Wake-structure interactions: analysis and experiments

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Project Statement

The objective of the research project is two-fold:

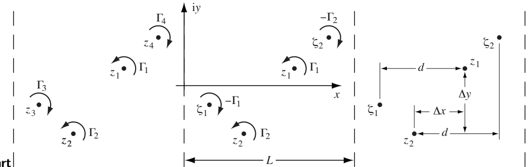
- Analysis of the effects of vortex-dominated wakes on the wake-shedding bluff body in terms of the forces induced by the vortices on it and its subsequent dynamic response
- Execution of wake experiments on flowing soap film with a freely oscillating structure and validation of the analytical calculations based on the experimental feedback

Point vortex model of complex vortex wakes

- Point vortex model (von Kármán, Rubach 1912) exists for the well-known 2S mode wake (von Kármán vortex street).
- For 2P mode wakes (with 2 counter-rotating vortex pairs per cycle), it has been possible to to do a point vortex approximation and after some symmetry containts, the problem can be reduced to the Hamiltonian form. This is one of the most common wake forms after the 2S mode.

$$\mathbb{H}(\Delta x, \Delta y) = -\frac{1}{2\pi}\left[\ln\left\{\frac{\sin^2(\pi\,\Delta x/L) + \sinh^2(\pi\,\Delta y/L)}{\cos^2(\pi\Delta x/L) + \sinh^2[\pi(\mathbb{P} + (1-2\gamma)\Delta y/L]}\right\}\right.$$

$$\left. -\frac{\gamma}{1-\gamma}\ln\{\cosh[\pi(\mathbb{P} + 2(1-\gamma)\Delta y/L)]\} - \frac{1-\gamma}{\gamma}\ln\{\cosh[\pi(\mathbb{P} - 2\gamma\,\Delta y/L)]\}\right]$$



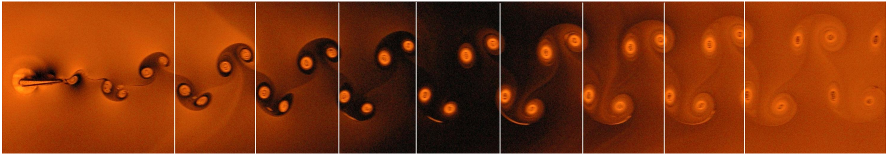
Canonical EOMS

$$\frac{2}{\mathbb{S}}\frac{\mathrm{d}(\Delta x)}{\mathrm{d}t} = \frac{\partial \mathbb{H}}{\partial(\Delta y)}$$

$$\frac{2}{\mathbb{S}}\frac{\mathrm{d}(\Delta y)}{\mathrm{d}t} = -\frac{\partial \mathbb{H}}{\partial(\Delta x)}$$

Feedback (or, main take-aways) from the point vortex model for our project

- The model provides a good representation of the dynamics in the mid-wake region.
- It is possible to predict the strengths of the individual vortices as well their evolving dynamics.
- The model prediction on vortex spatial evolution over time fits well with existing experiments.



Andersen, Bohr, and Schnipper (Theoretical & Computational Fluid Dynamics, 2011).

Unsteady point vortex model

- 2-dimensional coupled motion of a general body and a surrounding inviscid flow
- Shedding of point vortices with a varying intensity to satisfy regularity condition at the generating corner
- Intensity of the point vortices increases monotonically
- If the intensity reaches an extremum, it will henceforth be steady and new unsteady vortex is generated from the same corner.

Formulation for the force on the solid body

$$f = \rho e^{i\theta}\Big[(2\pi a^2 - S)\dot{\bar{U}} - 2\pi G_1 \dot{U} - 2\pi i R_1 \dot{\omega} + i\omega(2\pi a^2 - S)\bar{U} - 2\pi i\omega G_1 U + 2\pi R_1 \omega^2$$

$$+ i\frac{d}{dt}\sum_{n=1}^{N}\Gamma_n\left(\zeta_n - \frac{a^2}{\bar{\zeta}_n}\right) - \omega\sum_{n=1}^{N}\Gamma_n\left(\zeta_n - \frac{a^2}{\bar{\zeta}_n}\right)\Big].$$

Michelin and Llewellyn Smith (Theoretical & Computational Fluid Dynamics, 2009).

Analysis of the structural response

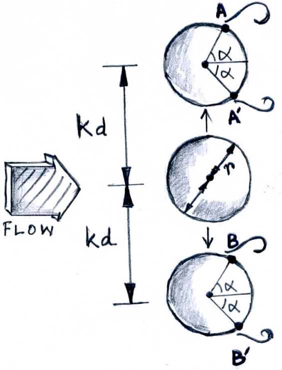


Figure: Wake generation from oscillating cylinder

From the unsteady point vortex model

$$f = f_x + i\, f_y = F(\dot{\Gamma}_n, \dot{z}_n, \dot{c}, \dot{\theta}, \ddot{\theta}, S, \Gamma_n, z_n, c),$$

where c is the body centroid location, S is the body area, $\dot{\theta}$ is the angular velocity of the body, and Γ_n and z_n are respectively the circulation and location of the n-th point vortex.

Separation Points:

$A = z_{1_0} \equiv (r\cos\alpha, kd + r\sin\alpha)$
$A' = z_{2_0} \equiv (r\cos\alpha, kd - r\sin\alpha)$
$B = z_{3_0} \equiv (r\cos\alpha, -kd + r\sin\alpha)$
$B' = z_{4_0} \equiv (r\cos\alpha, -kd - r\sin\alpha)$.

Oscilation amplitude $= kd$, where d is the cylinder diameter.

Analysis of the structural response

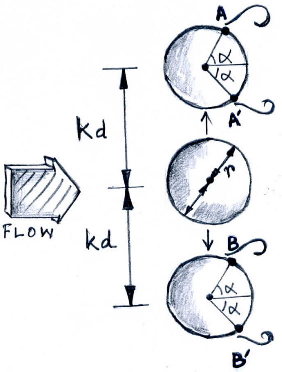


Figure: Wake generation from oscillating cylinder

The idea is to isolate the structural impulse from the unsteady point vortex equation,

$$\int_0^T f\,dt = a_1 \int_0^{\Gamma_n} d\Gamma_n + a_2 \int_{z_{n_0}}^{z_n} dz_n + other\ terms.$$

Upper integration limits (Γ_n, z_n) are fed back from the point vortex predictions for mid-wake and hence couple up the steady and unsteady methods.

Subsequently, from Newton's laws of motion,

$$\int_0^{kd} dy = \frac{1}{M}\left[\int_0^T Im\left[\int_0^T f\,dt\right] dt\right].$$

Hence, this gives the structural vibration response for a particular vortex street. Experiments would help us to validate this procedure.

Four-vortex wake formations

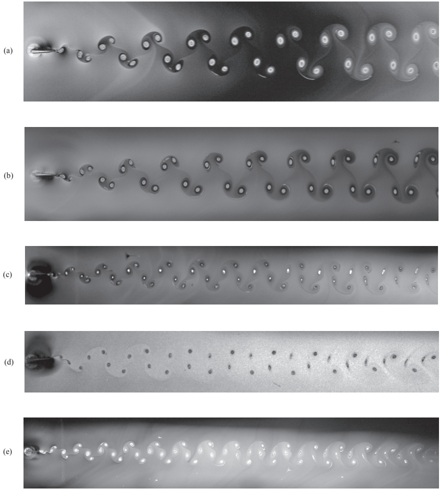


Figure: Soap film experiments with vibrating airfoil

Schnipper, Andersen, and Bohr (JFM, 2009). Andersen, Bohr, and Schnipper (Theoretical & Computational Fluid Dynamics, 2011).

Four-vortex wake formations

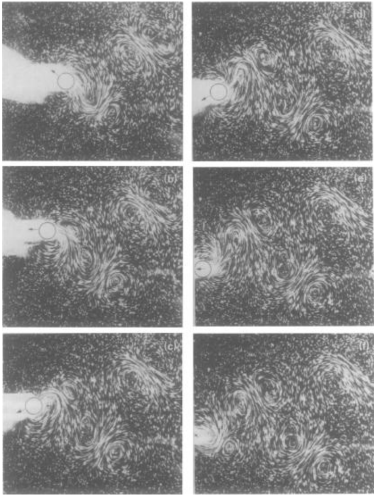


Figure: 2P mode vortex formation from forced oscillation of a cylinder

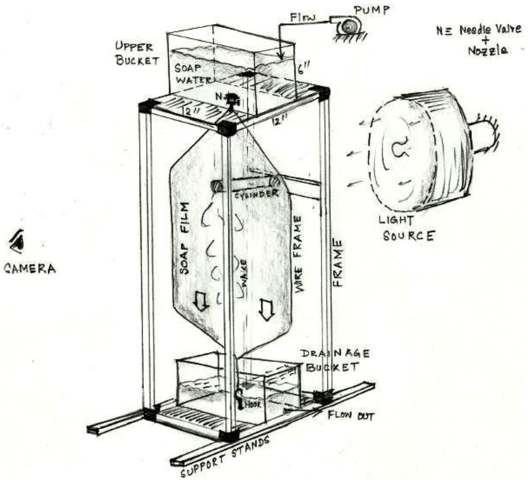
Williamson and Roshko (JFM, 1988)

Targets for the experiments

The main objectives of the soap-film experiments would be:

- Generation of wakes through free oscillation of the body
- 2P mode generation would be the target (this would also be a valuable addition to the existing literature).
- 2P wake, if generated, would serve as the feedback framework (using the model predictions) to validate and confirm the structural response formulation. Otherwise, similar calculations would be performed for the normal von Kármán mode to validate our work.

Experimental setup (vertically flowing soap film under pressure head)



Final configuration

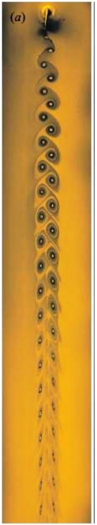
Current progress

Soap film experiments

Flowing soap films are used to study 2D turbulence, cylinder wakes, and wakes shed by foils.

Soap film characteristics

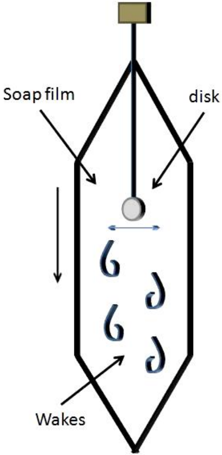
- Has uniform velocity profile and constant thickness
- Behaves as 2D incompressible fluid
- Flow structures are viewed using a monochrome light source



Schnipper, Andersen, and Bohr (JFM, 2009)

Experimental aims and methods

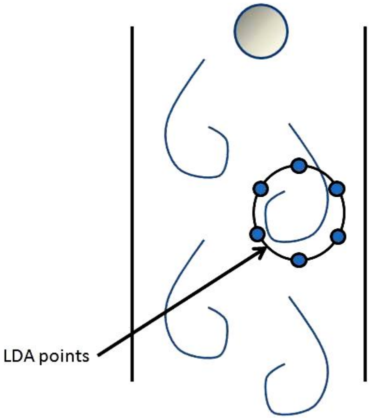
- View wake structures shed from a freely oscillating disk (or, ring)
- Suspend thin disks (or, rings) of varying density tethered through wires so that they are free to oscillate
- The disk (or, ring) will be immersed in the soap film to make the fluid forces dominant. This is essential for free oscillation.



Data analysis

- Use Laser Doppler anemometry (LDA) technique to measure film speed and circulation

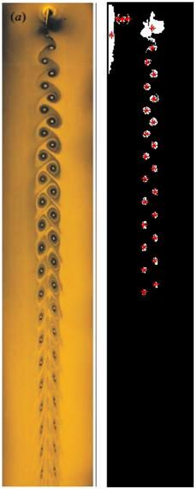
- A high speed camera will capture the wake and disk dynamics



Matlab image processing

Track wake and disk movement using Matlab image processing

- Changes image to binary (black and white only)
- Uses function to locate object centroids
- Plots and stores vortex coordinates



Questions...