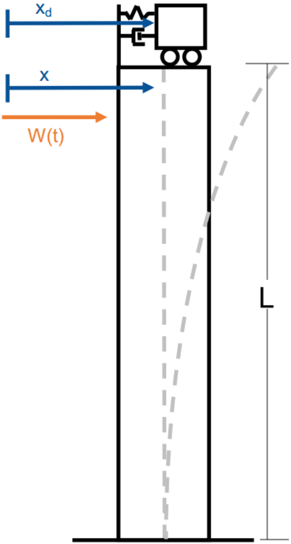
- Efforts in structutal health monitoring has been focussed on controlling response under earthquake as base excitations.



- Damper characteristics are controlled based on the feedback from the structure to optimize cost function of very high dimensional (100 or so) state vector.

- Response to wind load is calculated from empirical gust formulae and designed using probabilistic approach towards worst gust over the life span of the structure.

Reduced order model



- Model the structure as a cantilever beam with simple oscillation
- Assumptions:
  - Oscillation only in first mode
  - No damping coefficient for structure
  - Apply wind force entirely at top floor
- This system has just **two** degrees of freedom

Reduced order equations of motion

Use an *equivalent lumped mass* to derive equations of motion:

$$\mathbf{M}\ddot{\vec{x}} + \mathbf{C}\dot{\vec{x}} + \mathbf{K}\vec{x} = \vec{F}$$

with

$$\vec{x} = \begin{bmatrix} x \\ x_d \end{bmatrix},\ \mathbf{M} = \begin{bmatrix} m_L & 0 \\ 0 & m_d \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} c_d & -c_d \\ -c_d & c_d \end{bmatrix},$$

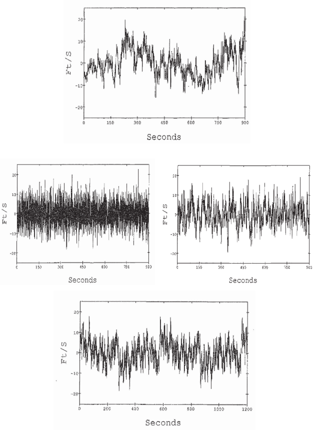
$$\mathbf{K} = \begin{bmatrix} k + k_d & -k_d \\ -k_d & k_d \end{bmatrix},\ \vec{F} = \begin{bmatrix} W(t) \\ 0 \end{bmatrix}$$

where

$$m_L = \tfrac{33}{140} mL \qquad \text{and} \qquad k = \tfrac{3EI}{L^3}.$$

Wind gust and related statistics

- Wind velocity is a **stochastic** variable which simulates physical wind effects
- Observations of wind speed follow a **gaussian** distribution
- Must consider **continuity** of wind, reprsented by the **autocorrelation**
- Simulate wind using **Markov chains** trained on experimental measurements



Time series of measured and synthetic wind speed [1]

Optimal control strategies

- **Control**: $c_d \in [0, c_{max}]$, the damping coefficient for the damper
- **Cost**: $J = \int_0^T \ddot{x}(t)dt$, the total acceleration of the top floor over time
- **Potential strategy**: Discretize the phase space and compute transition probabilities and costs for each state

References

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