Chaos in Space and Time

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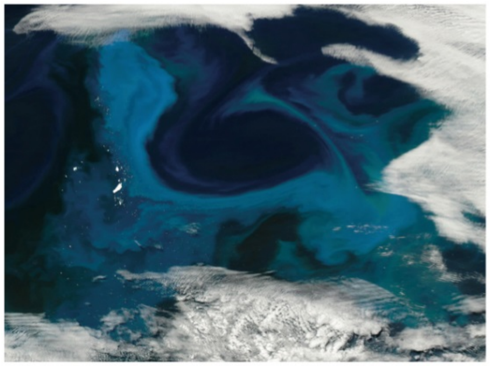
- Midterm Presentation
- ESM 6984 SS: Frontiers in Dynamical Systems

Why study chaos in space and time?

Many phenomena in nature are systems far-from equilibrium and exhibit chaotic dynamics in both space and time



NASA images



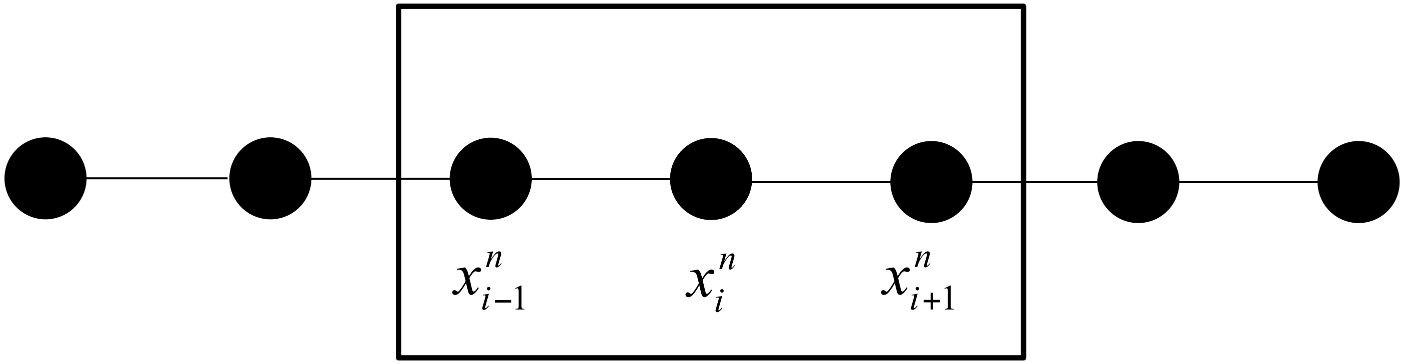
Falkowski Nature (2012)

NASA image

Presentation Outline

- Lattice map with “diffusive” coupling
  - – Difference equations
- Calculating Lyapunov vectors
  - – System of ODEs
- Transport in complex flow
  - – Governing PDEs
- Future work

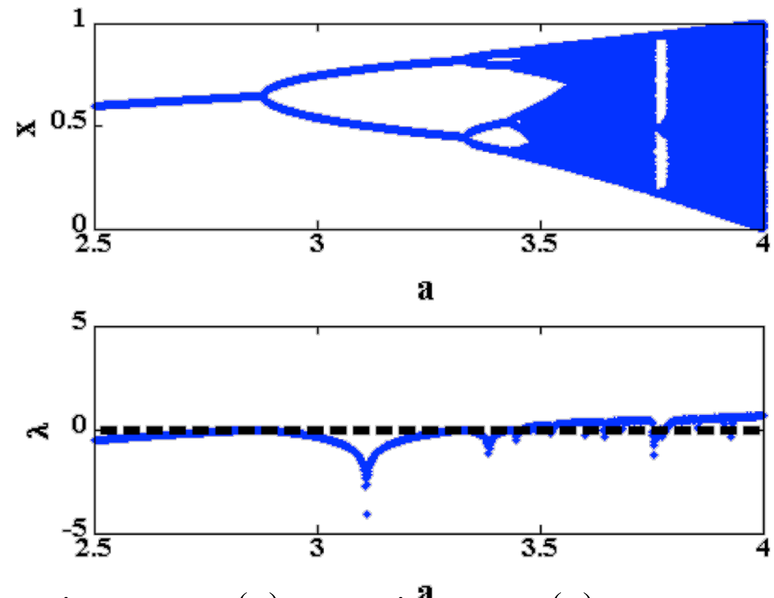
Coupled Map Lattice 1D



$$x_i^{(n+1)} = f(x_i^n) + D[\frac{1}{2}(f(x_{i+1}^n) + f(x_{i-1}^n)) - f(x_i^n)]$$

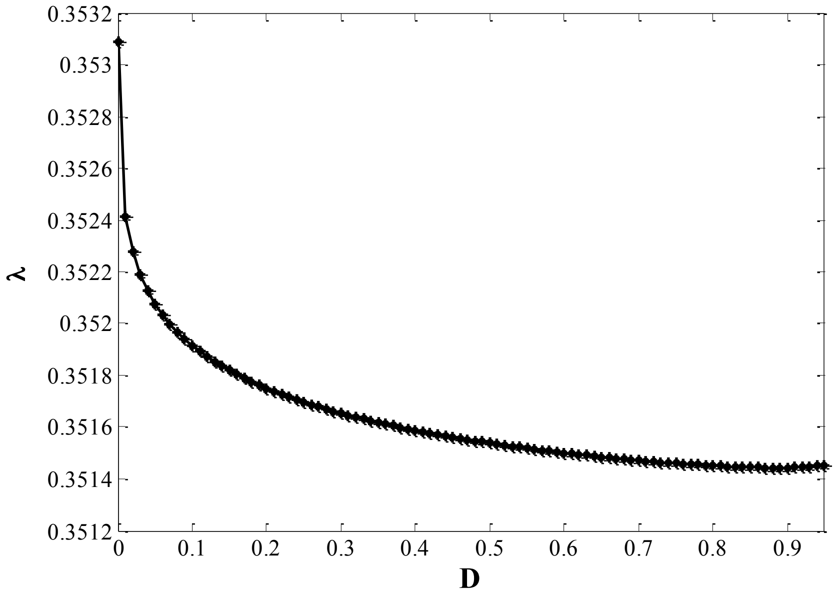
$$f(x_i^n) = a x_i^n (1 - x_i^n)$$

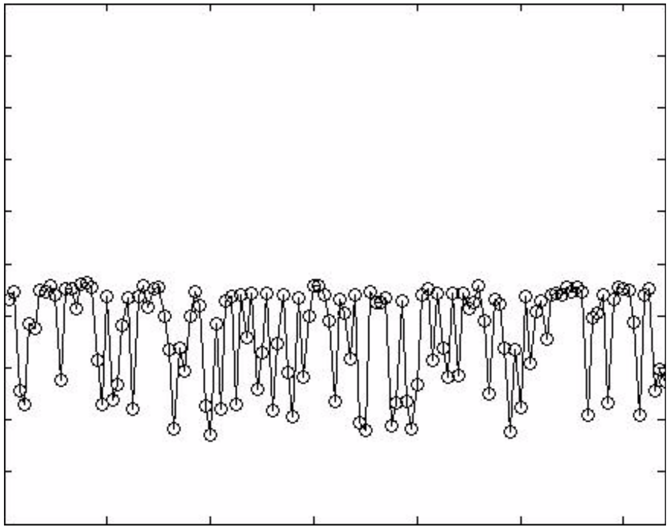
$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln\left|f'(x_i^n)\right|$$

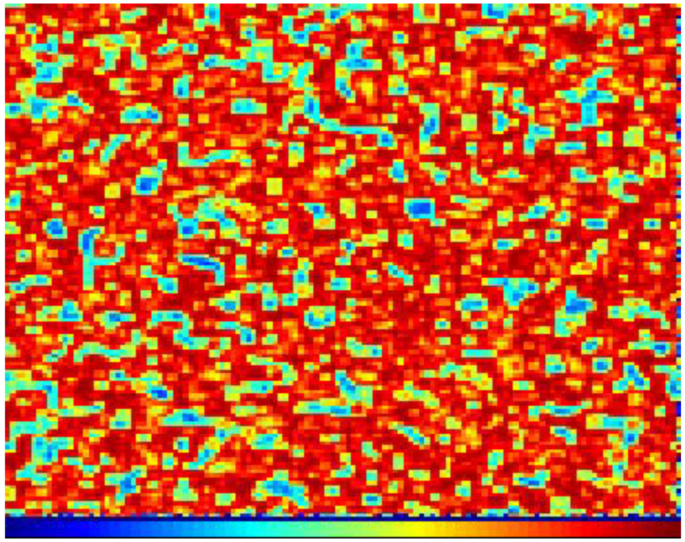


$$\delta x_i^{(n+1)} = f'(x_i^n)\delta x_i^{(n)} + D[\frac{1}{2}(f'(x_{i+1}^n)\delta x_{i+1}^{(n)} + f'(x_{i-1}^n)\delta x_{i-1}^{(n)}) - f'(x_i^n)\delta x_i^{(n)}]$$

Coupled Map Lattice 1D







$$\lambda > 0 \Rightarrow Chaos$$

Lyapunov Vectors

Covariant Lyapunov Vectors

Pros:

- True direction in phase space.
- Reflect the direction of perturbation
- Test hyperbolicity

Cons:

- Difficult to calculate
- Algorithm only recently available(Ginelli (2007) and Pazo (2007))

Orthogonal Lyapunov Vectors

Pros:

- Easy to calculate
- Leading order Lyapunov vector is in correct direction
- Can calculate fractal dimension

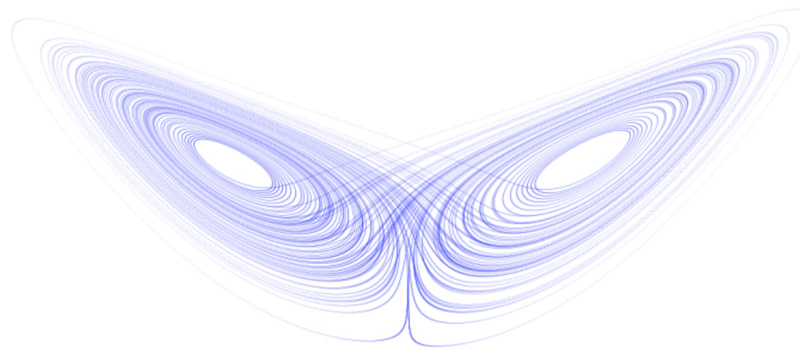
Cons

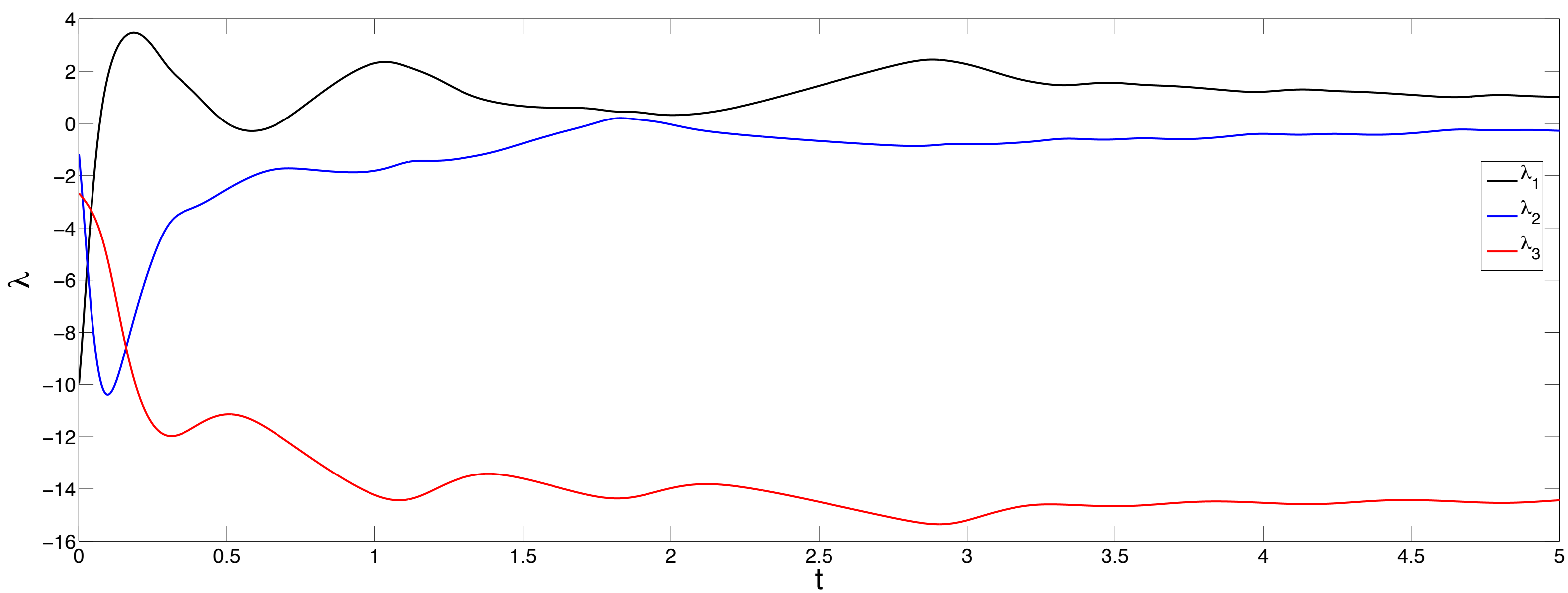
- Lose all direction except leading order

Lorenz System

$$dx/dt = \sigma(x - y)$$
$$dy/dt = x(\rho - z) - y$$
$$dz/dt = xy - \beta z$$

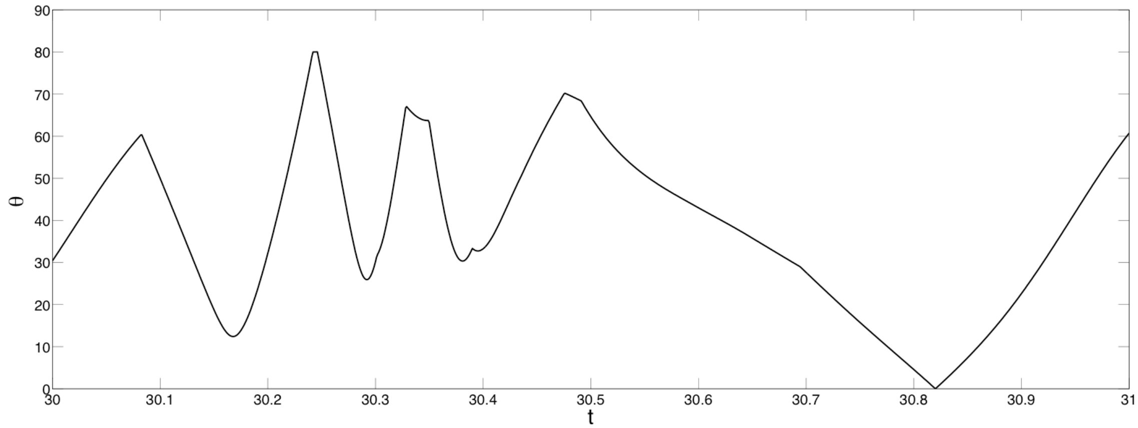
$\sigma = 10$
$\rho = 28$
$\beta = 8/3$

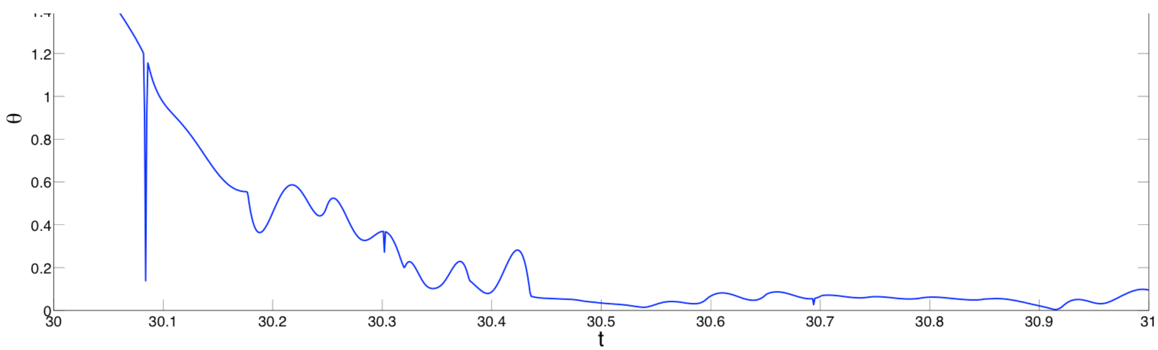




Result

The direction of the second covariant Lyapunov vector and the direction of the tangent vector should be same.





Transport in Complex Flows

Boussinesq Equations

$$\sigma^{-1}(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}) = -\vec{\nabla}p + \vec{\nabla}^2\vec{u} + RT\hat{z}$$

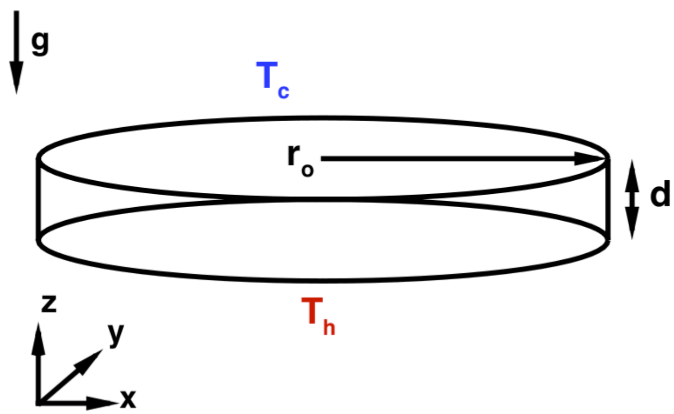
$$(\frac{\partial T}{\partial t} + (\vec{u}\cdot\vec{\nabla})T) = \vec{\nabla}^2 T$$

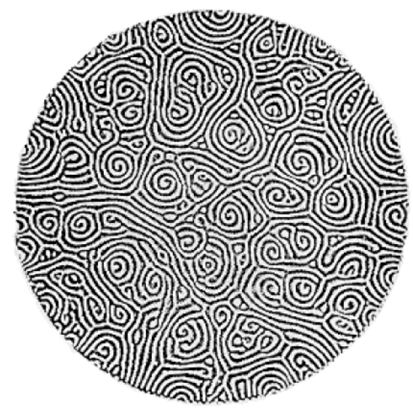
$$\vec{\nabla}\cdot\vec{u} = 0$$

Advection-Diffusion Equation

$$\frac{\partial c}{\partial t} + (\vec{u}\cdot\vec{\nabla})c = L\vec{\nabla}^2 c$$

$$R = \frac{\alpha g d^3}{\nu\kappa}\Delta T \qquad L = \frac{D}{\kappa}$$



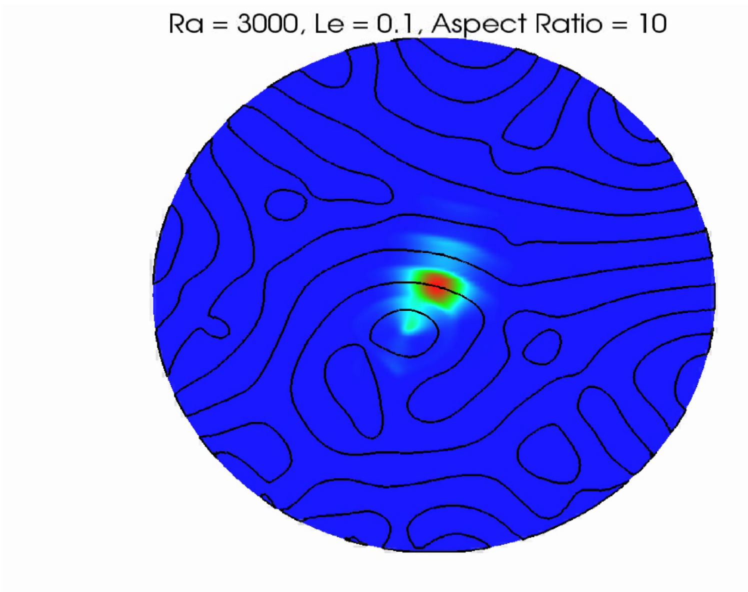


(a)

Ning et al. (2009)

Direct Numerical Simulations

Pr= 1



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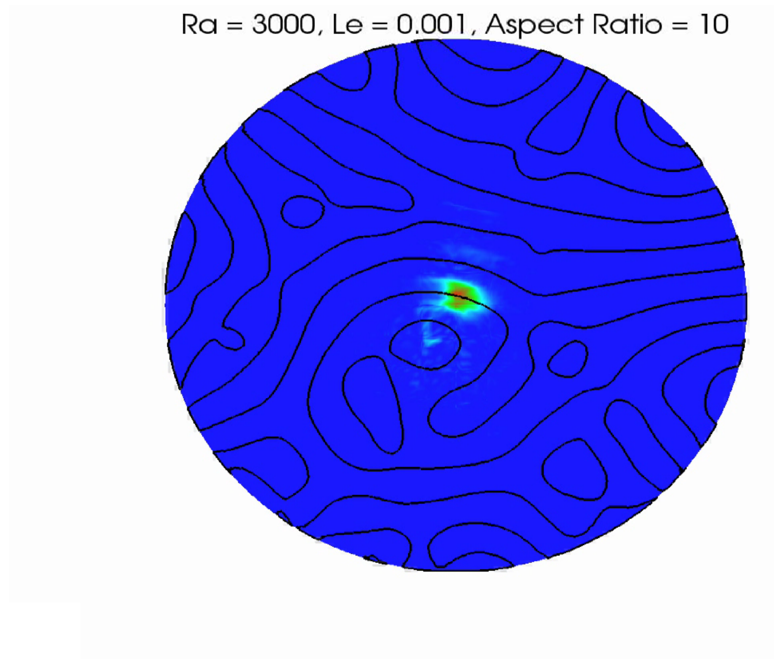

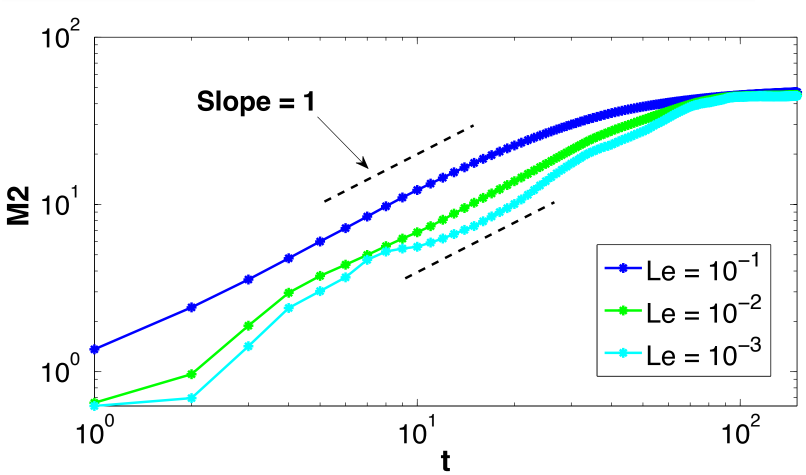

$$[L]^2 \propto D[T]$$

Diffusive Transport

Future Work

- Explore diagnostic tools for higher dimensional map lattice
  - Fractal dimension
  - Lyapunov spectrum
  - Covariant Lyapunov vectors
- Transport enhancement due to complex flow
- Active transport