BALANCE RECOVERY STRATEGY: ACROBOT VS. WOBBLE CHAIR

Frontiers in Dynamical Systems

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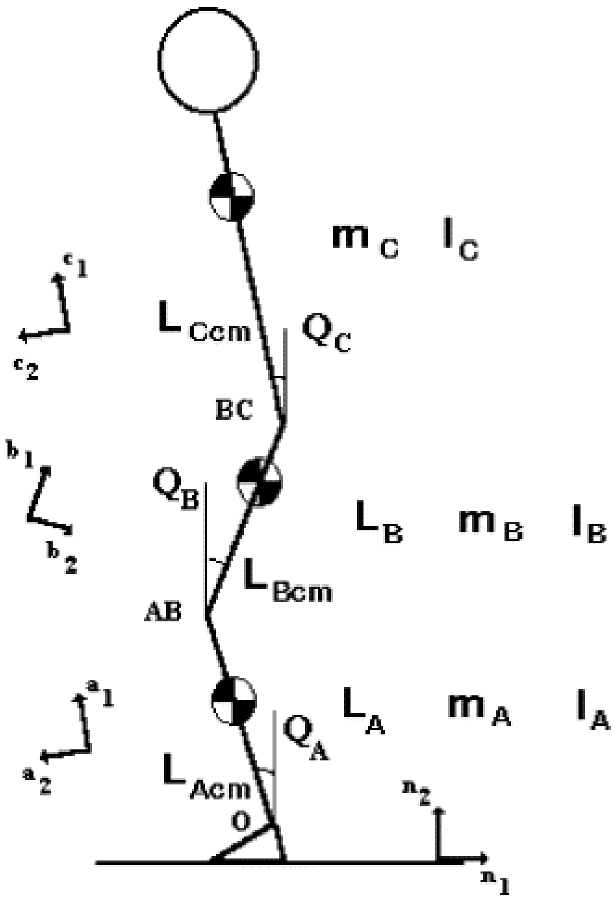
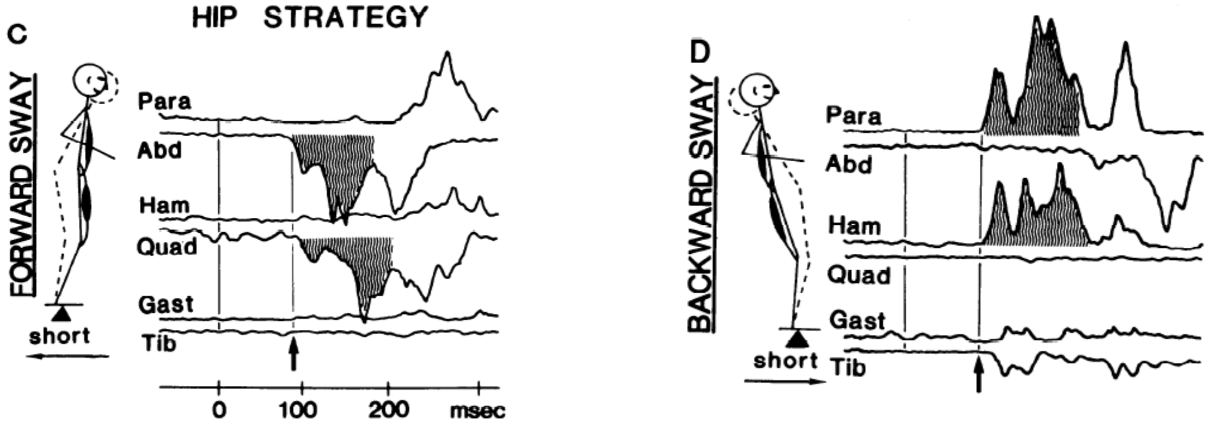
Introduction

- Multiple segment inverted pendulums
  - Used in biomechanics to model balance recovery and postural control in humans.
- Present Study
  - model two configurations of double segment inverted pendulums to analyze the strategy used to recover from external perturbation



Acrobot Model: Strategy 1

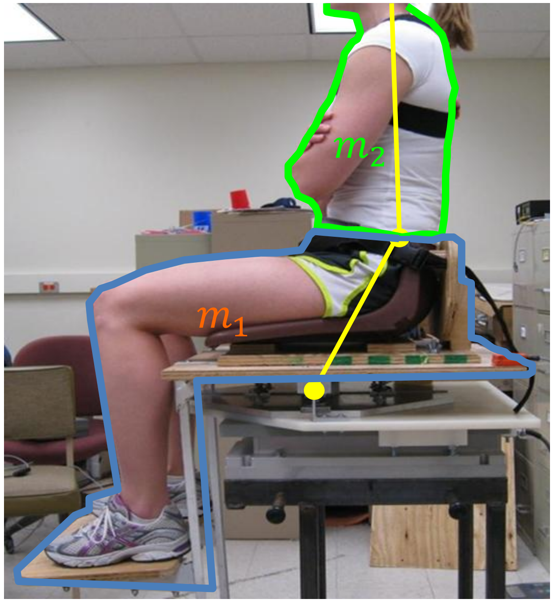
- Model:
  - Under actuated double segment inverted pendulum modeling "hip strategy"
- Strategy
  - Forward or backward sway displacement elicited rotation of the hip that moved the trunk in the direction of the initial body displacement.

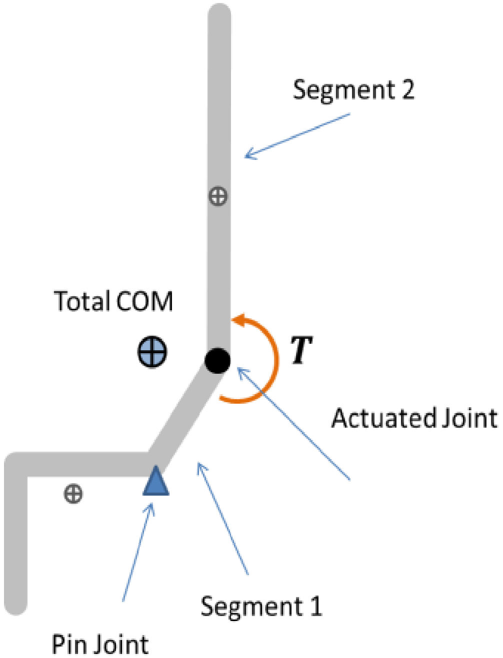


Figures: Experimental work showing use of hip strategy when subjects stand on a narrow beam.

Wobble Chair Model : Strategy 2

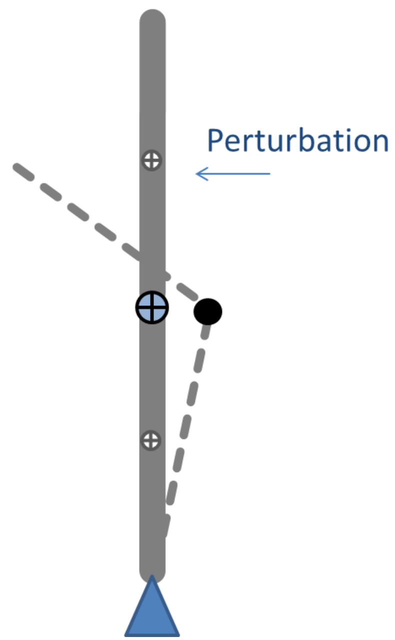
- Model:
  - Under actuated double segment inverted pendulum
- Strategy
  - Tanaka et al. (2010) recovery of COM location was achieved by causing flexion of trunk when overall center of mass was posterior to pivot point.

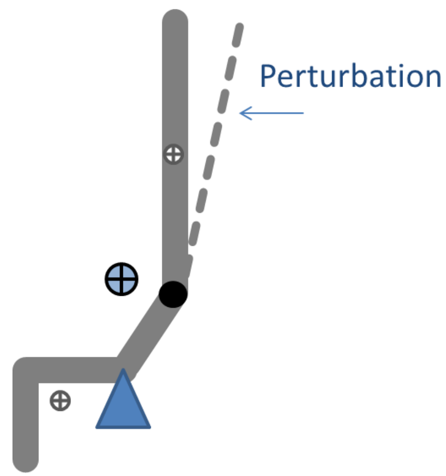




Strategy: Acrobot Vs. Wobble Chair

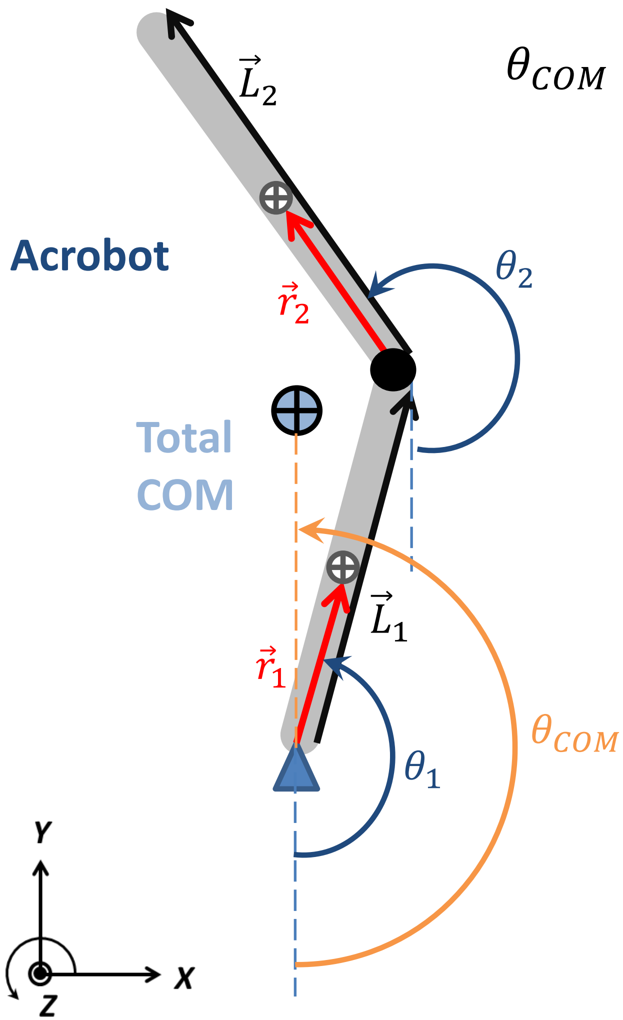
- Purpose
  - Attempt discovery of the opposing strategies used between the acrobot and the wobble chair to recover balance after an external perturbation

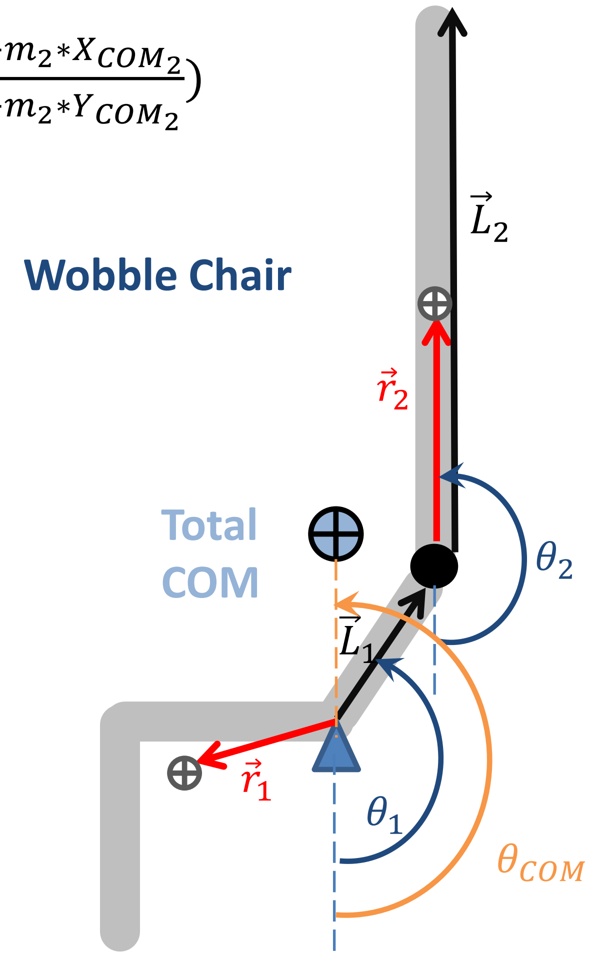




Models' Description

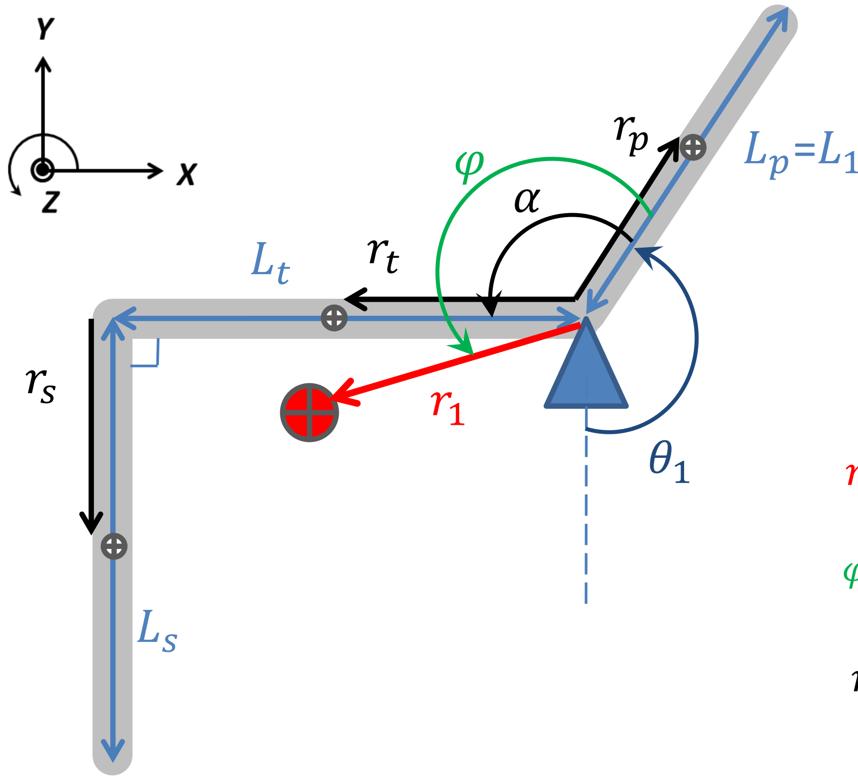
$$\theta_{COM} = \tan^{-1}\left(\frac{m_1 * X_{COM_1} + m_2 * X_{COM_2}}{m_1 * Y_{COM_1} + m_2 * Y_{COM_2}}\right)$$

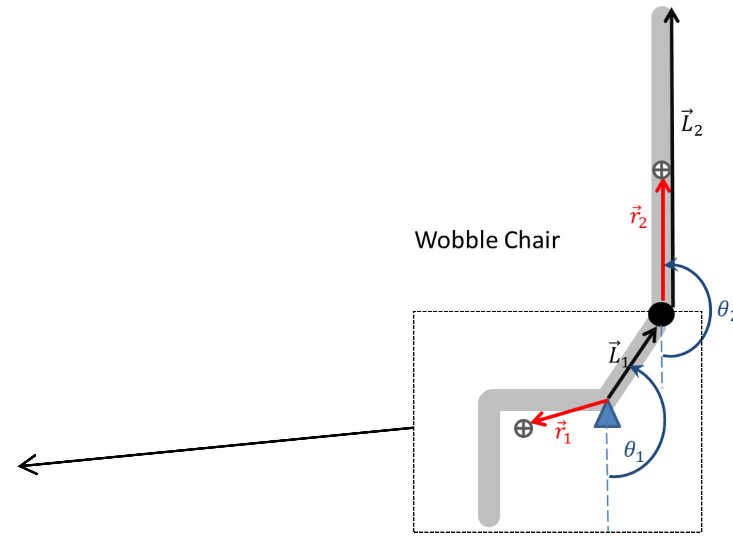




Wobble Chair

Segment 1





$$r_1 = \sqrt{X_{COM_1}{}^2 + Y_{COM_1}{}^2}$$

$$\varphi + \theta_1 = \tan^{-1}(X_{COM_1}/Y_{COM_1})$$

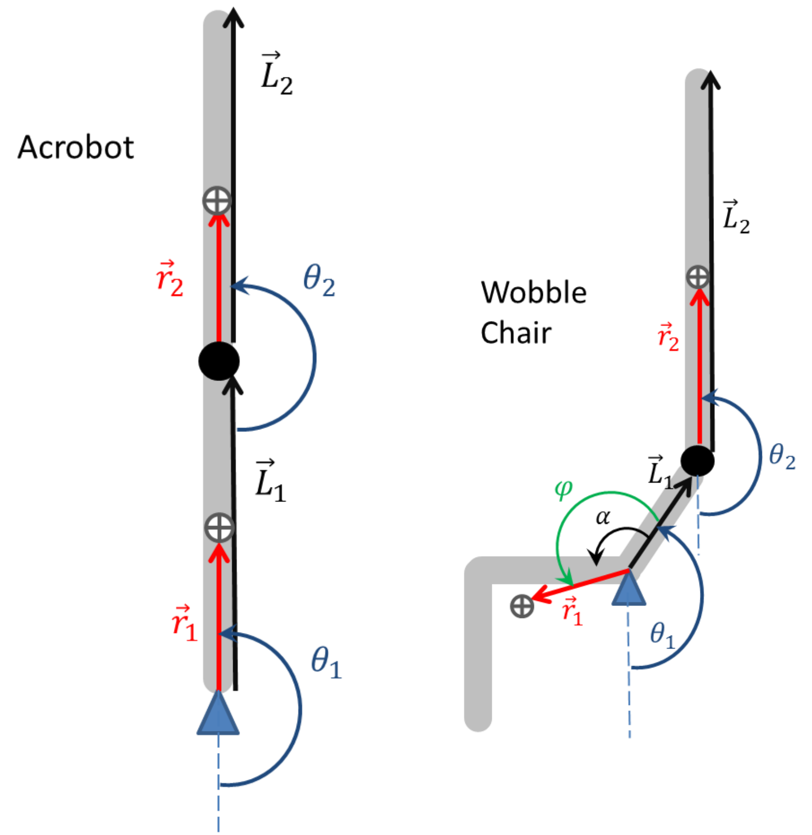
$$m_1 = m_p + m_t + m_s$$

$$X_{COM_1} = (m_p * r_p * \sin\theta_1 + m_t * r_t * \sin(\theta_1 + \alpha) + m_s * (L_t * \sin(\theta_1 + \alpha) + r_s * \cos(\theta_1 + \alpha)))/(m_p + m_t + m_s)$$

$$Y_{COM_1} = (-m_p * r_p * \cos\theta_1 - m_t * r_t * \cos(\theta_1 + \alpha) + m_s * (-L_t * \cos(\theta_1 + \alpha) + r_s * \sin(\theta_1 + \alpha)))/(m_p + m_t + m_s)$$

Models' Parameters

- Simplified models' Parameters



	m_p (kg)	m_t (kg)	m_s (kg)	m_1 (kg)	m_2 (kg)	r_1 (m)	r_2 (m)	L_1 (m)	L_2 (m)	I_1 $(\mathrm{kg.m\hat{}2})$	I_2 $(\mathrm{kg.m\hat{}2})$	α (deg)	φ (deg)
Acrobot	1	0	0	1	1	0.5	0.5	1	1	0.0833	0.0833	0	0
Wobble chair	0.2	0.4	0.4	1	1	0.19	0.5	0.2	1	0.0500	0.0833	150	176

Equations of motion (EQM)

- Lagrangian Method

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \left(\frac{\partial L}{\partial q_j}\right) = Q_j \qquad q_1 = \theta_1 \qquad q_2 = \theta_2 \qquad L = K - V$$

$$K = \frac{1}{2}m_1|\dot{\vec{r_1}}|^2 + \frac{1}{2}m_2|\dot{\vec{r_2}}|^2 + \frac{1}{2}I_1\dot{\theta_1}^2 + \frac{1}{2}I_2\dot{\theta_2}^2$$

$$V = -m_1 g\,|\vec{r_1}|\cos(\theta_1 + \varphi) - m_2 g\,|\vec{L_1}|\cos\theta_1 - m_2 g\,|\vec{r_2}|\cos\theta_2$$

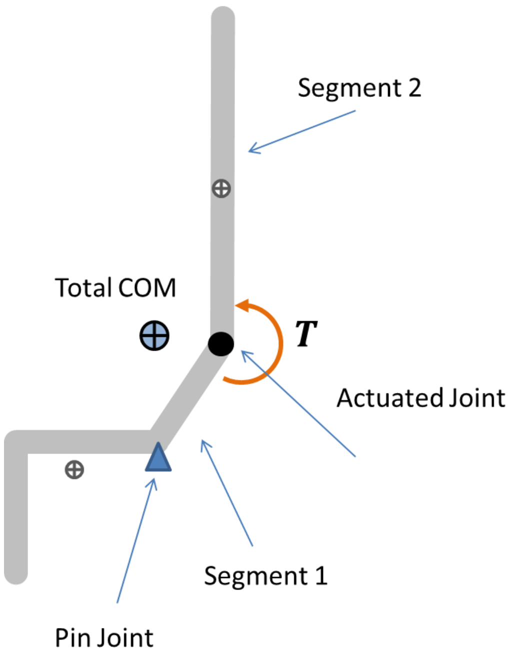
- EQM

$$\boldsymbol{M}\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \boldsymbol{G}(\boldsymbol{\theta}) = \boldsymbol{Q}$$

$$M = \begin{bmatrix} I_1 + m_1|\vec{r_1}|^2 + m_2|\vec{L_1}|^2 & m_2|\vec{L_1}||\vec{r_2}|\cos(\theta_1 - \theta_2) \\ m_2|\vec{L_1}||\vec{r_2}|\cos(\theta_1 - \theta_2) & I_2 + m_2|\vec{r_2}|^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & m_2|\vec{L_1}||\vec{r_2}|\sin(\theta_1 - \theta_2)\,\dot{\theta}_2 \\ -m_2|\vec{L_1}||\vec{r_2}|\sin(\theta_1 - \theta_2)\,\dot{\theta}_1 & 0 \end{bmatrix}$$

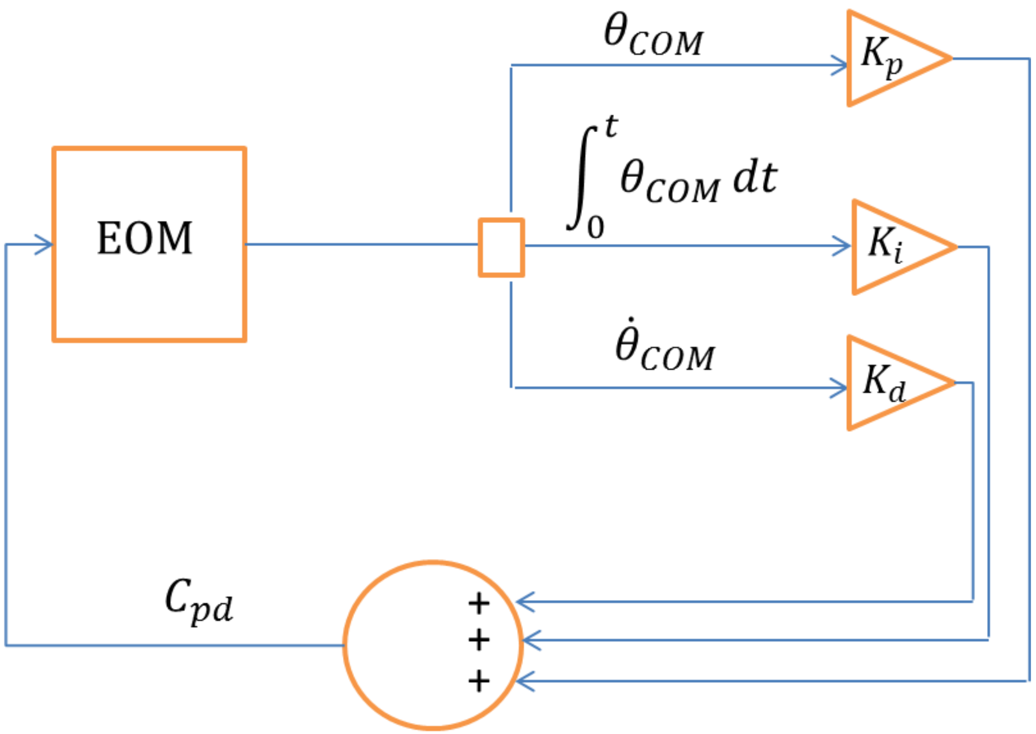
$$G = \begin{bmatrix} m_1 g|\vec{r_1}|\sin(\theta_1 + \varphi) + m_2 g|\vec{L_1}|\sin\theta_1 \\ m_2 g|\vec{r_2}|\sin\theta_2 \end{bmatrix} \qquad Q = \begin{bmatrix} -T + T_{sp} \\ T \end{bmatrix}$$

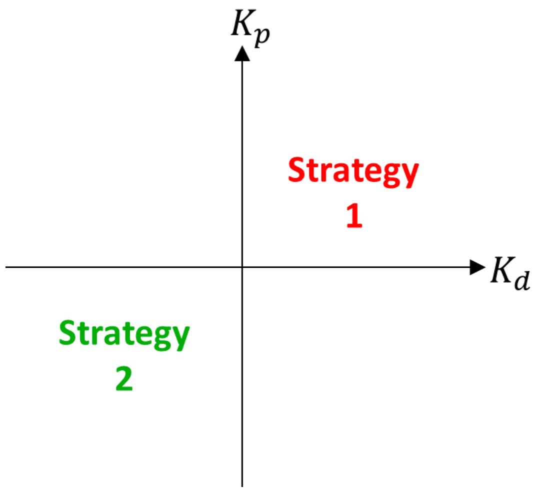


$$T_{sp} = k_s * d^2 * \sin(\theta_1)$$

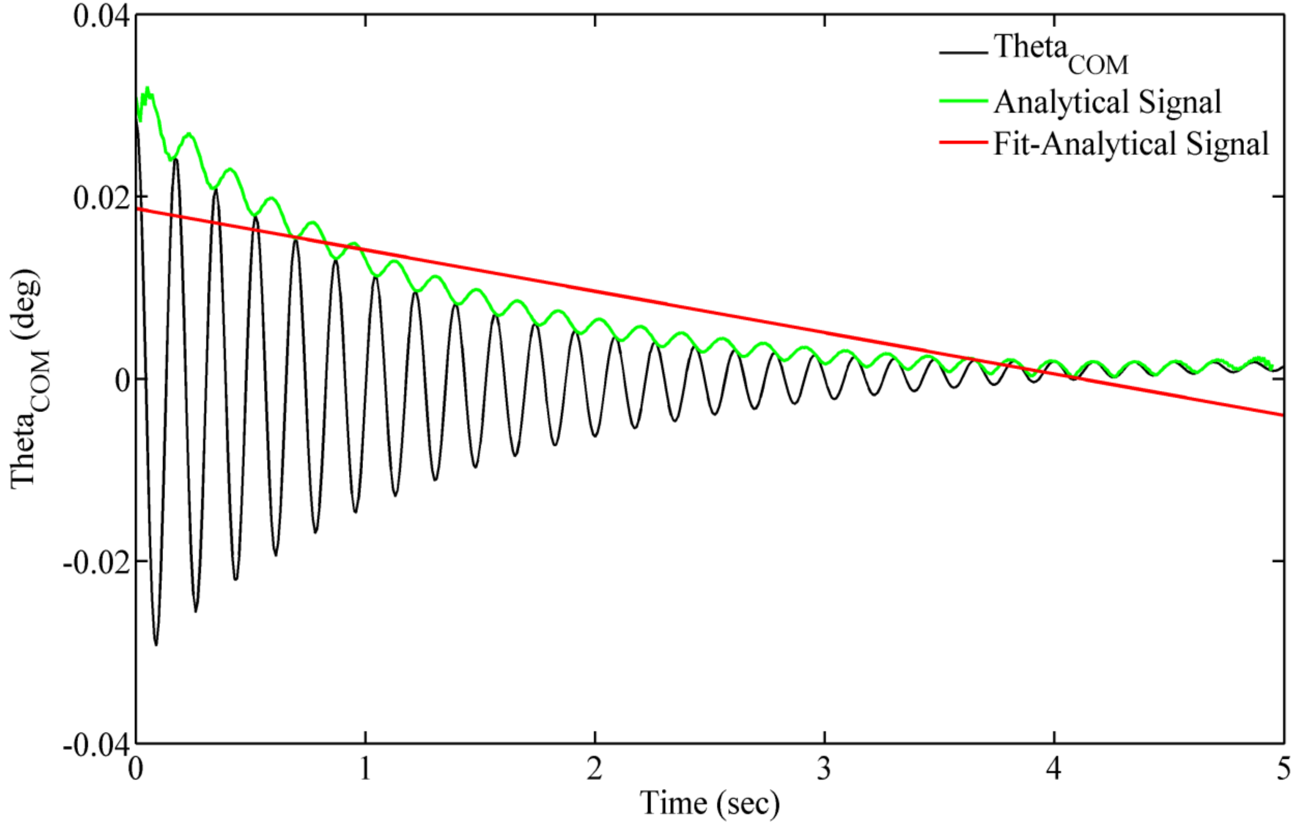
Controller: PID

$$C_{pd} = K_d * \dot{\theta}_{COM} + \begin{cases} K_p * \theta_{COM} + K_i * \int_0^t \theta_{COM}\, dt & if\ \ |\theta_{COM}| < \theta_{critical} \\ T_{pmax} & Otherwise \end{cases}$$



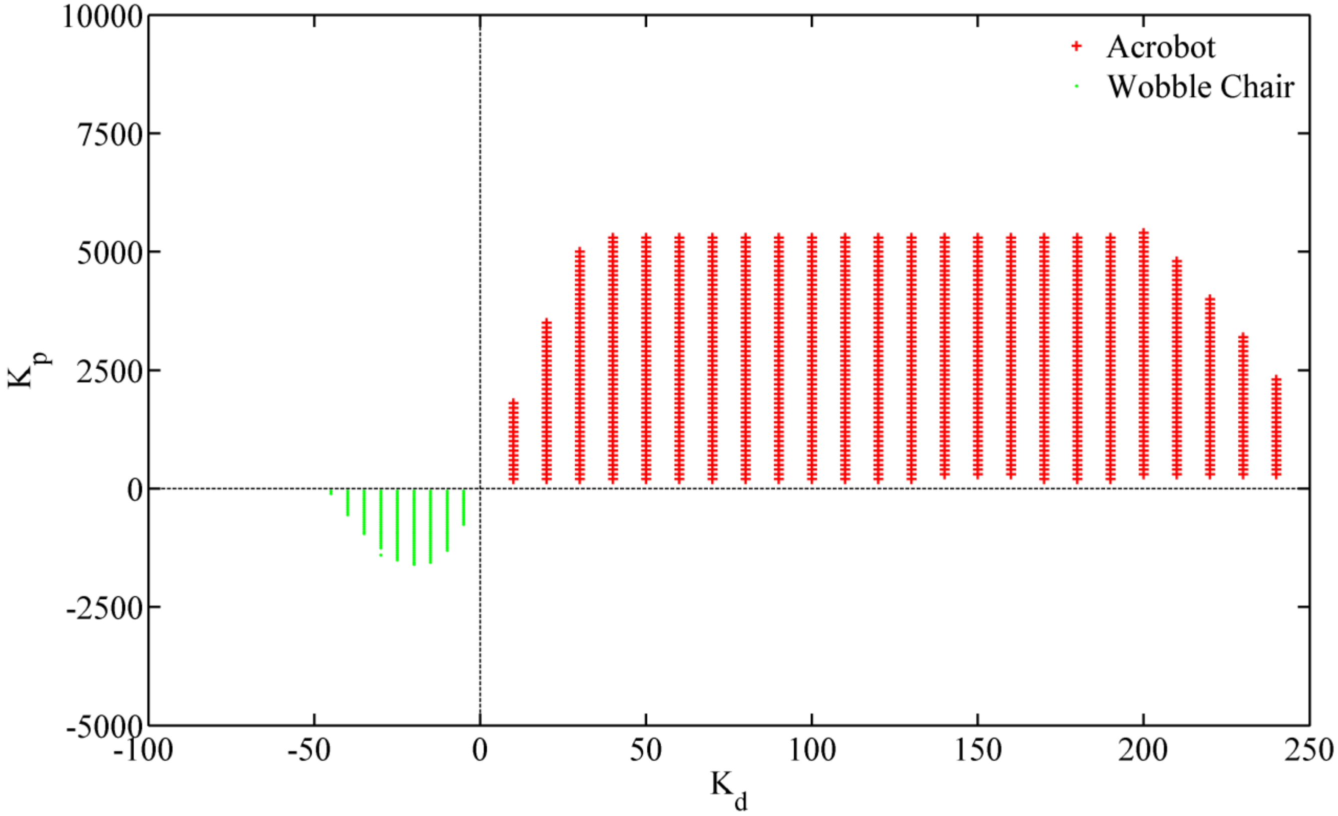


Stability Criteria: Hilbert Envelope

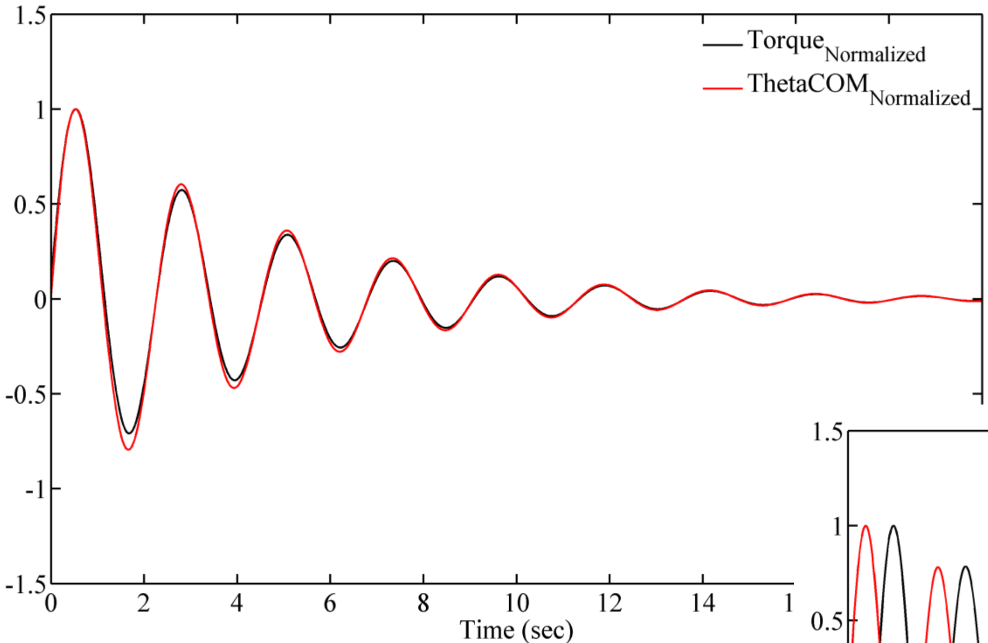


θ_{com} converging to zero indicating a stable system

Stability regions

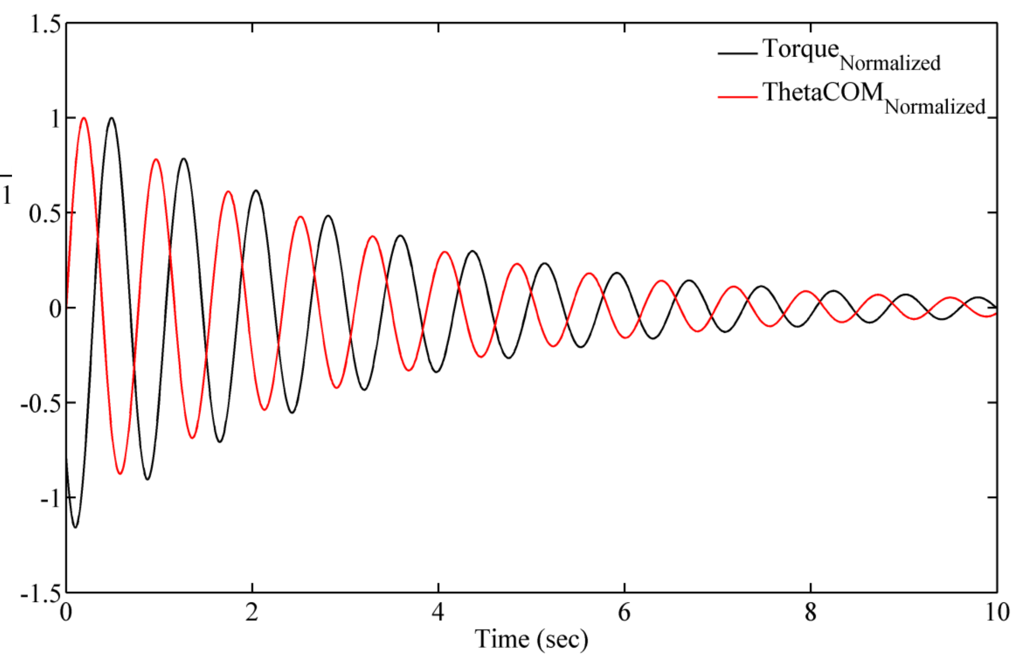


Different Strategies

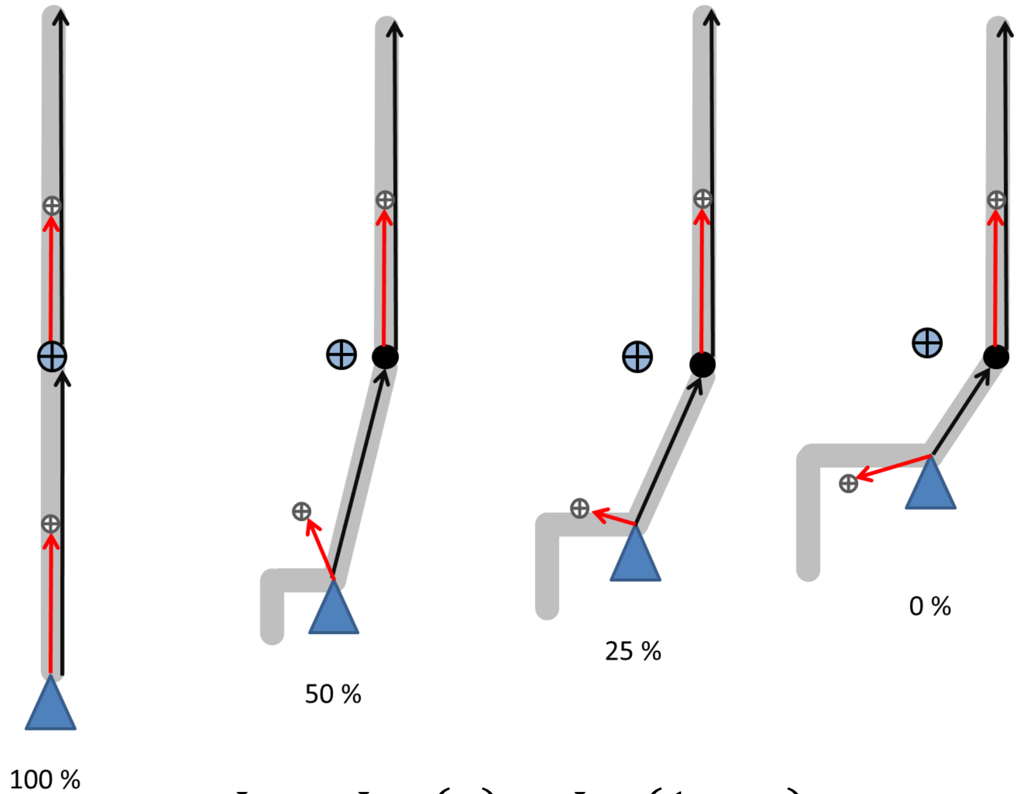


Acrobot

Wobble Chair



Transformation:



$$L_1 = L_{1A}(x) + L_{1S}(1 - x)$$

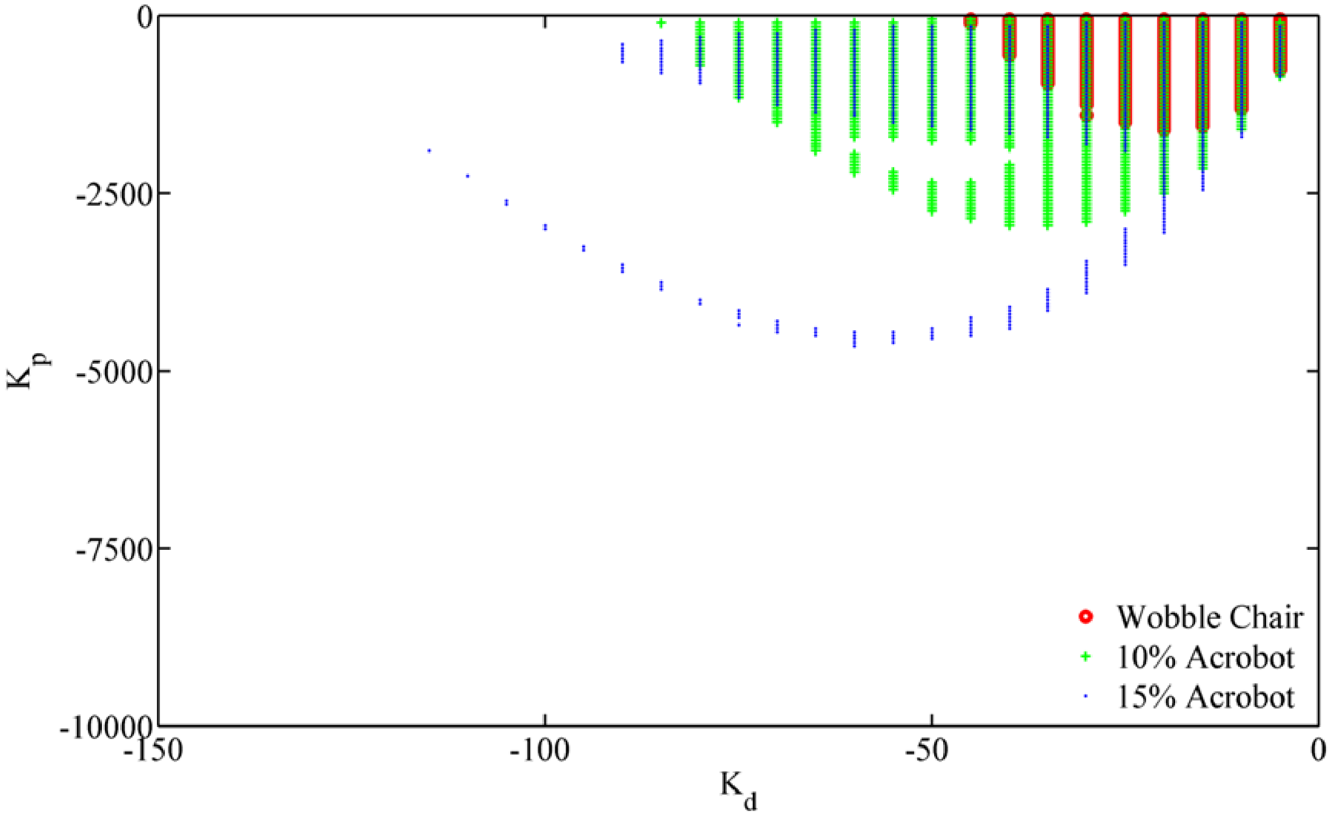
$$m_1 = m_{1A}(x) + m_{1S}(1 - x)$$

$$L_t = L_s$$
$$1 = L_p + L_s + L_t$$

$$M_t = M_s$$
$$1 = M_p + M_s + M_t$$

Transformation Results

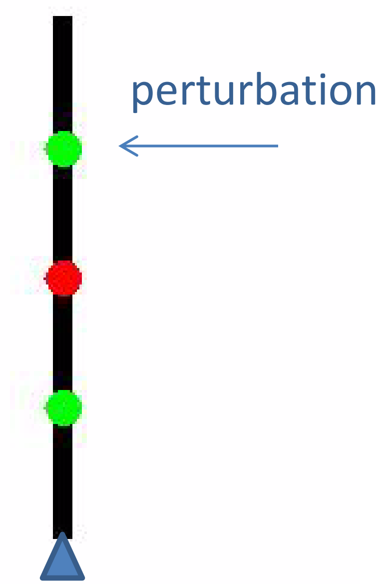
10% and 15% acrobot



Animation

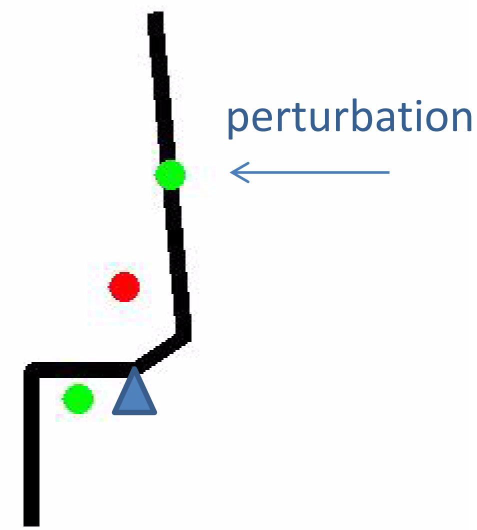
Acrobot

Startegy 1



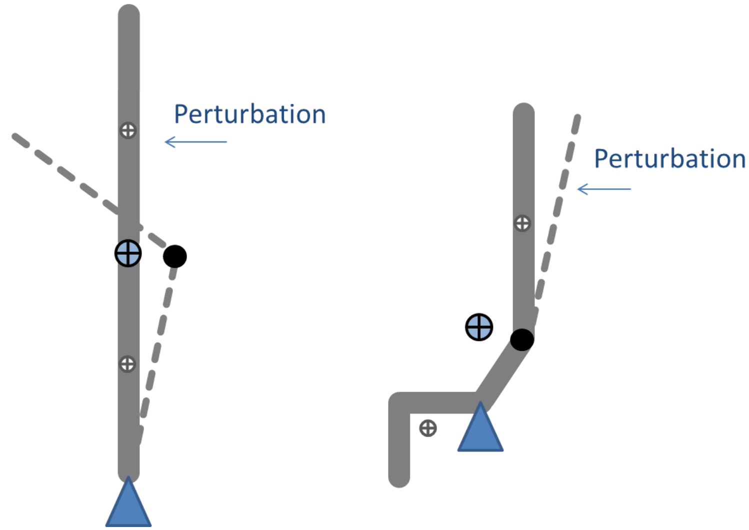
Wobble Chair

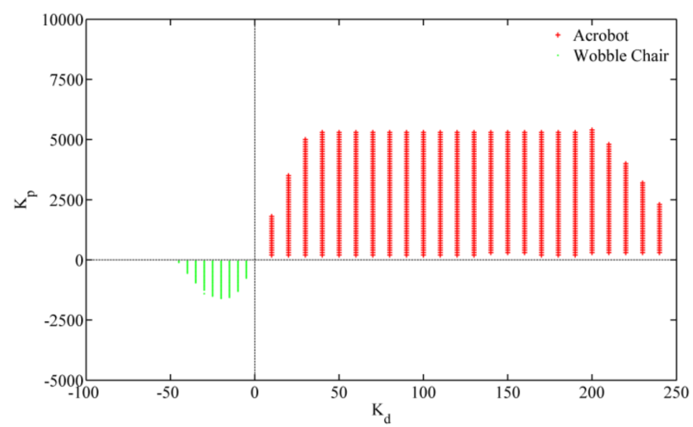
Startegy 2



Summary

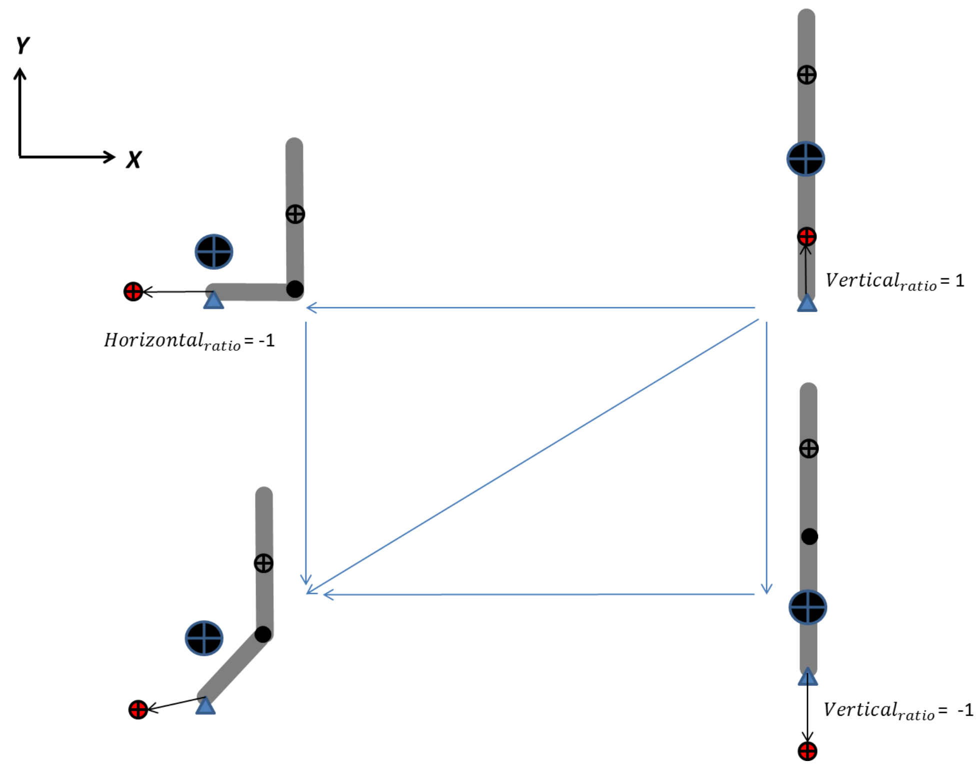
- Two segment inverted pendulum models were developed representing the acrobot and wobble chair configurations
- Two opposite recovery strategies were observed for the models subjected to an external perturbation
- Using a search method with variations in controller gains revealed the presence of these opposing strategies in our models





Future Work

- Transforming the model in x and y direction
  - The location of the COM of segment 1



- Find out why the models choose different strategies

Thank you