# Balance Recovery Strategy: Acrobot vs. Wobble Chair

## Frontiers of Dynamical Systems

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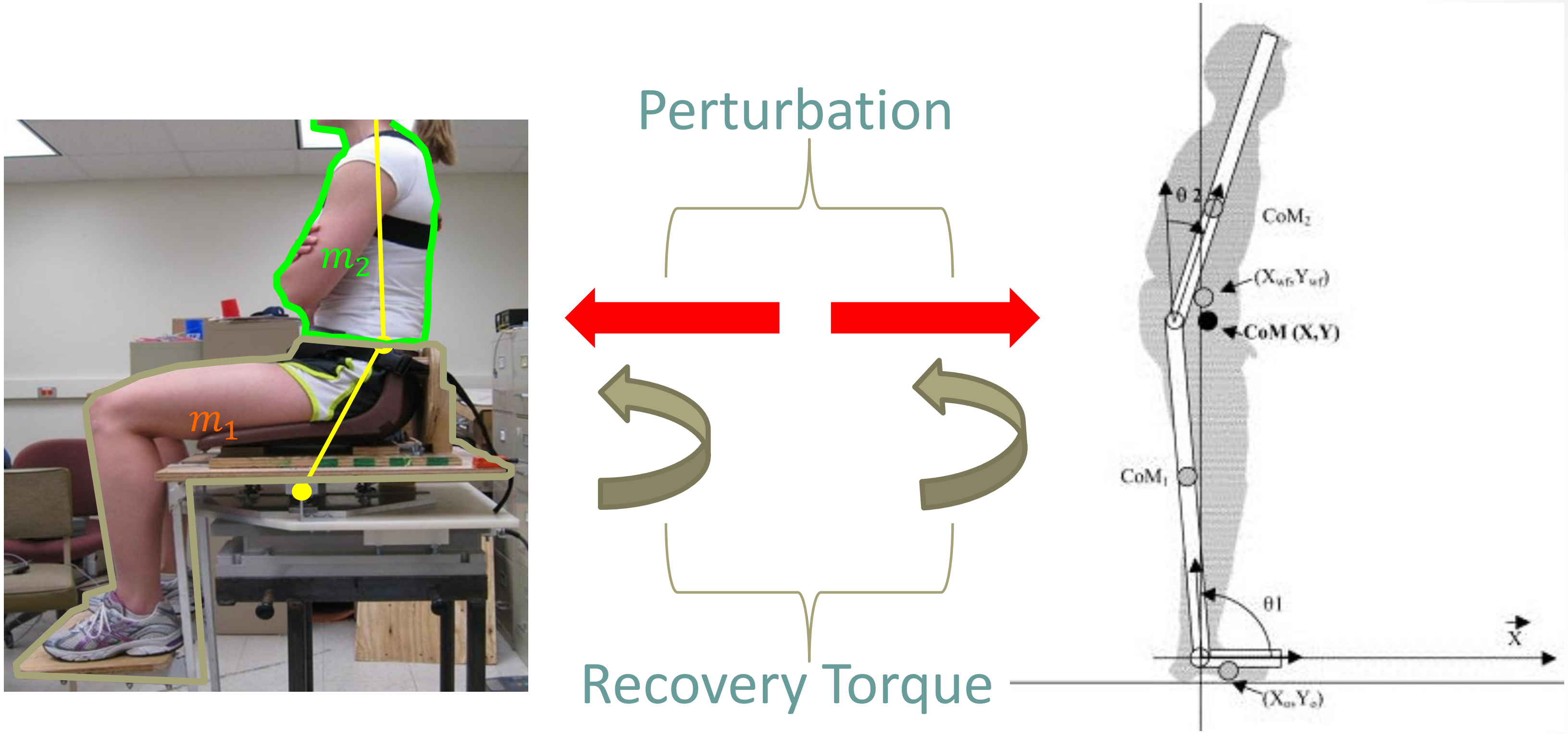
# The Question?




**Wobble chair model**

Recovery strategy: Recovery torque in same direction as perturbation.

**Acrobot model**

Recovery strategy: Recovery torque in opposite direction as perturbation.

# Model

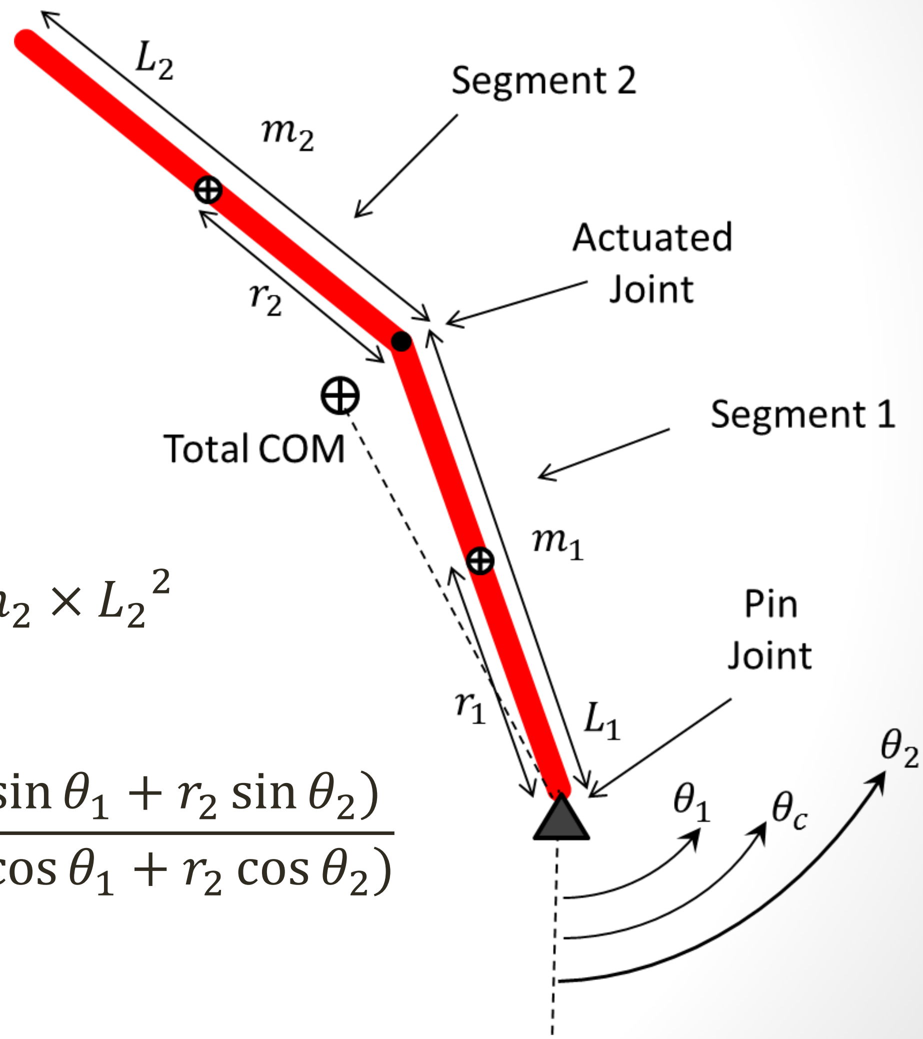
$$m_1 = m_2 = 1\,kg$$

$$L_1 = L_2 = 1\,m$$

$$r_1 = r_2 = 0.5\,m$$

$$I_1 = \frac{1}{12} m_1 \times {L_1}^2 \qquad I_2 = \frac{1}{12} m_2 \times {L_2}^2$$

$$\tan\theta_c = \frac{m_1 r_1 \sin\theta_1 + m_2(L_1 \sin\theta_1 + r_2 \sin\theta_2)}{m_1 r_1 \cos\theta_1 + m_2(L_1 \cos\theta_1 + r_2 \cos\theta_2)}$$



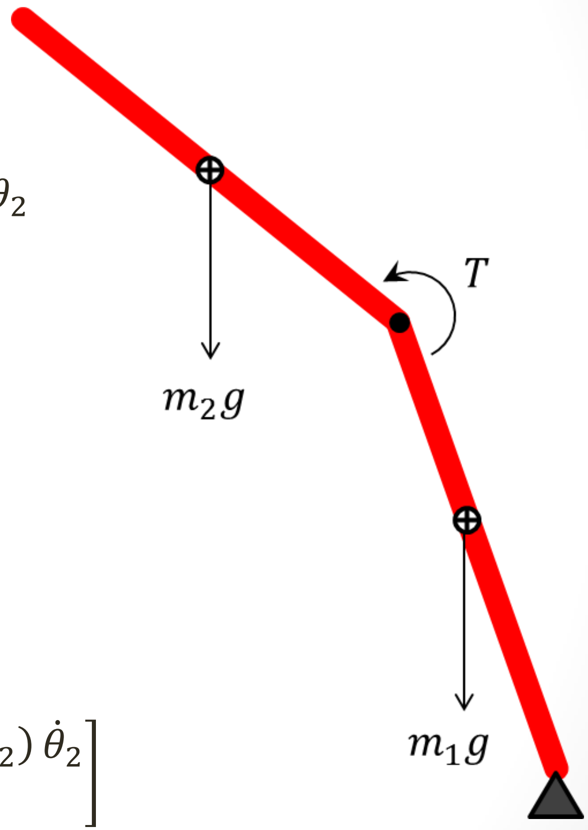
# Equations of Motion

- Lagrangian Method

$$T = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 + \frac{1}{2}I_1{\dot{\theta}_1}^2 + \frac{1}{2}I_2{\dot{\theta}_2}^2$$

$$V = -m_1g\,r_1\cos\theta_1 - m_2g\,L_1\cos\theta_1 - m_2g\,r_2\cos\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \left(\frac{\partial L}{\partial q_j}\right) = Q_j \qquad q_1 = \theta_1 \quad q_2 = \theta_2 \qquad L = T - V$$




$$\boldsymbol{M}\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \boldsymbol{G}(\boldsymbol{\theta}) = \boldsymbol{Q}$$

$$M = \begin{bmatrix} I_1 + m_1{r_1}^2 + m_2{L_1}^2 & m_2L_1r_2\cos(\theta_1-\theta_2) \\ m_2L_1r_2\cos(\theta_1-\theta_2) & I_2 + m_2{r_2}^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & m_2L_1r_2\sin(\theta_1-\theta_2)\,\dot{\theta}_2 \\ -m_2L_1r_2\sin(\theta_1-\theta_2)\,\dot{\theta}_1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} m_1gr_1\sin\theta_1 + m_2gL_1\sin\theta_1 \\ m_2gr_2\sin\theta_2 \end{bmatrix} \qquad Q = \begin{bmatrix} -T \\ T \end{bmatrix} \qquad T = C_T$$

# Controls

- PD control torque $C_T$ applied between segments

$$\theta_{critical} = \frac{T_{pmax}}{Gp} \quad \text{(Tanaka et al)}$$

$$C_T = G_p\, e_1 + G_d\, e_2 + G_{p2}\, e_3 \qquad If\ |\theta_c| < \theta_{critical}$$

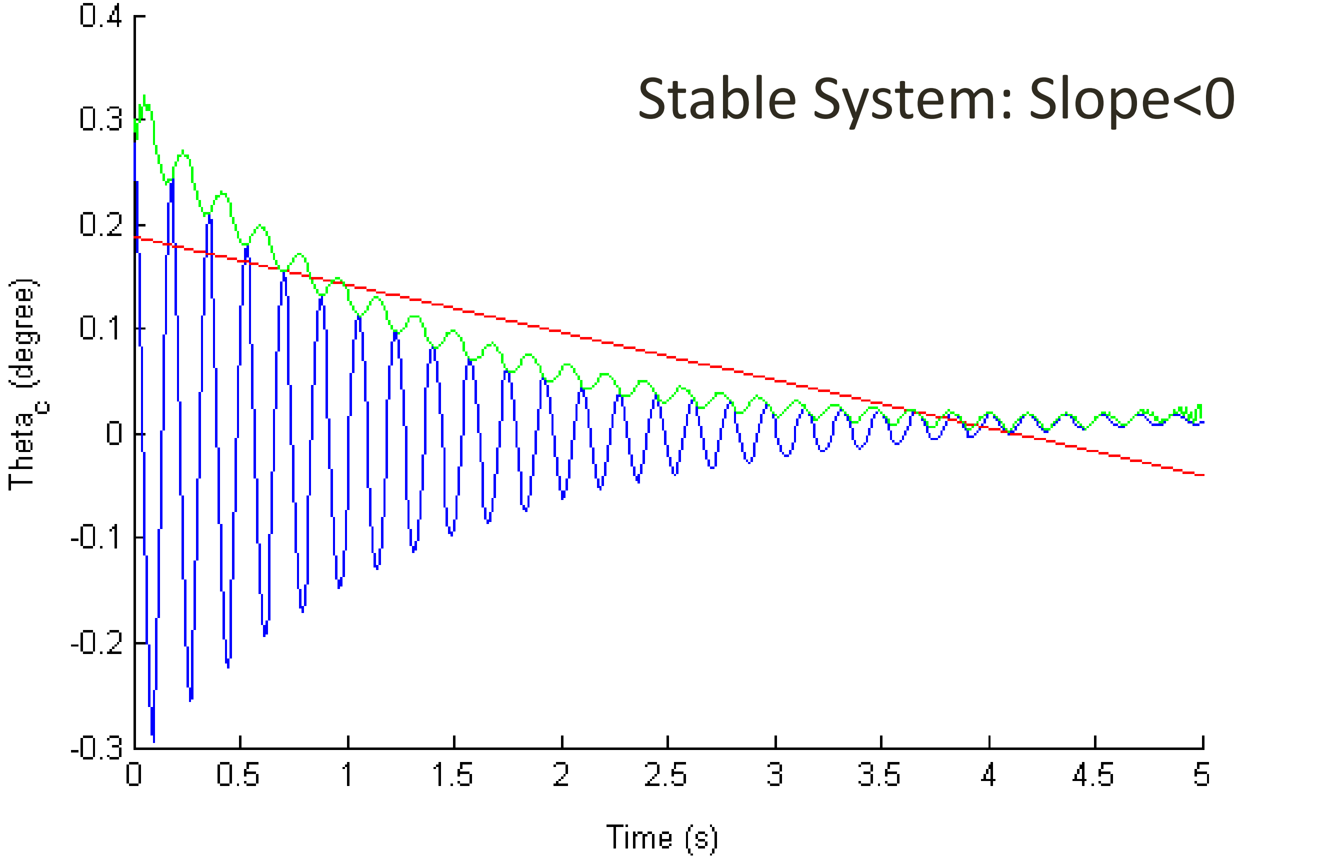
$$C_T = G_d\, e_2 + T_{pmax} \qquad otherwise$$

$$e1 = \theta_c \qquad e2 = \dot{\theta}_c \qquad e3 = \theta_2' = \theta_2 - \pi$$

- $G_{p2}$ drives pendulum to desired equilibrium position

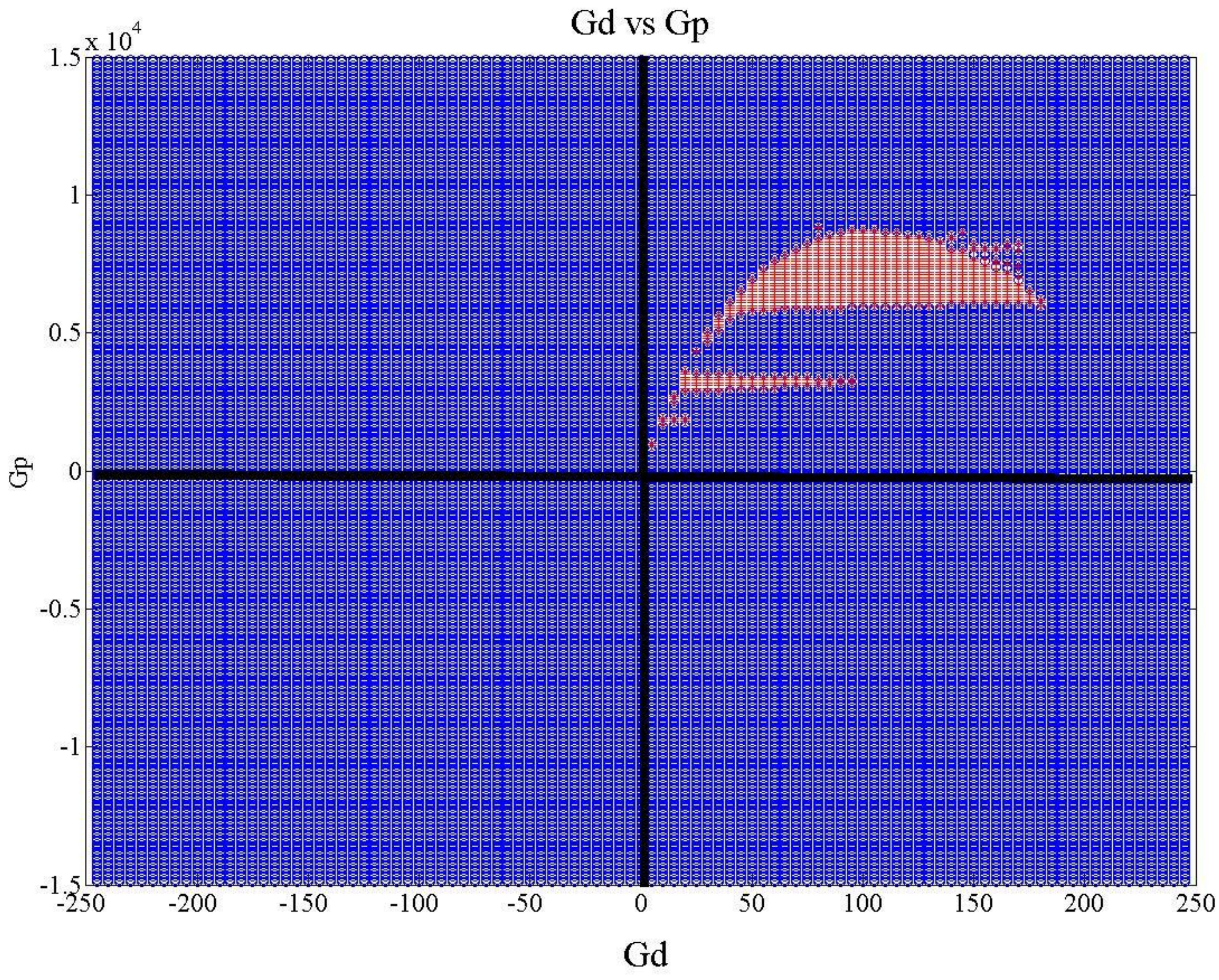
# Stability Criteria

- Hilbert Envelope is used to determine stability at various combinations of $G_p$ and $G_d$



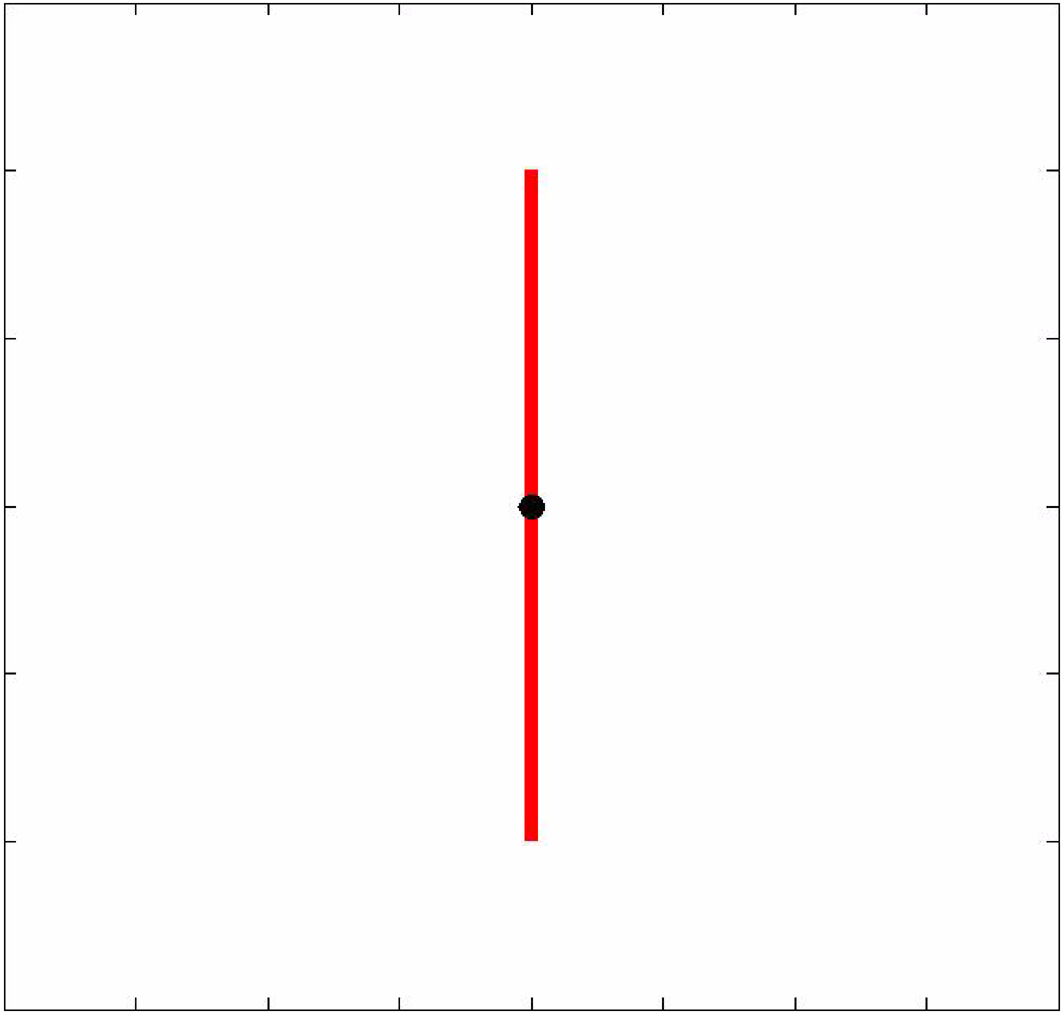
# Stability Charts

IC: $[\theta_1\ \dot{\theta_1}\ \theta_2\ \dot{\theta_2}] = [\pi\ 0\ \pi\ 0.1]$



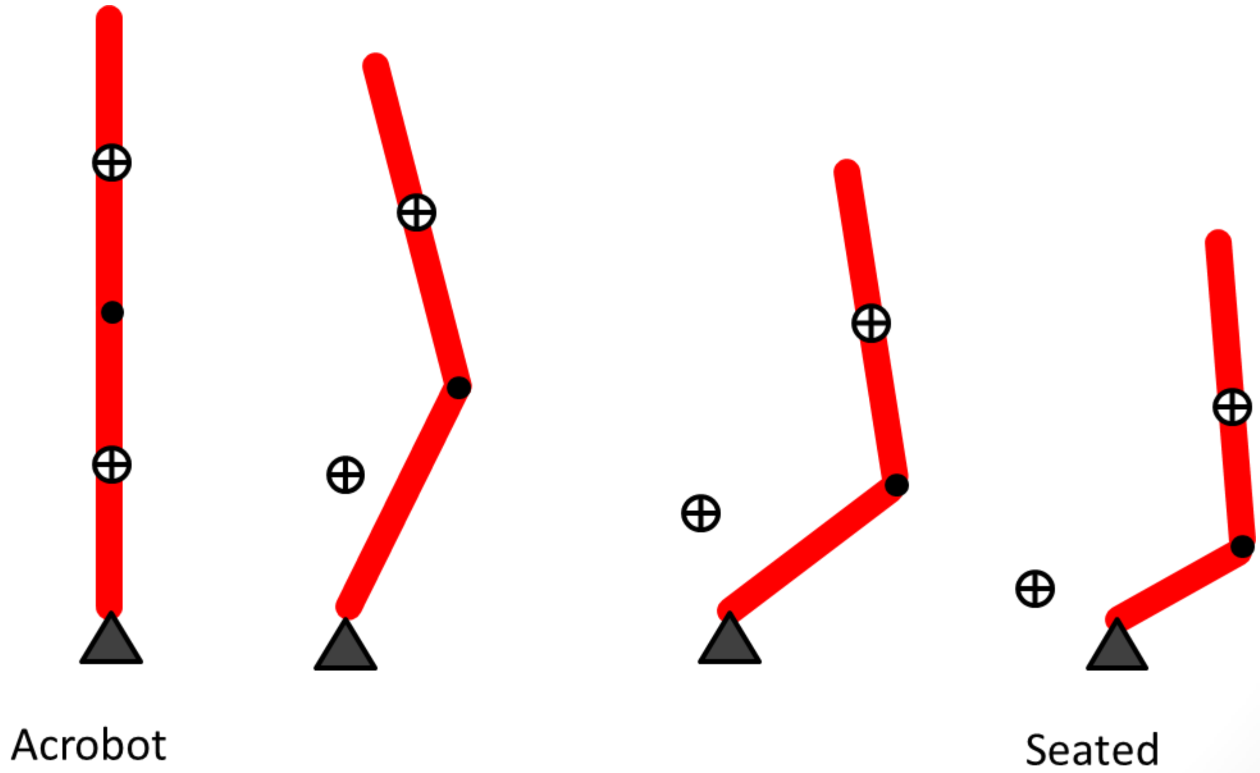
# Animation

Gp=1800 Gd=10 Gp2=5 IC: $[\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2] = [\pi\ 0\ \pi\ 0.1]$



# Future Work (I)

- Modify the model to simulate the transformation to the seated position and make it more human like.



# Future Work (II)

- At each step, run the simulation and reproduce stability charts.
- Determine at what point the model will start recovering to a stable position using the second strategy.
- If both strategies can be used at certain steps, find out why the body prefers one strategy to another.