Role of Magnus effect in cell separation using dielectrophoresis

ESM-6984: Frontiers of dynamical systems
Virginia Tech

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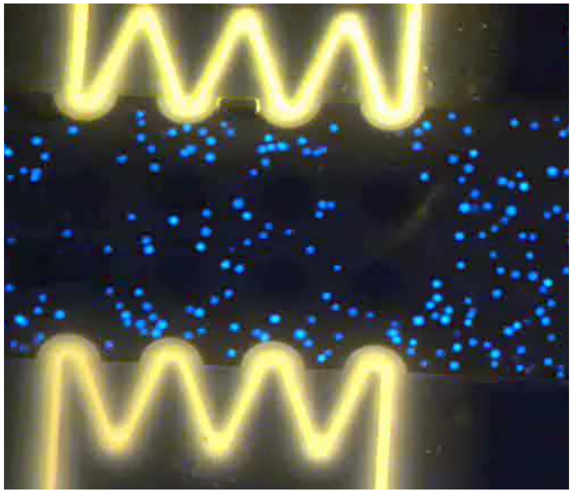

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Acknowledgement:
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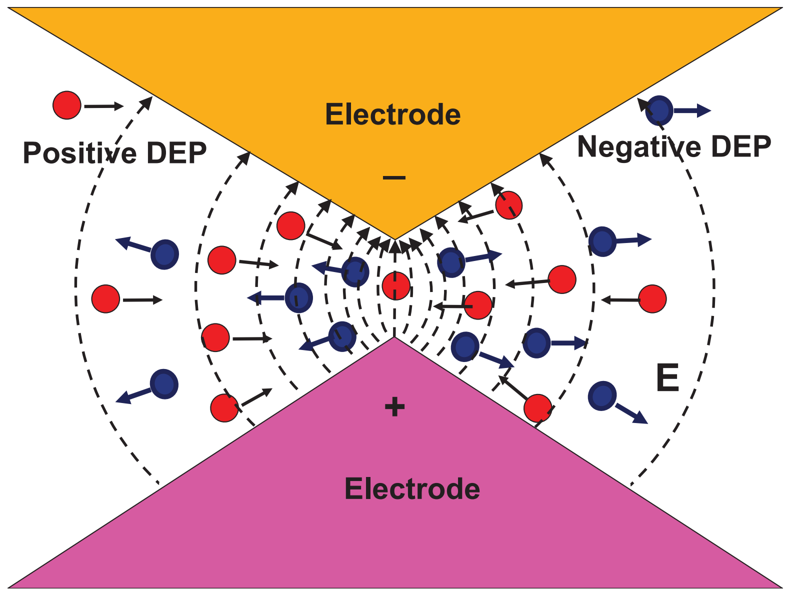
What is Dielectrophoresis?

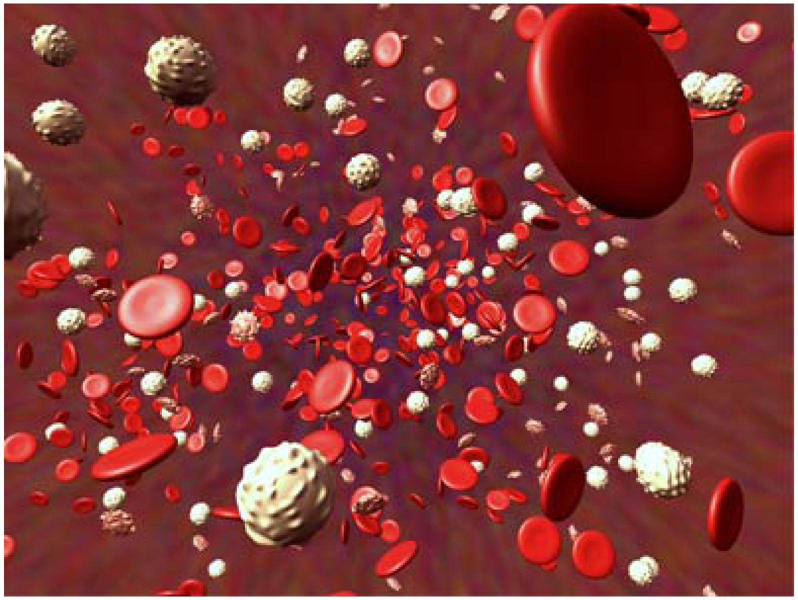
- Dielectrophoresis(DEP) is the motion of a particle due to the interaction between a non-uniform electric field and the induced dipole moment in the particle.



- The particles which are more polarizable than the surrounding medium is attracted towards the stronger field $\Rightarrow$ positive DEP

Why Dielectophoresis?

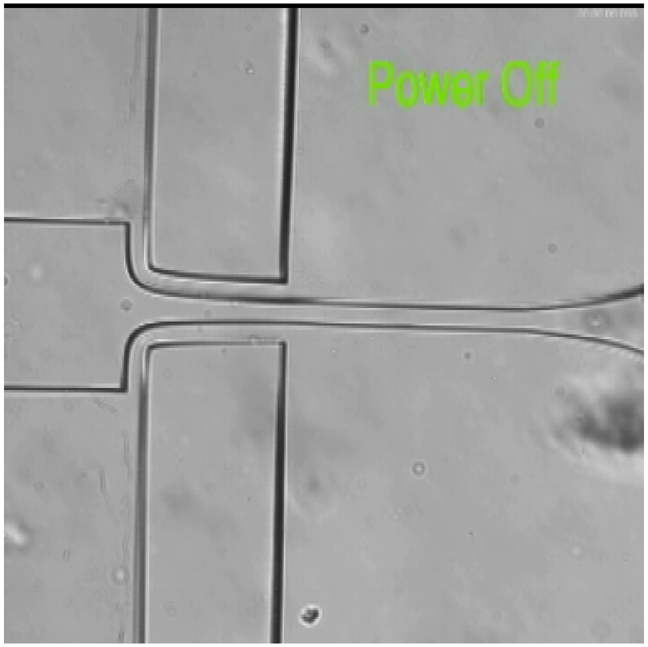
- Established technique to discriminate between distinct cellular identities in heterogeneous populations
  - Identify tumor stem cells
  - Isolate stem cells in adipose tissue



- Cell manipulation for drug targeting and lab on chip concept for safer and confident clinical trials.
- Common methods like flow cytometry, magnetic bead-coupled cell separation depend on specific cell-surface antigens.
- Of theoretical interest due to multiple physics involved like electrohydrodynamics, electro-osmosis, thermodynamics, elasticity of cells and as we shall see rotational dynamics.

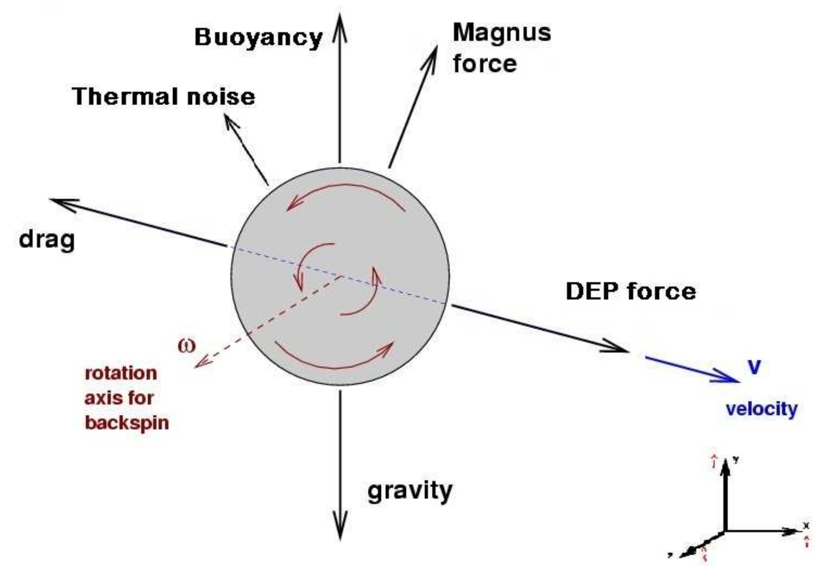
Motivation

- Present knowledge stands at the **what** causes electro-rotation and **which** properties it depends on.
- But **how** does it affect the trajectory and **how** does the rotational motion scale with a geometry is still unanswered.
- A significant contribution in contactless DEP where electro-osmosis and electro-phoresis are competing.



Primary forces

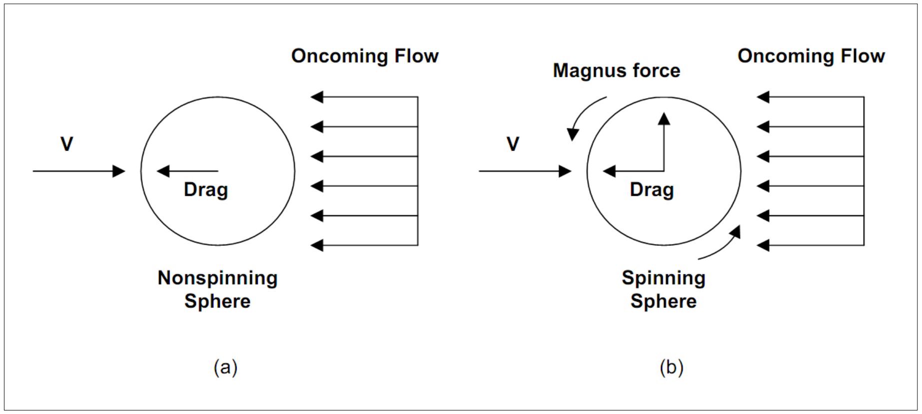
- Dielectrophoresis
  - Translational force
  - Electro-rotation Torque
- Drag force
  - Drag force
  - Rotational friction
- Gravitational force
- Buoyancy
- Magnus force
- Inertial force
- Thermal noise



- **Characteristic numbers**
  - Cell dim.: 15-30 μm
  - Domain dim.: 10-1000 μm
  - Typical velocity: $< 500 \mu m s^{-1}$
  - Knudsen < 0.1
  - Reynolds $<< 1$

Magnus force

- Consider a spinning sphere translating in a viscous fluid, this generates a force normal to the flow direction.



$$\mathbf{F}_M = 4\pi a^3 \rho_f \left[\vec{\omega} x (\vec{u}_p - \vec{u}_f)\right]$$

- An accidental phenomena of fluid flow around unstreamlined objects under which it curves away from its principal flight path.
- We discuss the effects on the motion of a cell which is restricted in a horizontal domain.

Hydrodynamics and brownian

- **Drag force:** Stokes' drag equation is valid for such flows.

$$F_{drag} = 6\pi\eta a(u_p - u_f)$$

$$T_f = 8\pi\eta a^3\Omega$$

- **Bouyancy:** To a first order approximation, the velocity due to buoyancy can be estimated as

$$u_p \approx 0.2\frac{d_p^2\rho_p g}{\eta}$$

The factor being still smaller in case of biological particles with density close to the DEP buffer.

- **Brownian motion:**

$$\Delta x = \sqrt{2Dt} = \sqrt{\frac{k_B T}{3\pi a\eta}t}$$

The Brownian effects become negligible in a dielectrophoretic system especially when the particle size $\sim$ 15-20 μm.

Dielectrophoresis:Force and torque

- **DEP force:** Particles are attracted or repelled from region of high electric fields and is governed by the the absolute permittivities of the particle and the DEP buffer. For oscillating electric-fields, time-averaged form for the translational force:

$$\langle \mathbf{F}_{DEP} \rangle = 2\pi\epsilon_f a^3 Re\left[\frac{\epsilon_p^* - \epsilon_f^*}{\epsilon_p^* + 2\epsilon_f^*}\right] \nabla[\mathbf{E}_{rms}^2(\vec{r}_0)]$$

  - **E** is inhomogeneous and hence gradient is non-zero.
- **DEP torque:** Time-averaged form for the electro-rotational torque:

$$\langle \mathbf{\Gamma}_{DEP} \rangle = -4\pi\epsilon_f a^3 Im\left[\frac{\epsilon_p^* - \epsilon_f^*}{\epsilon_p^* + 2\epsilon_f^*}\right] \mathbf{E}_{rms}^2(\vec{r}_0)$$

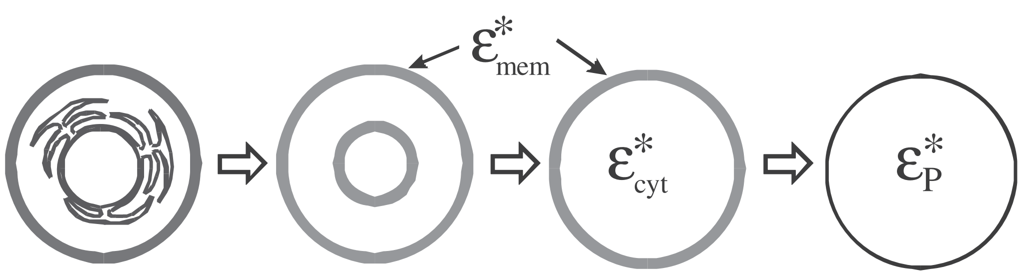
where, complex dielectric constant: $\epsilon^* = \epsilon + \frac{\sigma}{j\omega}$ and

Clausius-Mossotti factor: $K(\omega) = \frac{\epsilon_p^* - \epsilon_f^*}{\epsilon_p^* + 2\epsilon_f^*}$

Inertial effects and inhomogenity

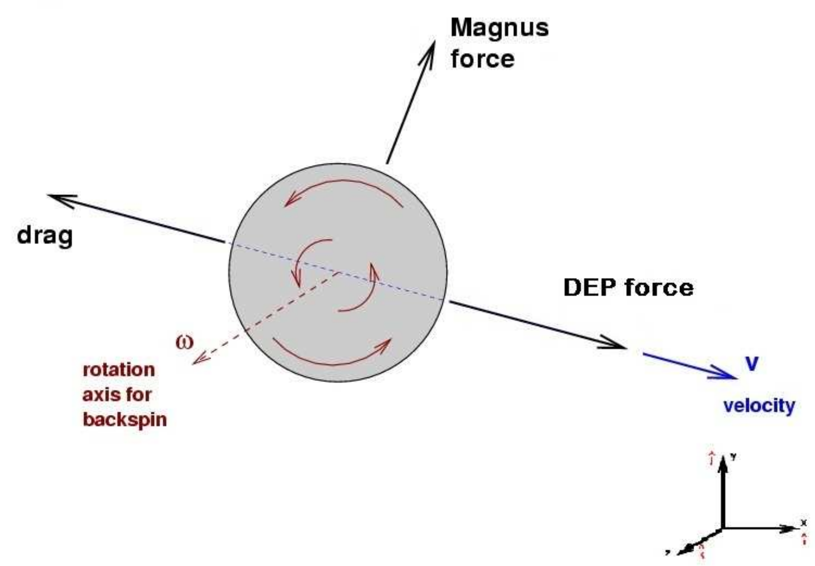
- **Inertia** We neglect the inertial effects of the motion which will also be recovered from the scaling analysis.
- **Multi-shell model:**The Clasius-Mossotti factor for biological particles is calculated using the effective values of the complex relative permittivity or conductivity. The relative permittivity of a cancer cell is expressed as:

$$\varepsilon_p = \varepsilon_{mem} \frac{\left(\frac{r_p+d}{r_p}\right)^3 + 2\left(\frac{\varepsilon_{cyt}-\varepsilon^*_{mem}}{\varepsilon_{cyt}+2\varepsilon^*_{mem}}\right)}{\left(\frac{r_p+d}{r_p}\right)^3 - 2\left(\frac{\varepsilon_{cyt}-\varepsilon^*_{mem}}{\varepsilon_{cyt}+2\varepsilon^*_{mem}}\right)}$$



Dominant forces

- **Dielectrophoresis**
  - **Translational force✓**
  - **Electro-rotation Torque?**
- **Viscous force**
  - **Drag force✓**
  - **Rotational friction?**
- Gravitational force✓
- Buoyancy✓
- **Magnus force?**
- **Inertial force?**
- Thermal noise



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  - Domain dim.: 10-1000 μm
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Equations of motion

- The equations for the motion of a cell suspended in a dielectric medium under applied potential:

$$m_p \frac{du_r}{dt} = F_{DEP} + F^r_{drag}$$

$$m_p \frac{du_t}{dt} = F_M + F^t_{drag}$$

$$I_p \frac{d\omega}{dt} = \Gamma_{DEP} + T_{drag}$$

- Inertial effects being negligible, we obtain a simplified EOM for the translational and rotational velocities:

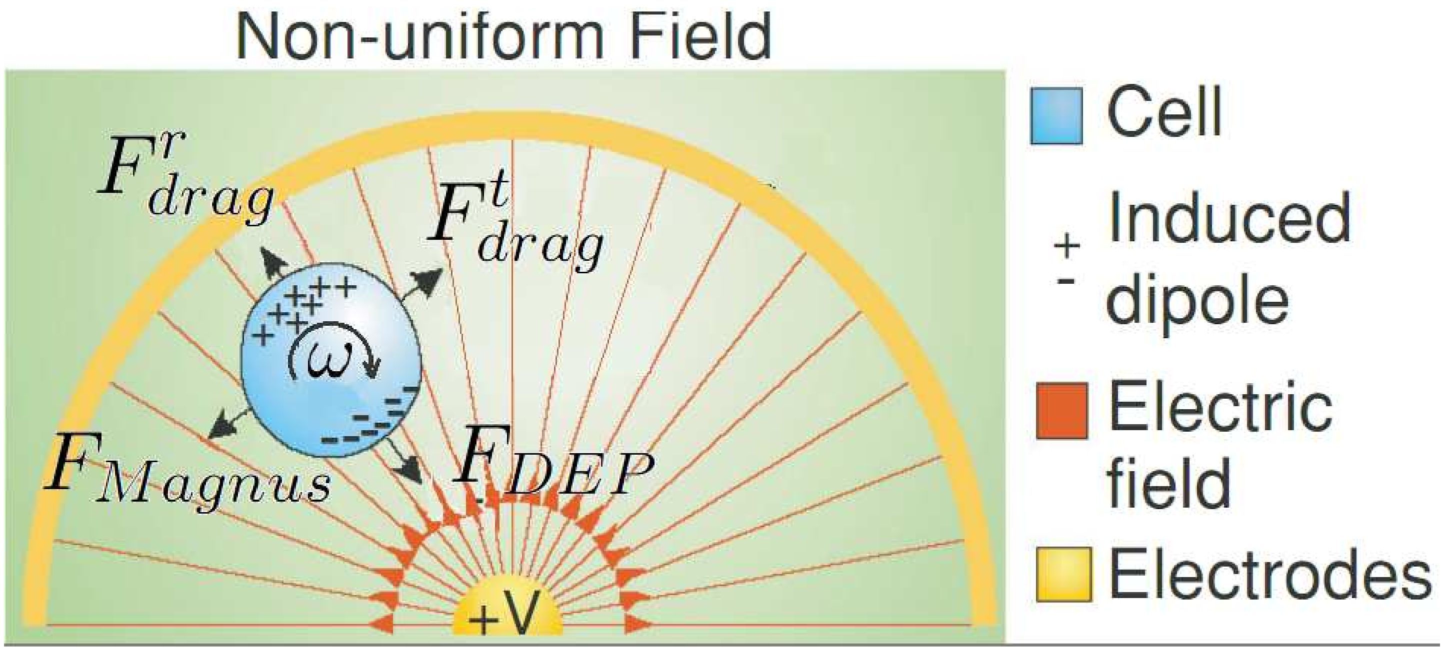
$$u_r = \frac{F_{DEP}}{6\pi\eta a}$$

$$u_t = \frac{F_M}{6\pi\eta a}$$

$$\omega = \frac{\Gamma_{DEP}}{8\pi\eta a^3}$$

Electrode configuration and field derivation

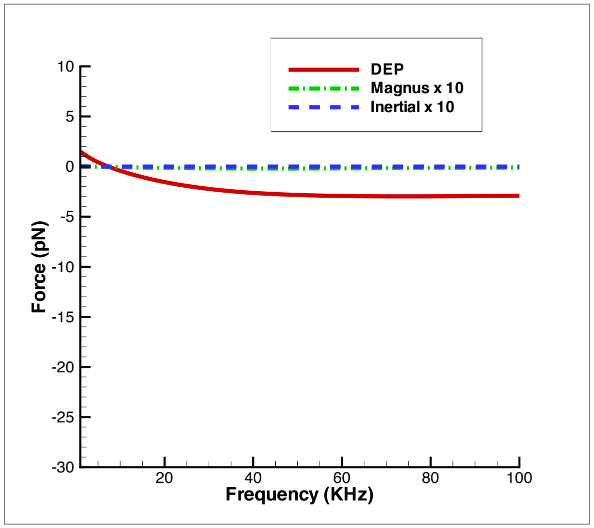
- For the present study, two coaxial cylinders are considered with positive voltages on the inner concentric with a grounded outer cylinder.



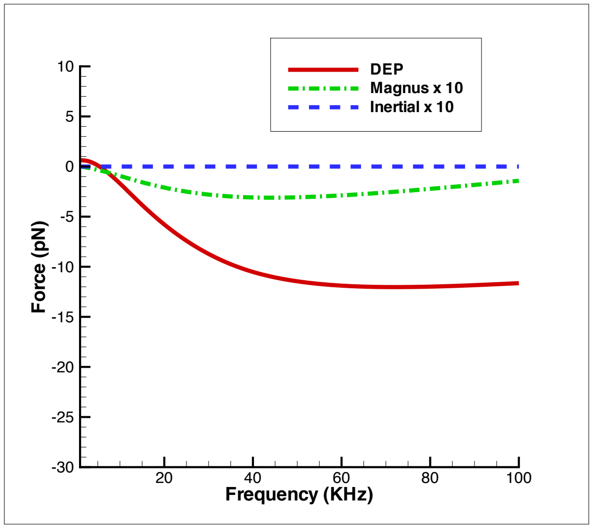
- The analytical solution for the electric field is given by

$$\mathbf{E}_{\mathbf{cyl}} = V \frac{\hat{\mathbf{r}}}{r \ln r_o / r_i}, \mathbf{E}_{\mathbf{AC}} = \mathbf{E}_{\mathbf{cyl}} \cos(2\pi\nu t)$$

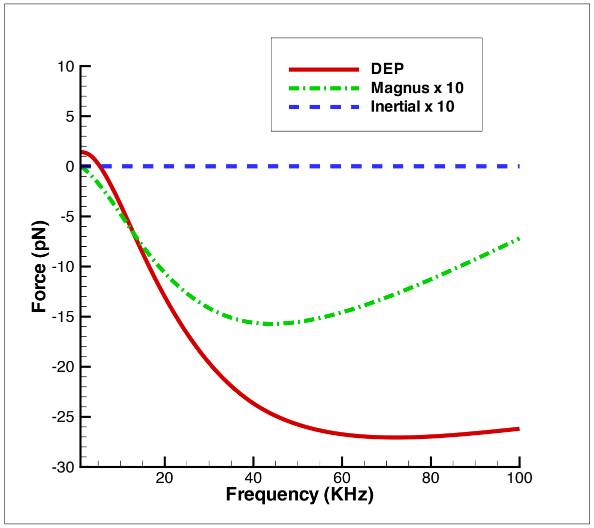
Results:Effect of frequency and voltage



(a) Voltage = 100V



(b) Voltage = 200V



(c) Voltage = 300V

Results:Effect of frequency on angular velocity

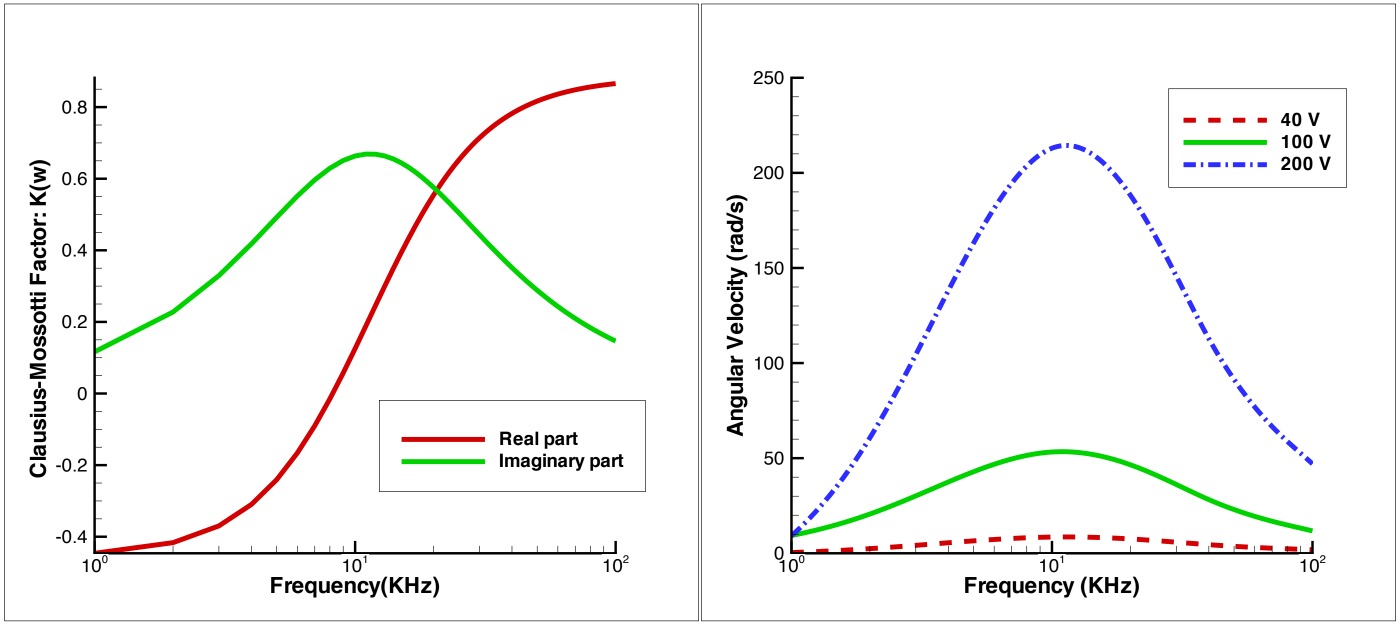


Figure : **Left:** A plot of the real and imaginary parts of the Clausius-Mossotti factor. **Right:** Angular velocity as a function of frequency plotted for various voltages. Diameter is kept fixed at $20\mu m$.

Results:Effect of size

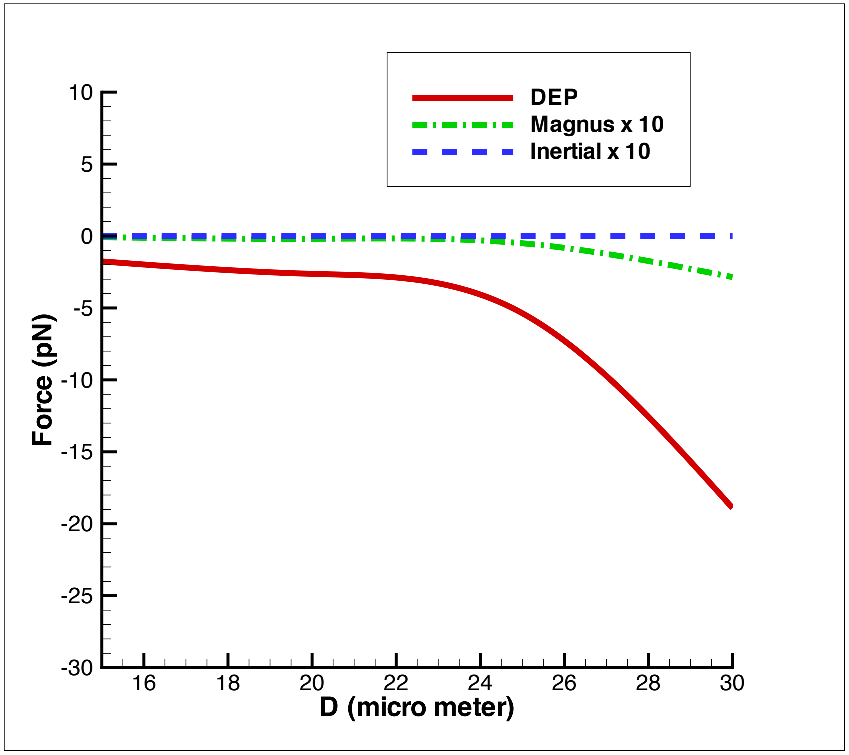
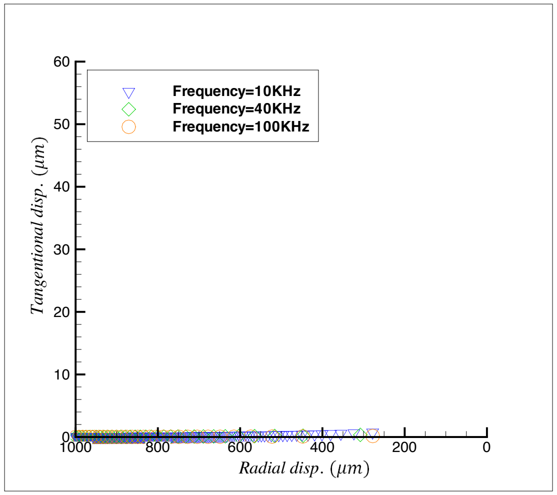
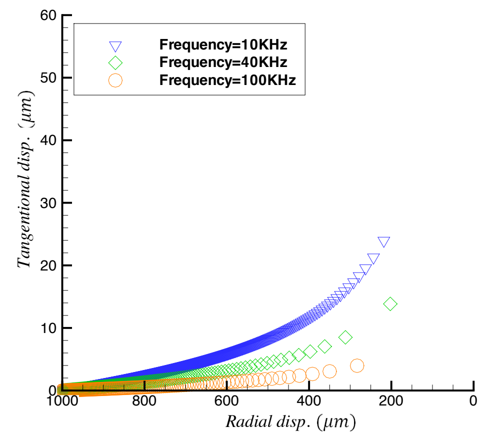


Figure : Effect of diameter on acting forces plotted for different voltages. Magnitudes of Magnus and DEP forces increase with diameter

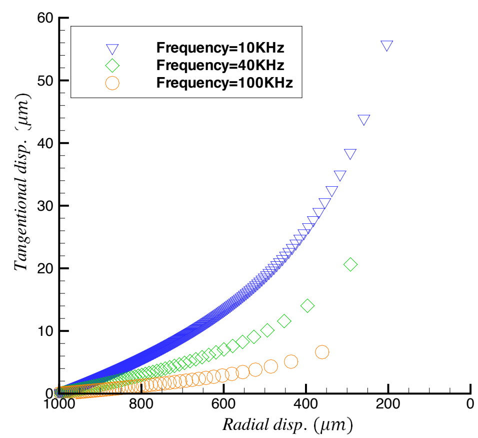
Results: Trajectories



(a) Voltage = 40V

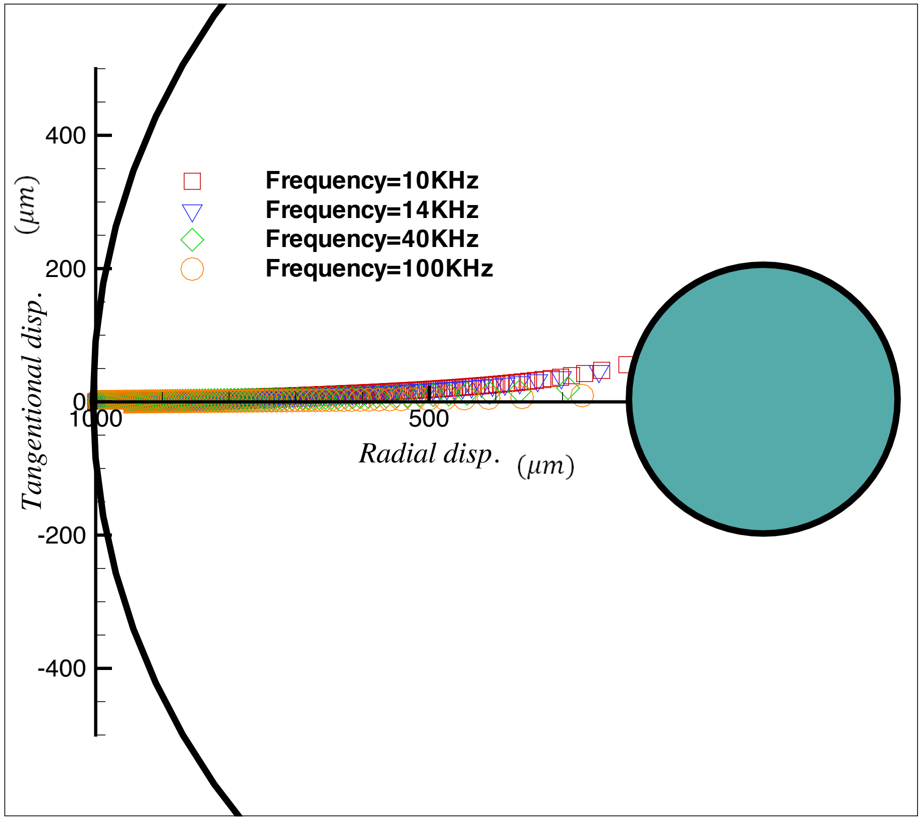


(b) Voltage = 200V



(c) Voltage = 300V

Results:Deviation compared to geometrical scale



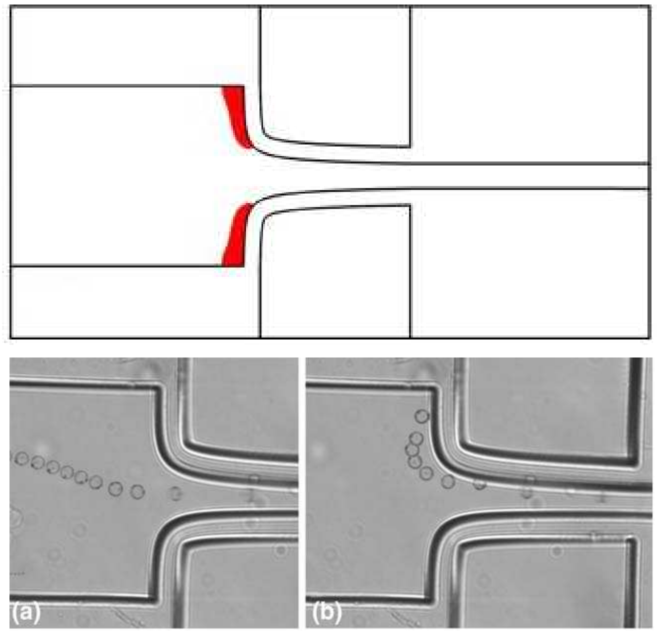


Figure : **Left:**Plot of trajectories from numerical solutions. **Right:** Experiments in Dr.Davalos's lab show similar behavior

Conclusion

- Magnus force can deviate the trajectory up to 10 % of the total displacement.
- Inertia is negligible compared to drag and Magnus forces.
- Although Magnus force is very small, it can become important in symmetrical electric fields.
- It can be useful for manipulating and separating cells with different rotative response.
- Can we apply these understandings to contactless DEP systems with electro-osmosis and electrophoresis are competing?

Future work

- Solving more realistic electrode configurations using finite element analysis
- Studying effects of pressure driven flow
- Studying the Magnus effect on non-spherical cells
- Experimental validation of the theory
- Validation of the dimensionless number:

$$IZINAIK = \frac{F_M}{F^r_{drag}} = \frac{\rho\omega a^2}{\eta}$$

Bibliography

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- THANK YOU FOR YOUR ATTENTION!!!